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Nonlinear dynamic response of a simply supported rectangular functionally graded material plate under the time-dependent thermalmechanical loads †

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Abstract

An analysis on nonlinear dynamic characteristics of a simply supported functionally graded materials (FGMs) rectangular plate subjected to the transversal and in-plane excitations is presented in the time dependent thermal environment. Here we look the FGM Plates as isotropic materials which is assumed to be temperature dependent and graded in the thickness direction according to the power-law distribution in terms of volume fractions of the constituents. The geometrical nonlinearity using Von Karman's assumption is introduced. The formulation also includes in-plane and rotary inertia effects. In the framework of Reddy's third-order shear deformation plate theory, the governing equations of motion for the FGM plate are derived by the Hamilton's principle. Then the equations of motion with two-degree-of-freedom under combined the time-dependent thermomechanical loads can be obtained by using Galerkin's method. Using numerical method, the control equations are analyzed to obtain the response curves. Under certain conditions the periodic and chaotic motions of the FGM plate are found. It is found that because of the existence of the temperature which relate to the time the motions of the FGM plate show the great difference. A period motion can be changed into the chaotic motions which are affected by the time dependent temperature.

Keywords: Functionally graded materials; Rectangular plates; Chaotic motion; Higher-order theory; The time-dependent thermalmechanical loads

1. Introduction

Functionally graded material (FGM) used initially as thermal barrier materials for aerospace structural applications and fusion reactors are now developed for the general use as structural components in high temperature environments and being strongly considered as a potential structural material candidate for the design of high speed aerospace vehicles [1]. FGMs are multi-phase materials with the phase volume fractions varying gradually in space, in a pre-determined profile. This results in continuously graded thermomechanical properties at the macroscopic structural scale. One of the advantages of the FGM is that it can be able to withstand high-temperature-environments. Because of their specially-tailored thermomechanical properties, they are well suited for thermal protection against large temperature gradients [2, 3]. Due to this superior thermomechanical property, FGM plate structures have found a wide range of applications in many industries, especially in space vehicles and aircrafts, where they are very often subjected to high levels of thermal and dynamic loading, such as large temperature gradients and acoustic pressure. This may result in complicated large amplitude, nonlinear vibration behavior of the FGM plate due to the bending–stretching coupling and combined external loads [4]. With the increased use of these materials for structural components in many engineering applications, it is necessary for us to understand the nonlinear dynamic characteristics of functionally graded plates in thermal environments.

There are many studies for isotropic or laminated composite plate and shell structures such as Refs. [5-11]. Among the research about the nonlinear dynamic behaviors of the FGM plates under thermo-mechanical environment available, Praveen and Reddy [2] adopting finite element procedure analyzed the nonlinear dynamic response of functionally graded ceramic metal plates subjected to mechanical and thermal loads. Sundararajan [3] studied the free vibration characteristics of functionally graded material (FGM) plates subjected to thermal environment. Temperature field was assumed to be a uniform distribution over the plate surface and varied in the thickness direction. Yang et al. [4] presented the

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large amplitude vibration of pre-stressed functionally graded material laminated plates that were composed of a shear deformable functionally graded layer and two surface-mounted piezoelectric actuator layers. Nonlinear governing equations of motion were derived within the context of Reddy's higher-order shear deformation plate theory to account for transverse shear strain and rotary inertia. Cheng and Batra [12] studied the steady state vibration of a simply supported functionally graded polygonal plate with temperature independent material properties. He et al. [13] presented finite element formulation based on thin plate theory for the shape and vibration control of FGM plate with integrated piezoelectric sensors and actuators under mechanical load. The constituent materials possess temperature-dependent properties. Ng et al. [14] investigated the parametric resonance of plates based on the Hamilton's principle and the assumed mode technique. Yang and Shen [15, 16] analyzed dynamic response of thin FGM plates subjected to impulsive loads using Galerkin procedure coupled with modal superposition method whereas, by neglecting the heat conduction effect, and examined such plates and panels based on shear deformation with temperature dependent material properties. Sills et al. [17] presented different modeling aspects and also simulated the dynamic response under a step load. By adopting Laplace transformation technique and power series method, Vel and Batra [18] analyzed the three-dimensional thermomechanical deformations of simply supported functionally graded plates subjected to time-dependent thermal loads on its top or bottom surface. Based on perturbation technique, Huang and Shen [19] dealt with the nonlinear vibration and dynamic response of FGM plates in thermal environment. Heat conduction and temperature-dependent material properties were both considered. The temperature field considered was assumed to be a uniform distribution over the plate surface and varied in the thickness direction only. Kim and Noda [20] discussed transient displacement of FGM plates due to heat flux by a Green's function approach based on the classical laminated plate theory. Jacob [21] analyzed the steady-state response of a functionally graded thick cylindrical shell subjected to thermal and mechanical loads. The functionally graded shell was simply supported at the edges and it is assumed to have an arbitrary variation of material properties in the radial direction. Reddy and Cheng [22] used the method of asymptotic expansion to study the three-dimensional thermoelastic deformations of functionally graded elliptic and rectangular plates. Qian and Batra [23] obtained results for the steady-state and transient thermoelastic response of functionally graded plates by the meshless local Petrov–Galerkin method.

