Nonlinear diffusion and beam self-trapping in diffraction-managed waveguide arrays

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Abstract: We study nonlinear propagation of light in diffraction-managed photonic lattices created by periodically-curved arrays of optical waveguides. We identify different regimes of the nonlinear propagation of light in such structures depending on the input power. We start from the regime of self-collimation at low powers and demonstrate that, as the beam power increases, nonlinearity destroys the beam self-imaging and leads to *nonlinear diffusion*. At higher powers, we observe a sharp transition to the self-trapping and the formation of discrete diffraction-managed solitons.

OCIS codes: (190.4420) Nonlinear optics, transverse effects in; (190.5940) Self-action effects.

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1. Introduction

Propagation of light in dielectric media with a periodically-varying refractive index is known to demonstrate many novel features in both linear and nonlinear regimes [1]. Over the recent years, different types of periodic photonic structures, including arrays of evanescently coupled optical waveguides, optically-induced lattices in photorefractive materials, and photonic crystals, have been employed to engineer and control many fundamental properties of light propagation. In particular, the idea to control the light spreading through diffraction management [2] has attracted a special attention. It was shown that both magnitude and sign of the beam diffraction can be controlled in periodic photonic structures. For example, diffraction can be made *negative* allowing for focusing of diverging beams [3], or it can even be suppressed leading to the self-collimation effect when the beam width does not change over hundreds of the free-space diffraction lengths [4]. Self-collimation of light beams was also realized experimentally in periodically curved waveguide arrays [5–7]. Recently, the concept of the broadband diffraction management of polychromatic light has been introduced for the periodically-curved waveguide arrays with special bending profiles [8].

The combination of tailored diffraction characteristics and light self-action opens new possibilities for the power-controlled beam shaping and switching in nonlinear photonic structures. Various schemes for active beam control based on the special properties of narrow self-localized beams in straight waveguide arrays, called discrete spatial solitons, have been suggested and demonstrated [9–11]. It was shown that solitons can exist in diffraction-managed lattices [12, 13] in the regime when beam exhibits effectively averaged diffraction. However, the analysis of narrow beam self-action in periodically-curved waveguide arrays, beyond the applicability of averaging procedures, remained largely unexplored. An especially intriguing problem is the nonlinear beam self-action under the condition of linear self-collimation, where diffraction is suppressed in *all* orders. In the latter case, the modulational instability is suppressed [6], suggesting that discrete soliton formation may demonstrate unusual features.

In this work, we investigate nonlinear propagation of light beams in the diffraction-managed periodically curved photonic lattices [such as sketched in Fig. 1(a)], and identify different regimes of the light propagation depending on the input power. In particular, when linear discrete diffraction is fully suppressed, we observe the transition from the regime of self-collimation, for low powers, to that of the discrete self-trapping and the formation of lattice solitons, for high powers. This occurs through the intermediate regime of the *nonlinear diffusion*, where limited nonlinear beam broadening takes place. We also observe the similar regime of nonlinear diffusion in other types of periodically-curved photonic lattices; this regime has *no analogies* with the nonlinear beam self-focusing or self-defocusing in a bulk medium [11], or discrete self-trapping of light in arrays of straight waveguides [1]. We also show, that the crit-

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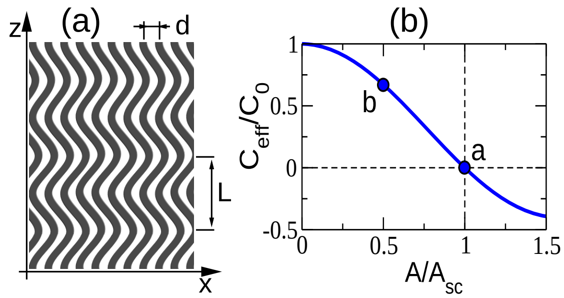

Fig. 1. (a) Schematic of an array of periodically curved waveguides. (b) Effective coupling C_{eff} in the curved waveguide array normalized to the coupling C_0 in the straight array, as a function of the bending amplitude A normalized to the amplitude A_{sc} which corresponds to the self-collimation condition. Point 'a' corresponds to the self-collimation, when the bending amplitude is $A = A_{\text{sc}}$, while for the point 'b' we take $A = 1/2A_{\text{sc}}$.

ical power required for the formation of lattice solitons in curved waveguide arrays is higher than that in the straight waveguide arrays, and it can be effectively controlled by changing the amplitude of the waveguide bending.

