Nonlinear Control of Hydraulic Robots

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Abstract— **This paper addresses the control problem of hydraulic robot manipulators. The backstepping design methodology is adopted to develop a novel nonlinear position tracking controller. The tracking errors are shown to be exponentially stable under the proposed control law. The controller is further augmented with adaptation laws to compensate for parametric uncertainties in the system dynamics. Acceleration feedback is avoided by using two new adaptive and robust sliding type observers. The adaptive controllers are proven to be asymptotically stable via Lyapunov analysis. Simulation and experimental results performed with a hydraulic Stewart platform demonstrate the effectiveness of the approach.**

Keywords— **Hydraulic robots, control of robot manipulators, adaptive/nonlinear control, backstepping.**

I. Introduction

HYdraulic robots and machinery are widely used in the construction and mining industries, as well as in motion simulators. They have rapid responses and high power-to-weight ratios suitable for many applications. High performance controllers can have a significant impact on the effectiveness of hydraulic robots. Furthermore, the potential complexity of such controllers is becoming less and less of an implementation issue due to the inexpensive and powerful processors available today for real-time control.

In general, the control of hydraulic manipulators is more challenging than that of their electrical counterparts. It might seem that a potentially effective way of increasing the performance of hydraulic robots is to consider control methods that neglect actuator dynamics but incorporate the manipulator rigid body dynamics, such as computed torque [1], passivity-based [2], [3], [4], adaptive [2], [5] and robust [6], [7], [8] control methods. However, this is not the case in general. Unlike their electrical counterparts that resemble force sources, hydraulic actuators resemble velocity sources. They also exhibit significant nonlinear characteristics. Therefore, the above control methods cannot be applied effectively to hydraulic manipulators as hydraulic actuators cannot accurately apply forces or torques over a significant dynamic range.

Actuator dynamic models have been successfully incorporated in the controller design for rigid link electrically driven (RLED) robots to improve position tracking performance. The complete dynamics of the manipulators, including their actuators, are third order nonlinear differential equations. [9] used feedback linearization to linearize and decouple these dynamics. [10] developed an adaptive controller for RLED manipulators that does not require acceleration feedback. [11] and [12] and other papers also considered the adaptive control of RLED robots based on models that include actuator dynamics.

While actuator dynamics are generally linear in RLED robots and can be ignored in many cases due to their fast time constants, they are highly nonlinear and dominant in hydraulic manipulators. Therefore, the incorporation of these dynamics in the design of controllers is of critical importance in hydraulic robots. Research in the area of hydraulic systems has mainly focused on the control of single-rod hydraulic actuators, e.g. see [13], [14], [15]. In particular, [13] developed a nonlinear position tracking controller for hydraulic servo-systems following the backstepping approach. There are only a few papers that address the control of robot manipulators driven by hydraulic actuators. In [16], the authors established a simplified model in a standard form suitable for the application of singular perturbation methods. No experimental or numerical results are presented in this work. A decentralized adaptive controller was proposed to control a hydraulic manipulator in [17], [18]. The use of pressure feedback in the control of a Stewart type hydraulic manipulator was proposed in [19]. However, these approaches lack stability proofs that are important from both theoretical and implementation points of view. Only recently, simultaneous to this work, [20] proposed a Lyapunov-based adaptive controller for hydraulic robots.

The *backstepping* design methodology [21], [22] has become increasingly popular in the control community. For some recent applications of this method see [23], [24]. In this paper, *backstepping* is adopted to develop a novel nonlinear controller for hydraulic manipulators. Both rigid body and actuator dynamics are incorporated into the design. The controller is also extended to compensate for parametric uncertainties in the system dynamics, including hydraulic and rigid body dynamics. Two types of observers are developed to avoid the use of acceleration feedback in the proposed adaptive control laws. The first observer is an extension of the passivity-based observers proposed by [3], to the case in which the system parameters are unknown. The concept of sliding observers [25] is also adopted to develop a robust acceleration observer. The tracking errors are proven to converge to zero asymptotically using Lyapunov analysis. It can be shown that these errors remain bounded in the presence of Coulomb friction in the actuators. The bounds on the tracking errors are adjustable by the controller gains.

The main differences between this work and the adaptive controller introduced in [20] are the following: (i) the adaptive controller / adaptive observer proposed here uses the same set of estimated rigid body parameters in the observer and controller, as opposed to the use of two distinct sets of parameter estimates and adaptation laws in [20]; and

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(ii) the introduction of an adaptive control method with a robust observer that is simpler to implement because it has reduced computational complexity. The form of the control laws and the observers are different from those of [20].

Position, velocity and hydraulic pressure measurements are required for the implementation of the proposed controllers. Simulation and experimental results for a hydraulic Stewart platform are presented to show the effectiveness of the approach.

The paper is organized as follows. System dynamics, including rigid body and hydraulic dynamics are presented in Section II. In Section III a nonlinear controller is proposed assuming that the dynamics are known exactly. The adaptive control of hydraulic robots is addressed in Section IV for the cases in which the robot dynamics are subject to parametric uncertainty. In Section V simulation results are presented. The experimental evaluation of the controllers is discussed in Section VI. Finally, conclusions are drawn in Section VII.

II. Manipulator/Actuators Dynamics

The dynamics of an n-link robot with rigid links are governed by a second-order nonlinear differential equation

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{1}$$

where $q \in R^n$ is a vector of generalized joint positions and $\tau \in R^n$ is a vector of generalized joint torques. $D(q) \in R^{n\times n}$ is the manipulator mass matrix, $C(q,\dot{q}) \in R^{n\times n}$ contains coriolis and centripetal terms and $G(q) \in R^n$ represents gravitational effects. Unlike electrically driven manipulators, hydraulic robots exhibit significant nonlinear actuator dynamics. Assuming a three-way valve configuration, these dynamics can be written in the following form,

$$\dot{\tau} = f(q,\dot{q}) + g(q,\tau,u) \tag{2}$$

where u is the control command vector and f, g are nonlinear functions of q, $\dot{q}$ and τ. The detailed expressions for f and g are given in *Appendix A*.

The matrices describing the rigid body dynamics in (1) satisfy the following properties [3]:

$$
\begin{array}{rll}
(i) & x^T\left(\dot{D}(q) - 2C(q,\dot{q})\right)x = 0 & \forall x \in R^n \\
(ii) & C(q,x)y = C(q,y)x & \forall q,x,y \in R^n \\
(iii) & \exists D_m, D_M \;\; s.t. \;\; 0 < D_m \le \|D(q)\| \le D_M < \infty & \\
 & & \forall q \in R^n \\
(iv) & \exists C_M \;\; s.t. \;\; \|C(q,x)\| \le C_M\|x\| & \forall q,x \in R^n \\
(v) & \exists G_M \;\; s.t. \;\; \|G(q)\| \le G_M & \forall q \in R^n
\end{array}
\tag{3}
$$

which are exploited in deriving the proposed control laws. According to (1) and (2), the overall actuator/manipulator dynamics are governed by a set of third-order nonlinear differential equations.

III. Nonadaptive Controller

In this section, the backstepping design methodology [21] is adopted to derive a nonlinear position tracking controller for hydraulic manipulators in the case in which the system parameters are known.

Theorem 1: Consider the system described by (1), (2) with the control law given by the solution u of the following algebraic equation:

$$g(q,\tau,u) = -f(q,\dot{q}) - \,,^{-1}s + \dot{\tau}_d - K_\tau\tilde{\tau} \tag{4}$$

and

$$\tau_d = D(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + G(q) - K_p e - K_d s \tag{5}$$

with

$$
\begin{array}{ll}
e = q - q_d, & \dot{q}_r = \dot{q}_d - \Lambda e, \\
s = \dot{q} - \dot{q}_r = \dot{e} + \Lambda e, & \tilde{\tau} = \tau - \tau_d
\end{array}
\tag{6}
$$

where K_p, K_d, K_τ, , and Λ are positive definite diagonal matrices, and $q_d \in R^n$ and $\dot{q}_d \in R^n$ are the desired joint position and velocity trajectories, respectively.

Then $\underline{0}$ is an exponentially stable equilibrium point for the state $\tilde{x} = \begin{bmatrix} e^T & s^T & \tilde{\tau}^T \end{bmatrix}^T$ of (1),(2),(4),(5).

Remark: From the expression of g from *Appendix A*, it can be seen that (4) can be easily solved for u.