However to the authors' knowledge, the studies of the bifurcation and chaos for the FGM plates under the time-dependent thermomechanical loads have been given quite a few investigations. Since the magnitudes of transient thermal stresses are usually larger than those of steady state stresses, it is important to quantify them for proper design of an FGM plate.

This paper aim focuses on a simply supported at the four-edge FGM rectangular plate subjected to in-plane and transversal excitation simultaneously in the time dependent thermal environment. Here we look the FGM Plates as isotropic materials which is assumed to be temperature-dependent and graded in the thickness direction according to the power-law distribution in terms of volume fractions of the constituents. The geometrical nonlinearity using Von Karman's assumption is introduced. The formulation also includes in-plane and rotary inertia effects. In the framework of Reddy's third-order shear deformation plate theory [24-27], the governing equations of motion for the FGM plate are derived by the Hamilton's principle. Then the equations of motion with two-degree-of-freedom under combined the time-dependent thermomechanical loads can be obtained by using Galerkin's method. Using numerical method, the control equations are analyzed to obtain the response curves. Under certain conditions the chaotic motions of the FGM plates are found. It is found that there exist different kinds of chaotic motions in the FGM plate.

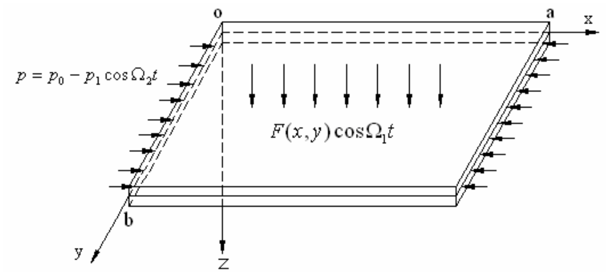

Fig. 1. The model of a FGMs rectangular plate and the coordinate system.

2. Formulation

An simply supported at the four-edges FGM rectangular plate subjected to in-plane and transversal excitations is considered, as shown in Fig. 1. The edge width and length of the FGM rectangular plate in the x and y directions are respectively a and b and the thickness is h. A Cartesian coordinate $Oxyz$ is located in the middle surface of the FGM rectangular plate. Assume that (u, v, w) and (u_0, v_0, w_0) represent the displacements of an arbitrary point and a point in the middle surface of the FGM plate in the x, y, and z directions, respectively. It is also assumed that ϕ_x and ϕ_y respectively represent the mid-plane rotations of two transverse normals about the x and y axes. The in-plane excitation of the FGM plate is distributed along the y direction at $x=0$ and $x=a$ and is of the form $P_0 - P_1\cos\Omega_2 t$. The transversal excitation subject to the FGM plate is represented by $F(x,y)\cos\Omega_1 t$. Here Ω_1, Ω_2 are the frequencies of the in-plane excitation and transversal excitation, respectively.

2.1 FGM material properties

Generally speaking, most of the FGM are employed in high-temperature environments and many of the constituent materials may possess temperature-dependent properties. We assume that the temperature variation occurs with non-uniform in-plane and the steady state temperature distribution along the thickness of the plate.

It is also supposed that the FGM plate is linear elastic throughout the deformation, and that the plate is initially stress free at T_0 and is subjected to a non-uniform temperature variation $\Delta T = T - T_0$.