2. Propagation of light beams in arrays of curved waveguides

We study the propagation of light beams in a one-dimensional array of coupled nonlinear optical waveguides with the transverse period d in the x direction, where the waveguide axes are periodically curved in the propagation direction z with the period $L \gg d$ [see Fig. 1(a)]. When the tilt of beams and waveguides at the input facet is less than the Bragg angle, the beam propagation is primarily characterized by coupling between the fundamental modes of the individual waveguides, and it can be described by the tight-binding equations taking into account the periodic waveguide bending [5,6], $id\Psi_n/dz + C_0\left(\Psi_{n+1} + \Psi_{n-1}\right) = \omega\ddot{x}_0(z)n\Psi_n - \gamma|\Psi_n|^2\Psi_n$, where $\Psi_n(z)$ is the amplitude of the n-th waveguide, $\omega = 2\pi \mathrm{n}_0 d/\lambda$ is the dimensionless frequency, λ is the vacuum wavelength, n_0 is the average refractive index of the medium, γ is an effective nonlinear coefficient which accounts for the Kerr-type nonlinear response of the waveguide material, and the dots stand for the derivatives. Transverse shift $x_0(z) \equiv x_0(z+L)$ defines the periodic longitudinal bending profile of the waveguide axis. Coefficient C_0 defines the coupling strength between the neighboring waveguides, and it characterizes diffraction in a straight waveguide array with $x_0 \equiv 0$ [14]. Then, the total electric field envelope $E(x,z)$ is represented as a superposition of the modes $E_0(x)$ of the individual waveguides, $E(x,z) = \sum_n \Psi_n E_0\left[x - nd - x_0(z)\right]$.

The distinctive features of discrete beam dynamics become most evident when only one waveguide is excited at the input. Then the light evolution for both positive and negative nonlinearities is fully equivalent in the framework of the tight-binding model [5,6], and we take $\gamma \geq 0$. In our simulations presented below, we use the following values which are typical for the experiments with optical waveguide arrays: $d = 9\ \mu$m, $\mathrm{n}_0 = 2.35$, $\lambda = 532$ nm, $C_0 = 0.13\ \text{mm}^{-1}$, $\gamma = 1.9$. Normalization chosen is such that x is measured in μm and z is measured in mm. We use the discrete model for the calculations presented in this paper, however we have confirmed the validity of our results by simulating the full parabolic equations for the continuous electric field envelopes. We note that in case of the strong coupling between the waveguides, the long range coupling between non-nearest neighbours becomes important. In this case, exact dynamical localization can also be ralized in arrays of curved waveguides [7], and effects similar to the presented in this paper may be expected to take place in the nonlinear regime.

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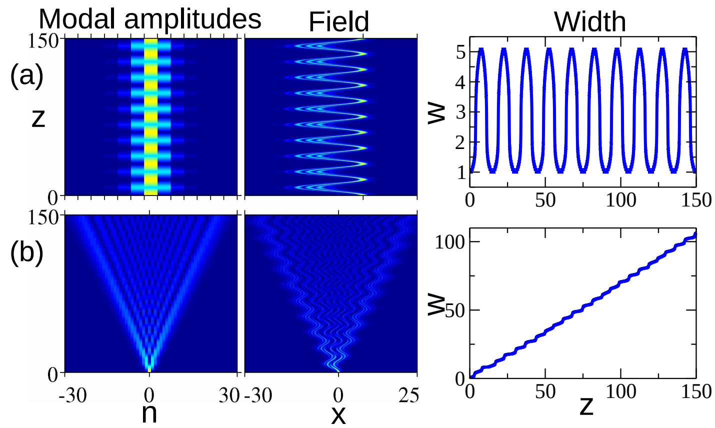

Fig. 2. Linear propagation of light in a curved waveguide array. Shown are the absolute values of the amplitudes of the modes of individual waveguides (left) and corresponding optical field distribution (center). Right: beam width w [normalized to the input width] as a function of the propagation distance. (a) Bending amplitude corresponds to the self-collimation condition [point 'a' in Fig. 1(b)]. (b) Bending amplitude is half of that in (a) [point 'b' in Fig. 1(b)]. Bending period is $L = 15$ mm. Waveguide array length is 150 mm.