Proof: Substituting (5) into (1) yields the following error dynamics,

$$D(q)\dot{s} + C(q,\dot{q})s + K_d s + K_p e = \tilde{\tau} \tag{7}$$

Note that the effect of actuator dynamics emerges as a non-zero $\tilde{\tau}$, as the controller reduces to a passivity-based controller [26] in the absence of actuator dynamics. Let V_1 be defined as

$$V_1 = \frac{1}{2}s^T D(q)s + \frac{1}{2}e^T K_p e \tag{8}$$

It can be shown that the derivative of V_1 along trajectories of the closed loop system becomes

$$\dot{V}_1 = -s^T K_d s - e^T \Lambda K_p e + s^T\tilde{\tau} \tag{9}$$

where (7) and the properties given in (3) have been used. Following the backstepping methodology, V_2, which is a Lyapunov function for the closed-loop system, is defined as

$$V_2 = V_1 + \frac{1}{2}\tilde{\tau}^T \,,\, \tilde{\tau} \tag{10}$$

with , > 0 diagonal. Note that:

$$\alpha_m\|\tilde{x}\|^2 < V_2 < \alpha_M\|\tilde{x}\|^2, \qquad \alpha_m, \alpha_M > 0 \tag{11}$$

By taking the derivative of (10) and using the control law (4), one can write

$$\dot{V}_2 = -s^T K_d s - e^T \Lambda K_p e - \tilde{\tau}^T \,,\, K_\tau\tilde{\tau} < -\beta\|\tilde{x}\|^2 \tag{12}$$

with $\beta > 0$. Therefore, the system is exponentially stable in the Lyapunov sense. This means that the position tracking error converges to zero exponentially. Furthermore, since $s = \dot{e} + \Lambda e$, the velocity tracking error is also exponentially stable. Note that in the realization of (4) one needs to compute $\dot{\tau}_d$ which is equal to

$$\begin{aligned} \dot{\tau}_d = \dot{D}(q)\ddot{q}_r + D(q)\dddot{q}_r + C(q,\dot{q})\ddot{q}_r + \dot{C}(q,\dot{q},\ddot{q})\dot{q}_r \\ + \dot{G}(q) - K_p\dot{e} - K_d\dot{s} \end{aligned} \tag{13}$$

Since $\dddot{q}_r = \dddot{q}_d - \Lambda\ddot{e}$, $\dot{s} = \ddot{e} + \Lambda\dot{e}$ and $\dot{C}$ are functions of $\ddot{q}$, link accelerations appear in the proposed control law. However, if τ is measured through pressure sensors, the link accelerations $\ddot{q}$ can be obtained from position, velocity and pressure measurements using

$$\ddot{q} = D(q)^{-1}\left[\tau - C(q,\dot{q})\dot{q} - G(q)\right] \tag{14}$$

Thus q, $\dot{q}$ and τ are required to implement the proposed control law that leads to exponentially stable tracking errors.

Remark: Since the system dynamics are fully known and the states are assumed to be measured, feedback linearization could also be used to derive a stabilizing controller. [9] adopted this approach to develop a controller for electrically driven manipulators in the presence of linear actuator dynamics.

IV. Adaptive Controller

The control law derived in the previous section requires full knowledge of the system parameters. However, the manipulator rigid body dynamics are uncertain and subject to changes, e.g. due to an unknown variable payload. It is also difficult to measure some of the manipulator's parameters. Moreover, the hydraulic parameters are usually unknown and time varying. In this section, the nonlinear controller proposed in Section III is extended to compensate for parametric uncertainties in the system dynamics. To deal with uncertainties in rigid body dynamics, the linear parameterization of manipulator dynamics is used [26]:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Y(q,\dot{q},\ddot{q})\theta \tag{15}$$

where $Y(q,\dot{q},\ddot{q})$ is a regressor matrix and $\theta \in R^m$ is the vector of unknown parameters. Similarly, as shown in *Appendix A*, the hydraulic dynamics (2) can be written as

$$\dot{\tau} = f_0(q,\dot{q})\gamma_1 + g_0(q,\tau,u)\gamma_2 \tag{16}$$

where $\gamma_1 = \begin{bmatrix}\gamma_1^1 & \cdots & \gamma_1^n\end{bmatrix}^T$, $\gamma_2 = \begin{bmatrix}\gamma_2^1 & \cdots & \gamma_2^n\end{bmatrix}^T$ are two sets of hydraulic parameters and f_0, g_0 are defined as

$$\begin{aligned} f_0(q,\dot{q}) = diag\{f_0^i(q^i,\dot{q}^i)\} \\ g_0(q,\tau,u) = diag\{g_0^i(q^i,\tau^i,u^i)\} \end{aligned} \tag{17}$$

In the non-adaptive controller, (14) was used to compute joint accelerations from joint positions and velocities and hydraulic pressure measurements. This can not be done if $D(q)$, $C(q,\dot{q})$ and $G(q)$ are not known. To deal with this problem, novel adaptive and robust observers are introduced. The following *Lemma* [27] will be used in the stability proofs.

Lemma 1: Consider the scalar function $\alpha = (\theta - \hat{\theta})^T(\rho - \dot{\hat{\theta}})$, with $\theta, \hat{\theta}, \rho \in R^n$ and $a^i \le \theta^i \le b^i$.

If $\dot{\hat{\theta}} = \kappa(a,b,\rho)\rho$, where $\kappa(a,b,\rho)$ is a diagonal matrix with entries

$$\kappa^i(a,b,\rho) = \begin{cases} 0 & \text{if } \hat{\theta}^i \le a^i,\ \rho^i \le 0 \\ 0 & \text{if } \hat{\theta}^i \ge b^i,\ \rho^i \ge 0 \\ 1 & \text{otherwise} \end{cases} \tag{18}$$

then $\alpha \le 0$.

A. Adaptive Controller/ Adaptive Observer

The first solution is an adaptive controller using an adaptive passivity-based observer. Before stating the result, the following notation must be defined:

$$\begin{aligned} \dot{q}_r &= \dot{q}_d - \Lambda_1(\hat{q} - q_d) = \dot{q}_d - \Lambda_1(e - \tilde{q}) \\ \dot{q}_o &= \dot{\hat{q}} - \Lambda_2(q - \hat{q}) = \dot{\hat{q}} - \Lambda_2\tilde{q} \\ s_1 &= \dot{q} - \dot{q}_r = \dot{e} + \Lambda_1(e - \tilde{q}) \\ s_2 &= \dot{q} - \dot{q}_o = \dot{\tilde{q}} + \Lambda_2\tilde{q} \end{aligned} \tag{19}$$

where $\hat{q} \in R^n$ is the estimated value of q, $e = q - q_d$, and $\tilde{q} = q - \hat{q}$ are position tracking and observation errors, respectively. $\Lambda_1, \Lambda_2 > 0$ are diagonal. Note that in the definitions of $\dot{q}_r$ and $\dot{q}_o$, $\dot{q}$ has been replaced by $\dot{q}_d$ and $\dot{\hat{q}}$. This will be shown to eliminate the need for acceleration feedback.

Theorem 2: Consider the system described by (1), (2), the observer dynamics

$$\begin{aligned} \dot{\hat{q}} &= z + \Lambda_2\tilde{q} \\ \dot{z} &= \hat{D}(q)^{-1}[\tau - \hat{C}(q,\dot{q})\dot{q}_o - \hat{G}(q) + L_p\tilde{q} + K_d s_1 \\ &\quad + K_d' s_2] \end{aligned} \tag{20}$$

and the controller obtained by solving the following algebraic equation

$$g_0(q,\tau,u)\hat{\gamma}_2 = \dot{\tau}_d - f_0(q,\dot{q})\hat{\gamma}_1 - \Gamma_\tau^{-1}s_1 - K_\tau\tilde{\tau} \tag{21}$$

where

$$\begin{aligned} \tau_d &= \hat{D}(q)\ddot{q}_r + \hat{C}(q,\dot{q}_r)\dot{q}_r + \hat{G}(q) - K_d(s_1 - s_2) - K_p e \\ &= Y_1(q,\dot{q}_r,\ddot{q}_r)\hat{\theta} - K_d(s_1 - s_2) - K_p e \end{aligned} \tag{22}$$

with unknown rigid body parameter adaptation law

$$\dot{\hat{\theta}} = -\kappa_\theta\Gamma^{-1}\left[Y_1^T(q,\dot{q}_r,\ddot{q}_r)s_1 + Y_2^T(q,\dot{q},\dot{q}_o,\ddot{q}_o)s_2\right] \tag{23}$$