The time-dependent thermal field contains two separable functions for the transient temperature variation and the spatial temperature distribution, respectively, i.e.,

$$T(x,y,z,t) = T_2 + T_1(x,y)\cos\Omega_3 t. \tag{1}$$

It is assumed that the plate is made from a mixture of the ceramics and metals with continuously varying such that the top surface of the plate is ceramic rich, whereas the bottom surface is metal rich. The material properties P such as Young's modulus E, the coefficient of thermal expansion α, can be expressed as a function of the temperature T_2, see Refs. [28, 29], as

$$P_i = P_0\left(P_{-1}T_2^{-1} + 1 + P_1T_2 + P_2T_2^2 + P_3T_2^3\right) \tag{2}$$

where P_0, P_{-1}, P_1, P_2 and P_3 are temperature coefficients.

The effective material properties P of the FGM can be expressed as

$$P = P_tV_c + P_bV_m \tag{3}$$

where subscript 't' and 'b' respectively represent the top and bottom surfaces of the FGM plate, V_c and V_m are the ceramic and metal volume fractions and add to unity

$$V_c + V_m = 1. \tag{4}$$

The metal volume fraction V_m is defined as

$$V_m(z) = \left(\frac{2z+h}{2h}\right)^N \tag{5}$$

where power law exponent N is a real number which characterizes the ceramic variation profile through the plate thickness. From Eqs. (2)-(4), the Young's modulus E, the coefficient of the thermal expansion α, the mass density ρ can be expressed as

$$E = (E_b - E_t)V_m + E_t,\ \ \alpha = (\alpha_b - \alpha_t)V_m + \alpha_t,$$
$$\rho = (\rho_b - \rho_t)V_m + \rho_t. \tag{6}$$

2.2 Theoretical equations

According to the Reddy's third-order shear deformation theory [24-27, 30] and the Hamilton's principle, the nonlinear governing equations of motion for the FGM rectangular plate are given as

$$N_{xx,x} + N_{xy,y} = I_0\ddot{u}_0 + (I_1 - c_1I_3)\ddot{\phi}_x - c_1I_3\frac{\partial\ddot{w}_0}{\partial x}, \tag{7a}$$

$$N_{yy,y} + N_{xy,x} = I_0\ddot{v}_0 + (I_1 - c_1I_3)\ddot{\phi}_y - c_1I_3\frac{\partial\ddot{w}_0}{\partial y}, \tag{7b}$$

$$\begin{aligned}
&N_{yy,y}\frac{\partial w_0}{\partial y} + N_{yy}\frac{\partial^2 w_0}{\partial y^2} + N_{xy,x}\frac{\partial w_0}{\partial y} + N_{xy,y}\frac{\partial w_0}{\partial x}\\
&+N_{xx,x}\frac{\partial w_0}{\partial x} + N_{xx}\frac{\partial^2 w_0}{\partial x^2} + c_1\left(P_{xx,xx} + 2P_{xy,xy} + P_{yy,yy}\right)\\
&+2N_{xy}\frac{\partial^2 w_0}{\partial y\partial x} = I_0\ddot{w}_0 + c_1I_3\left(\frac{\partial\ddot{u}_0}{\partial x} + \frac{\partial\ddot{v}_0}{\partial x}\right)\\
&+c_1(I_4 - c_1I_6)\left(\frac{\partial\ddot{\phi}_x}{\partial x} + \frac{\partial\ddot{\phi}_y}{\partial y}\right) - c_1I_6\left(\frac{\partial^2\ddot{w}_0}{\partial x^2} + \frac{\partial^2\ddot{w}_0}{\partial y^2}\right),
\end{aligned} \tag{7c}$$

$$\begin{aligned}
&M_{xx,x} + M_{xy,y} - c_1P_{xx,x} - c_1P_{xy,y} - (Q_x - c_2R_x)\\
&= (I_1 - c_1I_3)\ddot{u}_0 + \left(I_2 - 2c_1I_4 + c_1^2I_6\right)\ddot{\phi}_x - c_1(I_4 - c_1I_6)\frac{\partial\ddot{w}_0}{\partial x},
\end{aligned} \tag{7d}$$