3. Discrete diffraction and self-collimation

In order to specifically distinguish the effects due to diffraction management, we consider the light propagation in the waveguide arrays with symmetric bending profiles, i.e. $x_0(z - z_a) = x_0(z_a - z)$ for a given coordinate shift z_a, since asymmetry may introduce other effects due to the modification of refraction [15–19]. Then, after the full bending period ($z \to z + L$) the beam diffraction remains the same as in a straight waveguide array with the effective coupling coefficient [5, 6] $C_{\mathrm{eff}} = C_0 L^{-1} \int_0^L \cos[\omega \dot{x}_0(\zeta)]\, d\zeta$. As a specific example, we consider a curved waveguide array with the sinusoidal bending of the form $x_0(z) = A\{\cos[2\pi z/L] - 1\}$, for which the effective coupling coefficient is $C_{\mathrm{eff}} = C_0 \mathrm{J}_0\,(2\pi\omega A/L)$, where J_0 is the Bessel function of the first kind of zero order. The effective diffraction can be made either normal, zero, or anomalous depending on the value of the bending amplitude [see Fig. 1(b)].

Cancellation of the effective coupling and periodic beam self-collimation takes place at low powers when $A = A_{\mathrm{sc}} = \xi L [2\pi\omega]^{-1}$, where $\xi \simeq 2.405$ is the first root of the function J_0. For example, for the bending period $L = 15$ mm self-collimation occurs when $A_{\mathrm{sc}} = 23.0\ \mu$m [see Fig. 2(a), where we use the modal representation described above in order to reconstruct the optical field distribution]. The beam width is determined as the width of the transverse cross-section function centered at the current center of mass of the beam where 75% of the beam power is concentrated. When the bending amplitude differs from the self-collimation value, the beam experiences discrete diffraction at low powers, similar to the effect observed in straight waveguide arrays [1] [see Fig. 2(b)].

4. Nonlinear beam propagation and control

When the power of the input beam increases, *nonlinearity destroys the self-collimation condition* of light by changing the refractive index of the waveguide material. Initially, we observe that the beam shape experiences irregular distortion, such that the periodicity of the self-collimation is lost [see Fig. 3(a)]. However, the beam does not broaden significantly, and it still experiences approximate self-restoration at some points.

When the input power increases further, the beam no longer experiences self-restoration,

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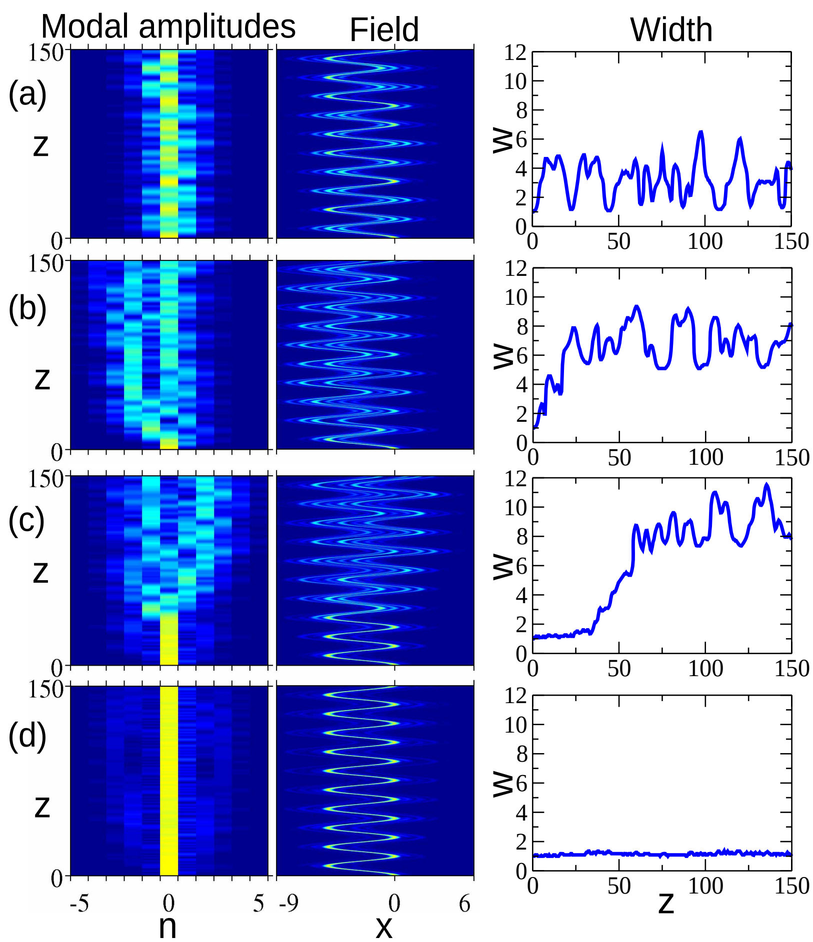