where

$$\begin{aligned} Y_1(q,\dot{q}_r,\ddot{q}_r)\hat{\theta} &= \hat{D}(q)\ddot{q}_r + \hat{C}(q,\dot{q}_r)\dot{q}_r + \hat{G}(q) \\ Y_2(q,\dot{q},\dot{q}_o,\ddot{q}_o)\hat{\theta} &= \hat{D}(q)\ddot{q}_o + \hat{C}(q,\dot{q})\dot{q}_o + \hat{G}(q) \end{aligned} \tag{24}$$

and with hydraulic parameter adaptation laws

$$\begin{aligned}\dot{\hat{\gamma}}_1 &= \kappa_{\gamma_1} ,_{\gamma_1}^{-1} ,_\tau f_0(q,\dot{q})\tilde{\tau}\\ \dot{\hat{\gamma}}_2 &= \kappa_{\gamma_2} ,_{\gamma_2}^{-1} ,_\tau \Pi(\frac{\tilde{\tau}}{\hat{\gamma}_2})\left(\dot{\tau}_d - f_0(q,\dot{q})\hat{\gamma}_1 - ,_\tau{}^{-1}s_1 - K_\tau\tilde{\tau}\right)\end{aligned} \tag{25}$$

where $\Pi(\frac{\tilde{\tau}}{\hat{\gamma}_2}) = diag\{\frac{\tilde{\tau}^i}{\hat{\gamma}_2{}^i}\}$. Then, if the conditions given below in (26) are satisfied, $\underline{0}$ is an asymptotically stable equilibrium point of the state $\tilde{x} = \begin{bmatrix} e^T & \tilde{q}^T & s_1^T & s_2^T & \tilde{\tau}^T\end{bmatrix}^T$.

$$\begin{aligned}&(a)\ \underline{\sigma}(K_p)\underline{\sigma}(L_p)\underline{\sigma}(\Lambda_1)\underline{\sigma}(\Lambda_2) > \frac{1}{4}\bar{\sigma}^2(K_p)\bar{\sigma}^2(\Lambda_1)\\ &(b)\ \|\tilde{x}(0)\| \le \sqrt{\frac{\alpha_m}{3\alpha_M}\left(\frac{\underline{\sigma}(K_d) - C_M\dot{q}_{dm}}{C_M\bar{\sigma}(\Lambda_1)}\right)^2 - \frac{V_{p_M} - V_{p_m}}{\alpha_M}}\end{aligned} \tag{26}$$

where

$$\begin{aligned}\alpha_m &= \frac{1}{2}\min\{D_m, \underline{\sigma}(K_p), \underline{\sigma}(L_p), \underline{\sigma}(,_\tau)\}\\ \alpha_M &= \frac{1}{2}\max\{D_M, \bar{\sigma}(K_p), \bar{\sigma}(L_p), \bar{\sigma}(,_\tau)\}\\ V_{p_m} &\le \frac{1}{2}\left(\tilde{\theta}^T ,\tilde{\theta} + \tilde{\gamma}_1^T ,_{\gamma_1}\tilde{\gamma}_1 + \tilde{\gamma}_2^T ,_{\gamma_2}\tilde{\gamma}_2\right) \le V_{p_M}\end{aligned} \tag{27}$$

Here, $\bar{\sigma}(.)$ and $\underline{\sigma}(.)$ denote the maximum and minimum singular values of their matrix argument, respectively, and $\dot{q}_{dm}$ is an upper bound on the norm of the desired velocity. The projection gains κ_θ, κ_{γ_1} and κ_{γ_2} are defined as in (18). All of the gains used in the controller and observer, i.e. $K_p, K_d, K_d', L_p, ,, ,_{\gamma_1}, ,_{\gamma_2}, ,_\tau, K_\tau$ are constant positive definite diagonal matrices.

Remark 1: The inequality given in (26.b) specifies the boundary of the attraction region that can be enlarged by adjusting the controller gains, i.e. K_d and K_p. Therefore, the closed loop system is semiglobally asymptotically stable. While the controller is guaranteed to be stable, in practice the parameters should be tuned to achieve the desired performance.

Remark 2: In the above formulation, $\hat{D}$, $\hat{C}$ and $\hat{G}$ are the estimated dynamical matrices corresponding to $\hat{\theta}$. Note that the controller and observer use the same set of estimated parameters which compares favorably to the approach proposed in [20], in which different parameter estimates are employed in the controller and observer.

Remark 3: The use of projection gains κ, in the adaptation laws guarantees that the estimate of each parameter remains in a predefined interval $[a, b]$. In particular, if $\hat{\gamma}_2{}^i$ becomes zero the control law u in (21) is undefined. This can be avoided by using $a^i > 0$ for the estimation of γ_2^i. Furthermore, the parameter estimates can not drift because of the upper and lower bounds on their values. Therefore, parameter adaptation is robust against unmodeled disturbances [27].

Proof : By substituting (22) into (1) the following error dynamics are obtained

$$\begin{aligned}D(q)\dot{s}_1 + C(q,\dot{q})s_1 + K_d s_1 + K_p e = \\ K_d s_2 - C(q,s_1)(\dot{q} - s_1) - Y_1(q,\dot{q}_r,\ddot{q}_r)\tilde{\theta} + \tilde{\tau}\end{aligned} \tag{28}$$

The observer closed-loop dynamics can also be written as

$$\begin{aligned}D(q)\dot{s}_2 + C(q,\dot{q})s_2 + K_d' s_2 + L_p\tilde{q} = \\ - K_d s_1 - Y_2(q,\dot{q},\dot{q}_o,\ddot{q}_o)\tilde{\theta}\end{aligned} \tag{29}$$

where (1) and (20) have been used in deriving (29).

Now, let the Lyapunov-like function V_1 be defined as:

$$\begin{aligned}V_1 =& \frac{1}{2}s_1{}^T D(q)s_1 + \frac{1}{2}e^T K_p e + \frac{1}{2}s_2^T D(q)s_2\\ &+ \frac{1}{2}\tilde{q}^T L_p\tilde{q} + \frac{1}{2}\tilde{\theta}^T ,\tilde{\theta}\end{aligned} \tag{30}$$

It can be shown that the derivative of V_1 along the trajectory of the closed loop system is then given by

$$\begin{aligned}\dot{V}_1 =& -s_1{}^T K_d s_1 - s_2{}^T K_d' s_2 - e^T K_p\Lambda_1 e - \tilde{q}^T L_p\Lambda_2\tilde{q}\\ &+ e^T K_p\Lambda_1\tilde{q} + s_1{}^T\tilde{\tau} + \tilde{\theta}^T[-,\dot{\hat{\theta}} - Y_1^T(q,\dot{q}_r,\ddot{q}_r)s_1\\ &- Y_2^T(q,\dot{q},\dot{q}_o,\ddot{q}_o)s_2] - s_1{}^T C(q,s_1)(\dot{q} - s_1)\end{aligned} \tag{31}$$

With the adaptation law given in (23) and using *Lemma 1*, we have that

$$\begin{aligned}\dot{V}_1 \le& -s_1{}^T K_d s_1 - s_2{}^T K_d' s_2 - e^T K_p\Lambda_1 e - \tilde{q}^T L_p\Lambda_2\tilde{q}\\ &+ e^T K_p\Lambda_1\tilde{q} + s_1^T\tilde{\tau} - s_1^T C(q,s_1)(\dot{q}_d - \Lambda_1 e + \Lambda_1\tilde{q})\\ \le& -(\underline{\sigma}(K_d) - C_M(\dot{q}_{dm} + \bar{\sigma}(\Lambda_1)\|e\| + \bar{\sigma}(\Lambda_1)\|\tilde{q}\|))\|s_1\|^2\\ &- \underline{\sigma}(K_d')\|s_2\|^2 - \underline{\sigma}(K_p)\underline{\sigma}(\Lambda_1)\|e\|^2 - \underline{\sigma}(L_p)\underline{\sigma}(\Lambda_2)\|\tilde{q}\|^2\\ &+ \bar{\sigma}(K_p)\bar{\sigma}(\Lambda_1)\|e\|\|\tilde{q}\| + s_1{}^T\tilde{\tau}\\ =& H(\|e\|,\|\tilde{q}\|,\|s_1\|,\|s_2\|) + s_1^T\tilde{\tau}\end{aligned} \tag{32}$$

Note that since

$$\begin{aligned}&-\underline{\sigma}(K_p)\underline{\sigma}(\Lambda_1)\|e\|^2 - \underline{\sigma}(L_p)\underline{\sigma}(\Lambda_2)\|\tilde{q}\|^2 + \bar{\sigma}(K_p)\bar{\sigma}(\Lambda_1)\|e\|\|\tilde{q}\|\\ &= -\begin{bmatrix}\|e\| & \|\tilde{q}\|\end{bmatrix}\begin{bmatrix}\underline{\sigma}(K_p)\underline{\sigma}(\Lambda_1) & -\frac{1}{2}\bar{\sigma}(K_p)\bar{\sigma}(\Lambda_1)\\ -\frac{1}{2}\bar{\sigma}(K_p)\bar{\sigma}(\Lambda_1) & \underline{\sigma}(L_p)\underline{\sigma}(\Lambda_2)\end{bmatrix}\begin{bmatrix}\|e\|\\ \|\tilde{q}\|\end{bmatrix}\end{aligned} \tag{33}$$

the condition given in (26.a) and the inequality

$$\|e\| + \|\tilde{q}\| < \frac{\underline{\sigma}(K_d) - C_M\dot{q}_{dm}}{C_M\bar{\sigma}(\Lambda_1)} \tag{34}$$

guarantee that

$$H(\|e\|,\|\tilde{q}\|,\|s_1\|,\|s_2\|) \le -\alpha(\|e\|^2 + \|\tilde{q}\|^2 + \|s_1\|^2 + \|s_2\|^2) \tag{35}$$

with $\alpha > 0$. It is not difficult to show that if (26.b) holds then (34) is also satisfied.

Following the backstepping approach, V_2, which is a Lyapunov function for the system dynamics, is defined as

$$V_2 = V_1 + \frac{1}{2}\tilde{\tau}^T \Gamma_\tau \tilde{\tau} + \frac{1}{2}\tilde{\gamma}_1^T \Gamma_{\gamma_1}\tilde{\gamma}_1 + \frac{1}{2}\tilde{\gamma}_2^T \Gamma_{\gamma_2}\tilde{\gamma}_2 \tag{36}$$

where $\tilde{\gamma}_1 = [\ \tilde{\gamma}_1^1 \ \ \ldots \ \ \tilde{\gamma}_1^n\ ]^T$ and $\tilde{\gamma}_2 = [\ \tilde{\gamma}_2^1 \ \ \ldots \ \ \tilde{\gamma}_2^n\ ]^T$ are the vectors of hydraulic parameter errors. By taking the derivative of (36) and employing the control law given in (21), one can show after some manipulation that

$$\begin{aligned} \dot{V}_2 \leq\ & H(\|e\|, \|\tilde{q}\|, \|s_1\|, \|s_2\|) - \tilde{\tau}^T \Gamma_\tau K_\tau \tilde{\tau} \\ & + \tilde{\gamma}_1^T \left[f_0(q,\dot{q}) \Gamma_\tau \tilde{\tau} - \Gamma_{\gamma_1} \dot{\hat{\gamma}}_1 \right] + \tilde{\gamma}_2^T [\Gamma_\tau \Pi(\frac{\tilde{\tau}}{\hat{\gamma}_2}) \\ & (\dot{\tau}_d - f_0(q,\dot{q})\hat{\gamma}_1 - \Gamma_\tau^{-1} s_1 - K_\tau \tilde{\tau}) - \Gamma_{\gamma_2} \dot{\hat{\gamma}}_2] \end{aligned} \tag{37}$$

Using the adaptation laws given in (25) and *Lemma 1*, the derivative of V_2 becomes

$$\begin{aligned} \dot{V}_2 & \leq H(\|e\|, \|\tilde{q}\|, \|s_1\|, \|s_2\|) - \tilde{\tau}^T \Gamma_\tau K_\tau \tilde{\tau} \\ & \leq -\gamma(\|e\|^2 + \|\tilde{q}\|^2 + \|s_1\|^2 + \|s_2\|^2 + \|\tilde{\tau}\|^2) \end{aligned} \tag{38}$$

Thus the position and velocity tracking errors converge to zero asymptotically.

Remark 1: For parameter convergence the condition of persistency of excitation must be satisfied [26].

Remark 2: Inspection of (22) reveals that τ_d does not contain $\dot{q}$. This means that acceleration term $\ddot{q}$ does not appear in $\dot{\tau}_d$ and hence in the control law. This is achieved by the particular definition of $\dot{q}_r$ in (19) and also by using $s_1 - s_2 = \dot{q}_o - \dot{q}_r$ instead of s_1 in (22). In summary, the proposed controller requires q, $\dot{q}$ and τ to be measured.

Remark 3: In order to implement the observer proposed in (20), $\hat{D}^{-1}(q)$ must exist. This can be guaranteed by choosing bounds on the estimates of the rigid body parameters.

B. Adaptive Controller/ Robust Observer

In this subsection, an adaptive/nonlinear controller utilizing a sliding type observer [25] is proposed that yields globally asymptotically stable tracking errors.

Before stating the result the following variables should be defined:

$$\dot{q}_r = \dot{q}_d - \Lambda e, \qquad s = \dot{q} - \dot{q}_r = \dot{e} + \Lambda e \tag{39}$$

and $\Lambda > 0$ is diagonal.

Theorem 3: Consider the system described by (1),(2) and the following observer:

$$\dot{z} = \Gamma_o \dot{\tilde{q}} + \Lambda_o sgn(\dot{\tilde{q}}) - W^T(q, \dot{q}_r, \hat{\theta})s + \ddot{\bar{q}} \tag{40}$$

with

$$W(q, \dot{q}_r, \hat{\theta}) = -\hat{D}(q)\Lambda + \hat{C}(q, \dot{q}_r) - K_d \tag{41}$$

$$\ddot{\bar{q}} = \bar{D}^{-1}\left[\tau - \bar{C}\dot{q} - \bar{G}\right] \tag{42}$$

where $z = \dot{\hat{q}}$ is the observed velocity. $\bar{D}$, $\bar{C}$ and $\bar{G}$ are constant matrices (rough estimates of dynamical matrices). Let the control law be given by the solution u of the following algebraic equation

$$g_0(q, \tau, u)\hat{\gamma}_2 = \dot{\tau}_d - f_0(q, \dot{q})\hat{\gamma}_1 - \Gamma_\tau^{-1} s - K_\tau \tilde{\tau} \tag{43}$$

where

$$\tau_d = \hat{D}(q)(\ddot{q}_r + \Lambda\dot{\tilde{q}}) + \hat{C}(q, \dot{\hat{q}})\dot{q}_r + \hat{G}(q) - K_d(s - \dot{\tilde{q}}) - K_p e \tag{44}$$

and let the adaptation laws be given by

$$\dot{\hat{\theta}} = -\kappa_\theta \Gamma_\theta^{-1} Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)s \tag{45}$$

and

$$\begin{aligned} \dot{\hat{\gamma}}_1 &= \kappa_{\gamma_1} \Gamma_1^{-1} \Gamma_\tau f_0(q, \dot{q})\tilde{\tau} \\ \dot{\hat{\gamma}}_2 &= \kappa_{\gamma_2} \Gamma_2^{-1} \Gamma_\tau \Pi(\frac{\tilde{\tau}}{\hat{\gamma}_2}) \left(\dot{\tau}_d - f_0(q, \dot{q})\hat{\gamma}_1 - \Gamma_\tau^{-1}s - K_\tau\tilde{\tau}\right) \end{aligned} \tag{46}$$

for the rigid body and hydraulic parameters, respectively. Then, $\underline{0}$ is an asymptotically stable equilibrium point of the state $\tilde{x} = \begin{bmatrix} e^T & s^T & \dot{\tilde{q}}^T & \tilde{\tau}^T \end{bmatrix}^T$. In the above equations, $K_p, K_d, \Lambda_o, \Gamma_\theta, \Gamma_1, \Gamma_2, \Gamma_\tau, K_\tau$ are positive definite diagonal matrices.