Fig. 3. (1.5MB) Nonlinear propagation of light in an array of periodically curved waveguides. Shown are the absolute values of the amplitudes of the modes of individual waveguides (left) and corresponding optical field patterns (center). Right: beam width w [normalized to the input width] as a function of the propagation distance. Waveguide array is the same as in Fig. 2(a). The input power is (a) $P/P_0 = 0.70$, (b) $P/P_0 = 1.7$, (c) $P/P_0 = 2.7$, and (d) $P/P_0 = 3.4$, where P_0 is the power required for the formation of one-site discrete soliton in the straight array. The animation shows the beam propagation dynamics and the output beam width as the input power increases from $P/P_0 = 0.00029$ to $P/P_0 = 3.4$.

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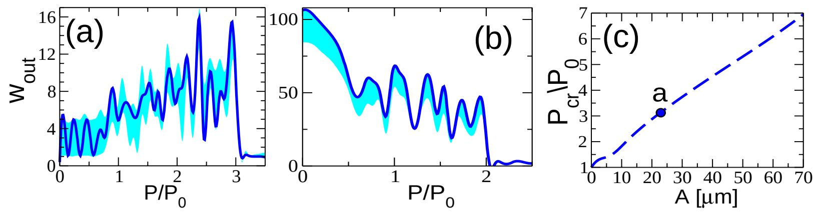

Fig. 4. (a, b) Output beam width vs. the input power. Waveguide arrays are the same as in Figs. 2(a) and 2(b), respectively. (c) Critical power $P_{\rm cr}/P_0$ required for the formation of a one-site lattice soliton in an array of periodically curved waveguides as a function of the bending amplitude A. Bending period and waveguide array length is the same as in (a) and (b). Points 'a' and 'b' correspond to the bending amplitudes in (a) and (b), respectively.

and the *nonlinear diffusion* takes place, where the beam experiences significant broadening and self-defocusing, as shown in Fig. 3(b). This self-defocusing is intrinsically limited due to the diffraction cancellation in the waveguide array. After propagation over some distance the beam broadens and its intensity is reduced accordingly. Therefore, the further beam spreading stops when the average beam width achieves a certain value. Such a peculiar nonlinear beam dynamics has no analogies in bulk media [11] or discrete systems [1] analyzed before.

At even higher input powers, transitional self-trapping of the beam is observed. The beam initially becomes self-trapped upon the launch into the array, but after propagation for some distance (which somewhat depends on the input power), it broadens rapidly and experiences again nonlinear diffusion [see Fig. 3(c)]. Finally, at some critical power we observe *a sharp transition from the nonlinear diffusion to the discrete self-trapping* over the whole length of the array, and the discrete lattice soliton is formed [see Fig. 3(d)].

From the practical point of view, the output width of the beam which exits the waveguide array is of the main interest. In Figs. 4(a) and 4(b) the beam width $w_{\rm out}$ at the output facet of the array is shown as a function of the input power P normalized to the power P_0 required for the formation of one-site discrete lattice soliton in the straight array, where $P_0 \simeq 5C_0/\gamma$ [20]. Shading shows the range between the minimum and the maximum beam widths during the propagation over the last two periods of the array [note that the maximum width in Fig. 4(b) is the same as the output width]. All the output, the minimum, and the maximum widths are normalized to the input beam width. In Fig. 4(a), one can clearly identify all the discussed above different nonlinear propagation regimes which happen in diffraction free waveguide arrays [compare this figure with the animation in Fig. 3]. In Fig. 4(b), where we take the bending amplitude to be half of that required for the self-collimation, the regime of the intermediate nonlinear beam diffusion can also be identified, however the critical power required for the formation of the lattice soliton is different. We find that this critical power is considerably higher in arrays of periodically curved waveguides than in straight waveguide arrays. It grows substantially with the magnitude of the bending amplitude, as shown in Fig. 4(c).

5. Conclusions

We have studied the nonlinear propagation of light beams in diffraction-managed photonic lattices. We have shown that the crossover between the regimes of the discrete diffraction and self-collimation of light, for low input powers, and nonlinear self-trapping, for high powers, occurs through a novel regime of nonlinear light diffusion. We have shown that the critical power required for the formation of lattice solitons can be controlled effectively by changing the amplitude of the waveguide bending.

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