Remark : τ_d does not contain any velocity terms. This can be seen from:

$$\begin{aligned} \ddot{q}_r + \Lambda\dot{\tilde{q}} &= \ddot{q}_d - \Lambda(\dot{q} - \dot{q}_d) + \Lambda(\dot{q} - \dot{\hat{q}}) = \ddot{q}_d - \Lambda(\dot{\hat{q}} - \dot{q}_d) \\ s - \dot{\tilde{q}} &= \dot{q} - \dot{q}_r - \dot{q} + \dot{\hat{q}} = \dot{\hat{q}} - \dot{q}_r \end{aligned} \tag{47}$$

Proof: By substituting (44) into (1) the following closed loop dynamics are obtained

$$\begin{aligned} D(q)\dot{s} + C(q, \dot{q})s + K_d s + K_p e = & \\ - Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{\theta} - W(q, \dot{q}_r, \hat{\theta})\dot{\tilde{q}} + \tilde{\tau} & \end{aligned} \tag{48}$$

Define the Lyapunov-like function V_1 to be

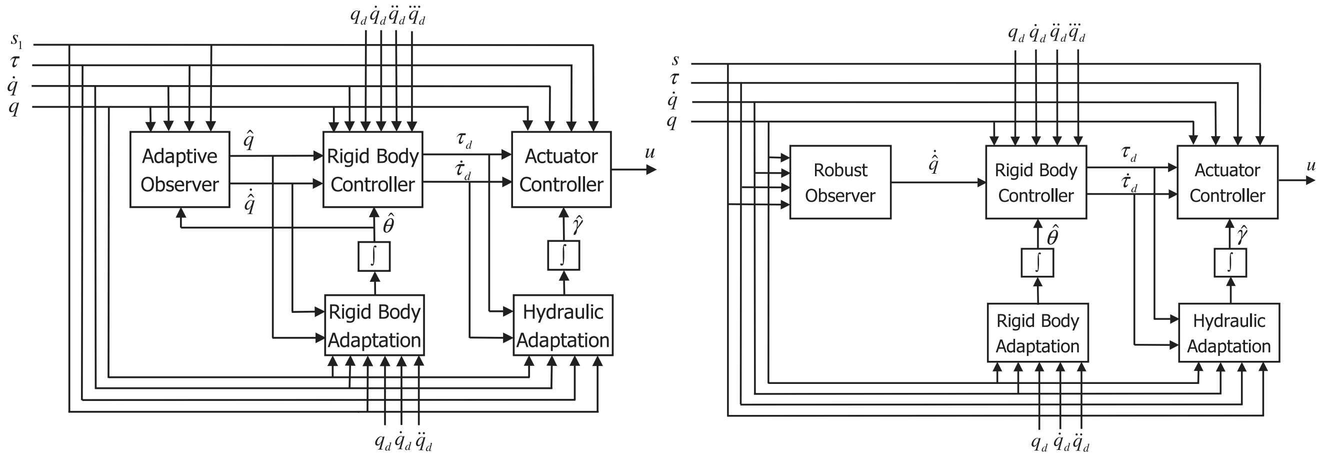
$$V_1 = \frac{1}{2}e^T K_p e + \frac{1}{2}s^T D(q)s + \frac{1}{2}\dot{\tilde{q}}^T\dot{\tilde{q}} + \frac{1}{2}\tilde{\theta}^T \Gamma_\theta \tilde{\theta} \tag{49}$$

The derivative of V_1 becomes

$$\begin{aligned} \dot{V}_1 = & s^T[-Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{\theta} - W(q, \dot{q}_r, \theta)\dot{\tilde{q}} - C(q, \dot{q})s \\ & - K_d s - K_p e + \tilde{\tau}] + \frac{1}{2}s^T \dot{D}(q)s + e^T K_p(s - \Lambda e) \\ & + \dot{\tilde{q}}^T \left[\ddot{q} - \Gamma_o\dot{\tilde{q}} - \Lambda_o sgn(\dot{\tilde{q}}) + W^T(q, \dot{q}_r, \hat{\theta})s - \ddot{\bar{q}}\right] + \tilde{\theta}^T \Gamma_\theta \dot{\tilde{\theta}} \end{aligned} \tag{50}$$

which can be written in the following form:

$$\begin{aligned} \dot{V}_1 = & -s^T K_d s - e^T K_p \Lambda e - \dot{\tilde{q}}^T \Gamma_o \dot{\tilde{q}} - \dot{\tilde{q}}^T[\ddot{q} - \ddot{\bar{q}} \\ & + \Lambda_o sgn(\dot{\tilde{q}})] + \tilde{\theta}^T[\Gamma_\theta\dot{\tilde{\theta}} - Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)s] + s^T\tilde{\tau} \end{aligned} \tag{51}$$



(a) (b)

Fig. 1. Controller implementation block diagrams. (a) Controller with adaptive observer. (b) Controller with robust observer.

With the adaptation law given in (45), $\dot{V}_1$ becomes

$$\dot{V}_1 = -s^T K_d s - e^T K_p \Lambda e - \dot{\tilde{q}}^T \Gamma_o \dot{\tilde{q}} + \Sigma + s^T \tilde{\tau} \tag{52}$$

where $\Sigma = -\dot{\tilde{q}}^T [\ddot{\hat{q}} - \ddot{q} + \Lambda_o sgn(\dot{\tilde{q}})]$. It is not difficult to show that

$$||\ddot{\hat{q}} - \ddot{q}|| \leq \sigma_0 + \sigma_1 ||\dot{q}||^2 + \sigma_2 ||\tau|| + \sigma_3 ||\dot{q}||. \tag{53}$$

with $\sigma_i > 0$. The properties given in (3) have been exploited in deriving (53). The following choice of Λ_o makes $\Sigma < 0$:

$$\Lambda_o = diag\{\Lambda_o^i\} \tag{54}$$

$$\Lambda_o^i = \lambda_0^i + \lambda_1^i ||\dot{q}||^2 + \lambda_2^i ||\tau|| + \lambda_3^i ||\dot{q}|| \tag{55}$$

and $\lambda_k^i > \sigma_k$ for $i = 1, \cdots, n$ and $k = 0, \cdots, 3$. The rest of the proof is the same as before and will not be presented here.

Note that there are no limitations on the norms of the initial state tracking errors in this approach. However, chattering phenomena, which are inherent in sliding mode systems, can affect stability. For example, if high frequency dynamics (e.g. valve dynamics) are excited, instability could result. The problem could be solved by using a piecewise linear approximation to $sgn(.)$.

The Effect of Friction

In the controllers proposed in this paper, friction in the hydraulic actuators has been neglected. It is easy to handle viscous friction since it acts as additional damping in the system. It can also be shown that in the presence of Coulomb friction, the tracking errors do not converge to zero but remain bounded. The error bounds can be reduced by increasing the gains. The proof will be omitted here.

V. Simulation Results

Simulations have been performed to investigate the effectiveness of the proposed controllers and to obtain guidelines for experimentation. For this purpose, a realistic model of the experimental setup, a hydraulic Stewart-type platform, has been used (see *Appendices A and B*). The system parameters were selected based upon their actual values and are given in Table I.

In the simulations and experiments conducted for this paper, a task-space control strategy has been followed. The advantage of this approach is that the dynamical matrices have simpler forms in these coordinates for parallel manipulators such as the Stewart platform. However, the forward kinematics problem must be solved on-line to convert the measured link positions to robot positions in task-space coordinates. Newton's method was utilized for this purpose. The control algorithms and the robot dynamics were all implemented using the *Matlab Simulink*^{TM} toolbox. The implementation block diagrams of the controllers are shown in Figures 1(a) and 1(b).

Extensive simulations showed similar performance for both of the proposed adaptive control methods, thus only the results obtained for the controller using the robust observer are presented here. The system parameters were initially set to values different from those used in the model to investigate the ability of the controllers to cope with parametric uncertainties. The reference trajectory was chosen to be $x_d = 0.02\sin(2\pi t) + 0.01\sin(4\pi t) + 0.01 sin(6\pi t)$, $y_d = 0$, $z_d = 0.02\sin(2\pi t) + 0.01\sin(4\pi t)$, $\psi_d = 0.0873\sin(2\pi t) + 0.0349\sin(4\pi t)$, $\theta_d = 0.0524\sin(2\pi t) + 0.0175\sin(4\pi t)$, $\phi_d = 0.0524\sin(2\pi t) + 0.0175\sin(4\pi t)$. Positions and angles are expressed in meters and radians, respectively. The tracking errors clearly converge to zero in all coordinates as shown in Figure 2. The profiles of the parameter estimates are given in Figure 3. The parameter adaptation laws were activated after $t = 0.5$ s. Both rigid body and hydraulic parameters converge to their actual values, even though the parameter convergence is not guaranteed in theory. The estimates of I_x, I_y reach their boundaries during some periods of the simulation as seen in Figure 3.

In summary, the controller with the robust observer com-

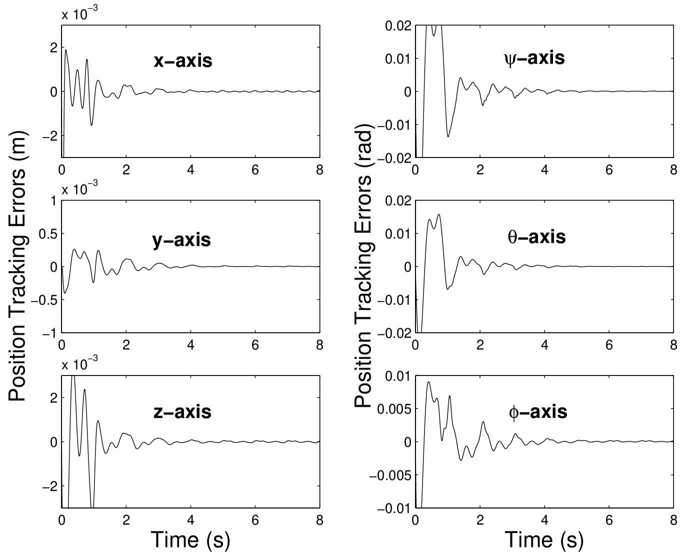


Fig. 2. Position tracking errors (simulation).

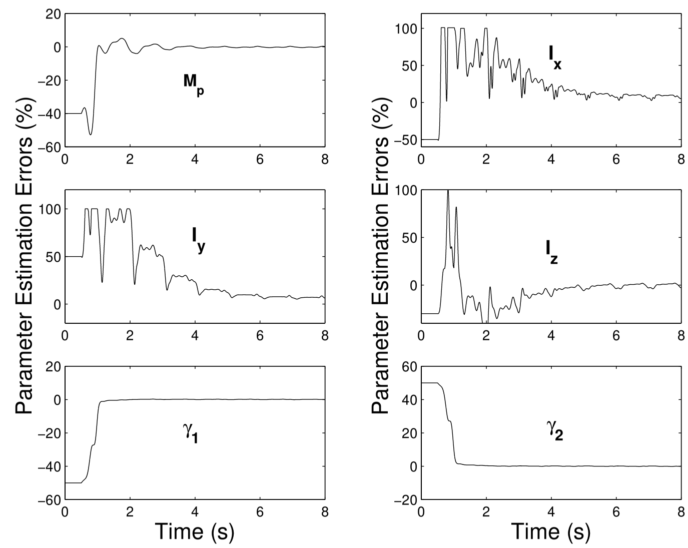


Fig. 3. Parameter estimation errors (simulation).

pares favorably to the one with adaptive observer, since it requires fewer computations and performs similarly.

VI. Experimental Results

The proposed control methods were experimentally evaluated using the motion simulator at the University of British Columbia [28] (see Figure 4). This simulator is driven by six hydraulic cylinders. Each cylinder is capable of exerting forces in excess of 4000 N at 1 m/s, and over 8000 N at zero rod speed. The hydraulic actuation system is equipped with *Rexroth 4WRDE* three-stage proportional valves connected in a three-way configuration. Low friction Teflon seals are used in the hydraulic cylinders. The installed sensors measure the actuator lengths, the valve spool positions and the pressures both in the control and supply sides of the cylinders. High bandwidth valves with a bandwidth of around 80Hz have been used in the setup so that the dynamics of the valves may be ignored. The actuator velocities that are required in the control laws are estimated from the measured actuator lengths using fixed gain Kalman filters. Off-line experiments were performed to identify the initial values of the parameter estimates.

The computational setup was a PC running $VxWorks^{TM}$ *5.4* and a Sparc 1e board running $VxWorks^{TM}$ *5.2* (see Figure 4). The Sparc 1e performs the I/O and safety monitoring functions and the controller runs on the PC. The controller was implemented using the *Matlab Real Time* $Workshop^{TM}$ toolbox targeting $Tornado^{TM}$ *2.0*. Data between the PC and the VME board are communicated through a custom parallel I/O communication protocol. Using this setup a control frequency of 512 Hz was successfully achieved. The same controller block used in the simulation studies was utilized to control the platform.

Only the results of the experiments with adaptive controller/robust observer are presented here, while similar performance was observed for the other controller. Figure 5 shows the tracking behavior of the nonlinear controller compared with that of a well-tuned P controller in tracking the reference trajectory $z_d = -2.34 + 0.05\sin(2\pi t)$ m (the bias is not shown). The maximum tracking errors are 4% and 43% for the nonlinear and P controller, respectively. The response of the system to a 2Hz reference trajectory was also examined and is presented in Figure 6. In this case $z_d = -2.34+0.02\sin(4\pi t)$ m and the maximum tracking errors are 14% and 69%. Similar results were obtained in the other coordinates. For example, Figure 7 shows the tracking results along the ψ axis where $\psi_d = 0.09\sin(2\pi t)$ rad with 4% and 41% maximum tracking error for the nonlinear and P controller, respectively. In all of these cases the proposed adaptive nonlinear controller clearly outperforms the well-tuned P controller and exhibits excellent tracking performance. Note that due to the friction in the actuators, tracking errors do not converge to zero but remain bounded as claimed in the paper.

During the experiments, the estimated parameters did not converge to fixed values, contrary to what was observed in the simulations. Friction is an important factor that introduces tracking errors and prevents the parameters from converging. The proposed controllers may be interpreted as cascade combinations of passivity-based position controllers and actuator force controllers. The very stiff dynamics of hydraulic actuators make the force (pressure) control loop sensitive to velocity estimation errors (or velocity measurement noise) and pressure measurement noise. This limits the level of the pressure feedback gains and may deteriorate force tracking and subsequently parameter estimation, especially for the hydraulic parameters. Other factors such as insufficient excitation and unmodeled dynamics, e.g. valve and leg dynamics, could also prevent the parameters from converging. Moreover, it should be stressed that the parameter convergence is not even guaranteed in theory, therefore the experimental results do not contradict the theoretical arguments. The

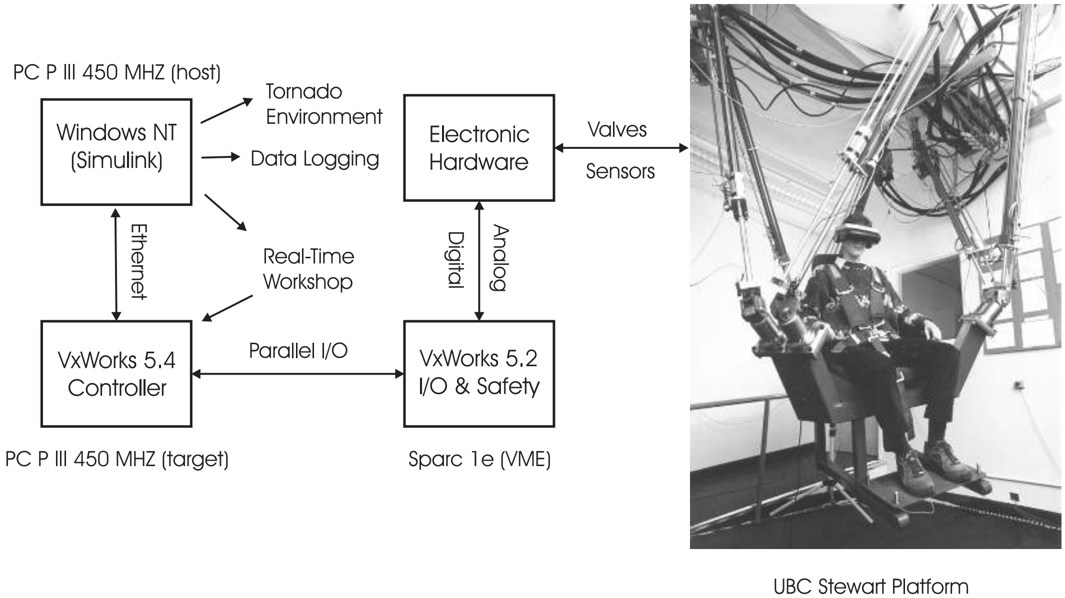


Fig. 4. The experimental setup.

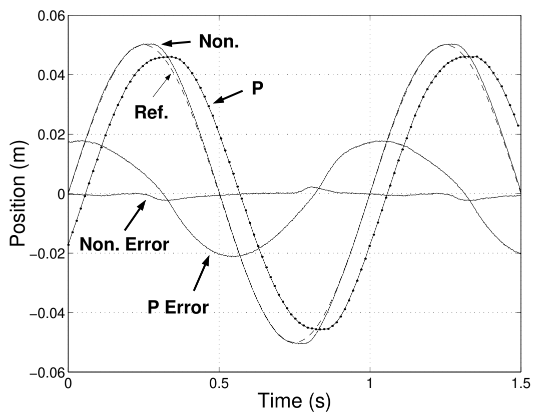


Fig. 5. Position tracking (1Hz) along z coordinate (experiment).

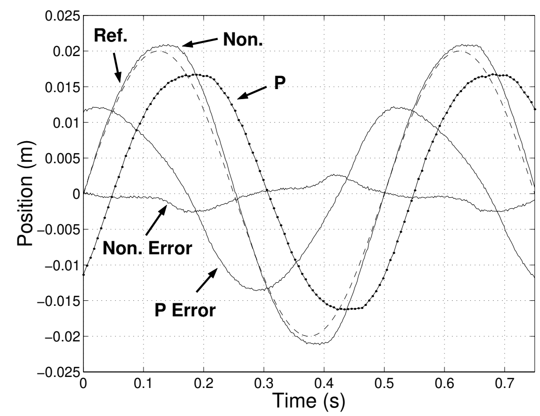


Fig. 6. Position tracking (2Hz) along z coordinate (experiment).

adaptation was found to be quite helpful in improving the tracking performance. The projection gains used in the adaptation laws proved effective in preventing the large parameter swings that can occur especially during start-up transients. The step response of the controller along the z axis is also compared with that of the P controller in Figure 8. As it can be seen, the nonlinear controller exhibits a much faster response with some overshoot.

VII. Conclusions

This paper addresses the control problem for hydraulically driven manipulators. The highly nonlinear dominant actuator dynamics prevents the use of standard robot control methods. In fact, inclusion of actuator dynamics in the design is of critical importance in hydraulic robots. While most of the reported work in the literature consider the control of single-rod hydraulic actuators, this paper proposed novel nonlinear controllers for hydraulic manipulators using backstepping. A realistic model of the system was utilized in developing these Lyapunov-based controllers. To deal with parameter uncertainties, the controllers were augmented with adaptation laws. Acceleration feedback was avoided by proposing adaptive and sliding-type observers. Simulations and experiments were carried out with a hydraulic Stewart platform to investigate the effectiveness of these approaches. The results demonstrated excellent tracking position tracking behavior and satisfactory transient responses for these new controllers.

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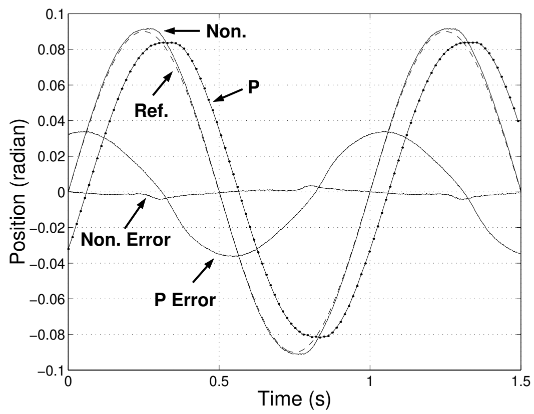

Fig. 7. Position tracking (1Hz) along ψ coordinate (experiment).

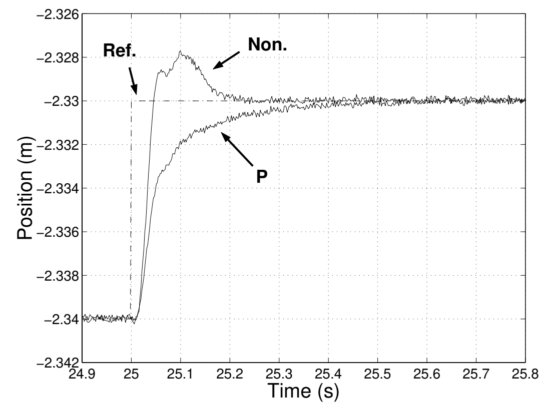

Fig. 8. Step response along z coordinate (experiment).

Appendix A

The dynamics of a typical hydraulic actuator are presented in this Appendix. A three-way valve configuration is assumed to be used in the actuators, as shown in Figure 9. For such a configuration, the control pressure dynamics are governed by [29]

$$\frac{V_t}{\beta}\dot{p}_c = q_l + c_l(p_s - p_c) - \dot{V}_t \tag{56}$$

where V_t is the trapped fluid volume in the control side, β is the effective bulk modulus, p_c is the control pressure acting on the control side, p_s is the supply pressure acting on the rod side, q_l is the load flow, and c_l is the coefficient of total leakage. The load flow, q_l, is a nonlinear function of the control pressure and the valve spool position and is given by

$$q_l = \begin{cases} c(u-d)\sqrt{p_c} & u < -d \\ c(u+d)\sqrt{p_s - p_c} + c(u-d)\sqrt{p_c} & -d \le u \le d \\ c(u+d)\sqrt{p_s - p_c} & u > d \end{cases} \tag{57}$$

and $c = c_d w\sqrt{\frac{2}{\rho}}$, where c_d is the effective discharge coefficient, w is the port width of the valve, ρ is the density of the fluid, d is the valve underlap length and u is the valve spool position which is the control command. Note that the actuator output force is $\tau = p_c A - p_s a$. Therefore, using (56) and (57), the dynamics of the i'th hydraulic actuator can be written in the following form (assuming $c_l \approx 0$)

$$\dot{\tau}^i = -\frac{A\beta^i \dot{q}^i}{q^i - l^i} + \frac{\beta^i}{q^i - l^i} q_l^i(\tau^i, u^i) = f^i(q^i, \dot{q}^i) + g^i(q^i, \tau^i, u^i) \tag{58}$$

where l is the actuator stroke length. For a Stewart platform, there are six actuators driving the system. The actuator subsystem dynamics can be represented in matrix form as in (2).

Note that (58) can be rewritten in the following form which is suitable for adaptive control:

$$\dot{\tau}^i = \gamma_1^i f_0^i(q^i, \dot{q}^i) + \gamma_2^i g_0^i(q^i, \tau^i, u^i) \tag{59}$$

where $\gamma^i = \begin{bmatrix}\beta^i & \beta^i c^i\end{bmatrix}^T$, $f_0^i = -\frac{A^i \dot{q}^i}{q^i - l^i}$, and $g_0^i = \frac{q_l^i}{c_i(q^i - l^i)}$ (does not depend on c_i^s, see (57)). These equations can be written in matrix form as in (16).

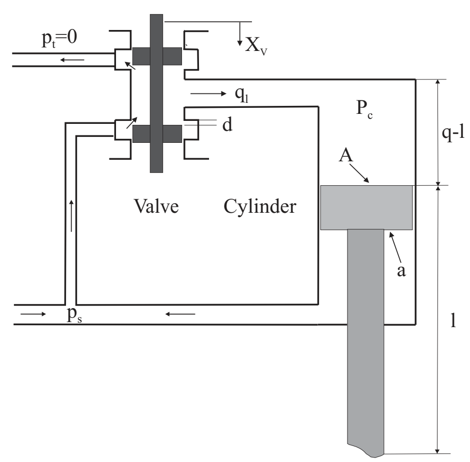

Fig. 9. A typical three-way valve configuration.

Appendix B

The Stewart platform is a parallel manipulator widely used in conventional motion simulators. The dynamics of an inverted, ceiling-mounted Stewart platform (Figure (10)) are presented here [19].

In task-space coordinates, the dynamics of the platform are governed by:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G = (JL)^T\tau \tag{60}$$

where $q = \begin{bmatrix}x & y & z & \psi & \theta & \phi\end{bmatrix}^T$ and ϕ, θ and ψ are roll-pitch-yaw angles, respectively. Furthermore, J is the manipulator Jacobian matrix and L is defined as

$$L = \begin{bmatrix} I_{3\times 3} & 0 \\ 0 & T \end{bmatrix} \tag{61}$$

TABLE I
THE SYSTEM PARAMETERS USED IN THE SIMULATIONS AND EXPERIMENTS.

Hydraulic Parameters							
Parameter	A (m^2)	a (m^2)	L (m)	P_s (psi)	d (m)	c	β (Mpa)
Value	1.14×10^{-3}	6.33×10^{-4}	1.37 m	1500	55.4×10^{-6}	1.5×10^{-4}	700
Rigid Body Parameters							
Parameter	M_p (kg)	I_x (kg.m^2)	I_y (kg.m^2)	I_z (kg.m^2)	–		
Value	250	45	45	43	–		

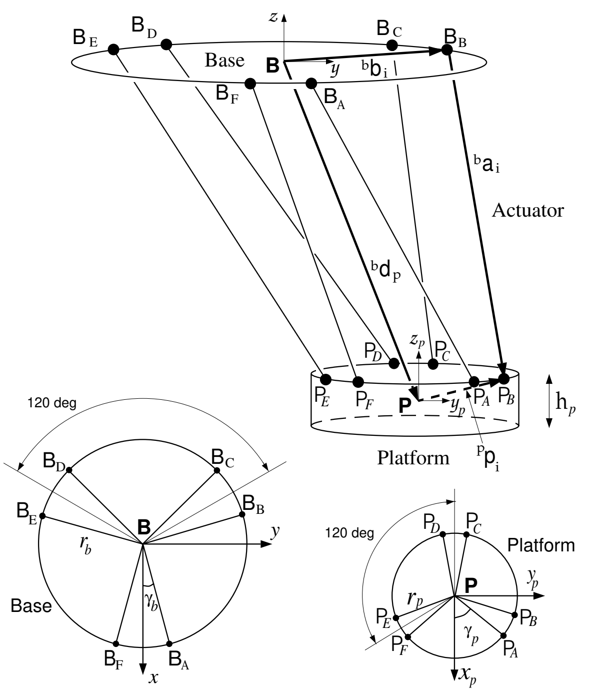

Fig. 10. The schematic of the Stewart platform.

with

$$T = \begin{bmatrix} \cos(\theta)\cos(\phi) & -\sin(\phi) & 0 \\ \cos(\theta)\sin(\phi) & \cos(\phi) & 0 \\ -\sin(\theta) & 0 & 1 \end{bmatrix} \quad (62)$$

Finally, $D(q)$, $C(q,\dot{q})$ and G have the following forms:

$$D(q) = \begin{bmatrix} M_p I_{3\times3} & 0 \\ 0 & T^T\, {}^bI_p T \end{bmatrix} \quad (63)$$

$$C(q,\dot{q}) = \begin{bmatrix} 0 & 0 \\ 0 & c_{22} \end{bmatrix}$$
$$c_{22} = T^T S(\omega)^b I_p T + T^{T\,b} I_p \dot{T} \quad (64)$$

$$G = \begin{bmatrix} 0 & 0 & M_p g & 0 & 0 & 0 \end{bmatrix}^T \quad (65)$$

where ω is the angular velocity vector of the platform and

$$S(\omega) = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \quad (66)$$

In the above equations, ${}^bI_p$ is the platform inertia matrix with respect to the base frame and is given by

$${}^bI_p = R\, {}^pI_p R^T \quad (67)$$

where

$${}^pI_p = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \quad (68)$$

and R is a rotation matrix giving the coordinates of the platform-attached basis vectors in a base frame. Note that (60) is not exactly as (1). However, since J is a function of platform position and is known, the controllers can be easily modified to be used in this case. Moreover, the rigid body dynamics may be written in so-called *linear in parameters* form.

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G = Y_{6\times4}(q,\dot{q},\ddot{q})\theta \quad (69)$$

where $\theta = \begin{bmatrix} M_p & I_x & I_y & I_z \end{bmatrix}^T$ is the vector of unknown parameters. The detailed expressions of the elements of Y are long and fairly straightforward and will not be presented here.

References

[1] J.Y.S. Luh, M.W. Walker, and R.P.C. Paul, "Resolved acceleration control of mechanical manipulators," *IEEE Tran. Automat. Cont.*, vol. 25, pp. 468–474, 1980.
[2] R. Ortega and M.W. Spong, "Adaptive motion control of rigid robots: A tutorial," *Automatica*, vol. 25, pp. 877–888, 1989.
[3] H. Berhuis and H. Nijmeijer, "A passivity approach to controller-observer design for robots," *IEEE Tran. Robot. Automat.*, vol. 9, no. 6, pp. 740–754, 1993.
[4] B. Paden and R. Panja, "Globally asymptotically stable 'PD+' controller for robot manipulators," *Int. J. Control*, vol. 47, pp. 1697–1712, 1988.
[5] J. J.-E Slotine and W. Li, "On the adaptive control of robot manipulators," *Int. J. Robot. Research*, vol. 6, pp. 49–59, 1987.
[6] M. W. Spong, "On the robust control of robot manipulators," *IEEE Tran. Automat. Cont.*, vol. 37, pp. 1782–1786, 1992.
[7] S. Zenieh and M. Corless, "Simple robust tracking controllers for robotic manipulators," in *SYROCO'94*, September 1994, pp. 193–198.
[8] C. Abdallah, D. Dawson, P. Dorato, and M. Jamshidi, "Survey of robust control for rigid robots," *IEEE Control Systems*, pp. 24–30, 1991.
[9] T. J. Tarn, A. K. Bejczy, X. Yun, and Z. Li, "Effect of motor dynamics on nonlinear feedback robot arm control," *IEEE Tran. Robot. Automat.*, vol. 7, no. 1, pp. 114–122, February 1991.
[10] J. Yuan, "Adaptive control of robotic manipulators including motor dynamics," *IEEE Tran. Robot. Automat.*, vol. 11, no. 4, pp. 612–617, 1995.
[11] T. Burg, D. Dawson, J. Hu, , and M. de Queiroz, "An adaptive partial state-feedback controller for RLED robot manipulators,"

IEEE Tran. Automat. Contr., vol. 41, no. 7, pp. 1024–1030, July 1996.
[12] C. Y. Su and Y. Stepanenko, "Redesign of hybrid adaptive/robust motion control of rigid-link electrically-driven robot manipulator," *IEEE Tran. Robot. Automat.*, vol. 14, no. 4, pp. 651–655, 1998.
[13] M. R. Sirouspour and S. E. Salcudean, "On the nonlinear control of hydraulic servo-systems," in *Int. Conf. Robot. Automat.*, April 2000, pp. 1276–1282.
[14] G. A. Sohl and J. E. Boborow, "Experiments and simulations on the nonlinear control of a hydraulic servosystem," *IEEE Tran. Cont. Syst. Tech.*, vol. 7, no. 2, pp. 238–247, March 1999.
[15] G. Vossoughi and M. Donath, "Dynamic feedback linearization for electrohydraulically actuated control systems," *J. Dyn. Sys., Meas. and Contr.*, vol. 117, pp. 468–477, Dec. 1995.
[16] B. Novel, M.A. Garnero, and A. Abichou, "Nonlinear control of a hydraulic robot using singular perturbations," in *Int. Conf. Syst. Man and Cyber.*, 1994, pp. 1932–1937.
[17] K. A. Edge and F. Gomes de Almeida, "Decentralized adaptive control of a directly driven hydraulic manipulator Part I: theory," in *Instn. Mech. Engr.*, 1995, vol. 209, pp. 191–196.
[18] K. A Edge and F. Gomes de Almeida, "Decentralized adaptive control of a directly driven hydraulic manipulator Part II: experiments," in *Instn. Mech. Engr.*, 1995, vol. 209, pp. 197–202.
[19] D. Li and S. E. Salcudean, "Modeling, simulation and control of a hydraulic Stewart platform," in *IEEE Int. Conf. Robot. Automat.*, April 1997, pp. 3360–3366.
[20] F. Bu and B. Yao, "Observer based coordinated adaptive robust control of robot manipulators driven by single-rod hydraulic actuators," in *IEEE Int. Conf. Robot. Automat.*, April 2000, pp. 3034–3039.
[21] M. Krstic, I. Kanellakopoulos, and P. Kokotovic, Eds., *Nonlinear and Adaptive Control Design*, Wiley and Sons, New York, 1995.
[22] Z. Pan and T. Basar, "Adaptive controller design for tracking and disturbance attenuation in parameteric strict-feedback nonlinear systems," *IEEE Tran. Automat. Control*, vol. 43, no. 8, pp. 1066–1083, August 1998.
[23] H. J. Shieh and K K Shyu, "Nonlinear sliding-mode torque control with adaptive backstepping approach for induction motor drive," *IEEE Tran. Indus. Elect.*, vol. 46, no. 2, pp. 380–389, April 1999.
[24] W.E. Dixon, E. Zergeroglu, D.M. Dawson, and M.W. Hannan, "Global adaptive partial state feedback tracking control of rigid-link flexible-joint robots.," *Robotica*, vol. 18, pp. 325–336, May-June 2000.
[25] C. C. de Wit and J. J. E. Slotine, "Sliding observers for robot manipulators," *Automatica*, vol. 27, no. 5, pp. 859–864, 1991.
[26] J. E. Slotine and W. Li, Eds., *Applied Nonlinear Control*, Prentice-Hall Inc., New Jersey, 1991.
[27] W. H. Zhu and J. De Schutter, "Adaptive control of mixed rigid/flexible joint robot manipulators based on virtual decomposition," *IEEE Tran. Robot. Automat.*, vol. 15, no. 2, pp. 310–317, April 1999.
[28] S. E. Salcudean, P.A. Drexel, D. Ben-Dov, A.J. Taylor, and P.D. Lawrence, "A six degree-of-freedom, hydraulic, one person motion simulator," in *Int. Conf. Robot. Automat.*, May 1994, pp. 859–864.
[29] H. E. Merrit, Ed., *Hydraulic control systems*, Wiley and Sons, New York, 1967.

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