$$\begin{aligned}
&M_{yy,y} + M_{xy,x} - c_1P_{yy,y} - c_1P_{xy,x} - (Q_y - c_2R_y)\\
&= (I_1 - c_1I_3)\ddot{v}_0 + \left(I_2 - 2c_1I_4 + c_1^2I_6\right)\ddot{\varphi}_y - c_1(I_4 - c_1I_6)\frac{\partial\ddot{w}_0}{\partial x}
\end{aligned} \tag{7e}$$

where the stress resultants are given as follows:

$$\begin{Bmatrix} N_{xx}\\ N_{yy}\\ N_{xy}\end{Bmatrix} = \{[A][B][E]\}\begin{Bmatrix}\varepsilon^0\\ \varepsilon^1\\ \varepsilon^3\end{Bmatrix} + \begin{Bmatrix} N_{xx}^T\\ N_{yy}^T\\ N_{xy}^T\end{Bmatrix},$$

$$\begin{Bmatrix} M_{xx}\\ M_{yy}\\ M_{xy}\end{Bmatrix} = \{[B][D][F]\}\begin{Bmatrix}\varepsilon^0\\ \varepsilon^1\\ \varepsilon^3\end{Bmatrix} + \begin{Bmatrix} M_{xx}^T\\ M_{yy}^T\\ M_{xy}^T\end{Bmatrix},$$

$$\begin{Bmatrix} P_{xx}\\ P_{yy}\\ P_{xy}\end{Bmatrix} = \{[E][F][H]\}\begin{Bmatrix}\varepsilon^0\\ \varepsilon^1\\ \varepsilon^3\end{Bmatrix} + \begin{Bmatrix} P_{xx}^T\\ P_{yy}^T\\ P_{xy}^T\end{Bmatrix},$$

$$\begin{Bmatrix} Q_y\\ Q_x\end{Bmatrix} = \{[A][D]\}\begin{Bmatrix}\gamma^0\\ \gamma^2\end{Bmatrix},\ \ \begin{Bmatrix} R_y\\ R_x\end{Bmatrix} = \{[D][F]\}\begin{Bmatrix}\gamma^0\\ \gamma^2\end{Bmatrix}. \tag{8}$$

From Eq. (8), it is known that the thermal stress resultants are represented as

$$\begin{Bmatrix} N_{xx}^T\\ N_{yy}^T\\ N_{xy}^T\end{Bmatrix} = \int_{-h/2}^{h/2}\begin{bmatrix} Q_{11} & Q_{12} & Q_{16}\\ Q_{21} & Q_{22} & Q_{26}\\ Q_{61} & Q_{62} & Q_{66}\end{bmatrix}\left[-\begin{Bmatrix}\alpha\\ \alpha\\ 0\end{Bmatrix}\Delta T\right]dz,$$

$$\begin{Bmatrix} M_{xx}^T \\ M_{yy}^T \\ M_{xy}^T \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \left[-\begin{Bmatrix} \alpha \\ \alpha \\ 0 \end{Bmatrix} z\Delta T \right] dz ,$$

$$\begin{Bmatrix} P_{xx}^T \\ P_{yy}^T \\ P_{xy}^T \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \left[-\begin{Bmatrix} \alpha \\ \alpha \\ 0 \end{Bmatrix} z^3\Delta T \right] dz . \qquad (9)$$

Substituting the stress resultants of Eq. (9) into Eq. (7), we can write Eq. (7) in terms of generalized displacements $\left(u_0, v_0, w_0, \phi_x, \phi_y\right)$

$$A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{66}\frac{\partial^2 u_0}{\partial y^2} + \left(A_{12} + A_{66}\right)\frac{\partial^2 v_0}{\partial x \partial y}$$
$$+\left(B_{11} + c_1 E_{11}\right)\frac{\partial^2 \phi_x}{\partial x^2} + \left(B_{66} + c_1 E_{66}\right)\frac{\partial^2 \phi_x}{\partial y^2}$$
$$+\left(B_{12} - c_1 E_{12} + B_{66} - c_1 E_{66}\right)\frac{\partial^2 \phi_y}{\partial x \partial y}$$
$$+A_{11}\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2} + A_{66}\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2} + \left(A_{12} + A_{66}\right)\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial x \partial y}$$
$$-c_1 E_{11}\frac{\partial^3 w_0}{\partial x^3} - c_1\left(E_{12} + 2E_{66}\right)\frac{\partial^3 w_0}{\partial x \partial y^2} - N_{xx1,x}^T$$
$$= I_0 \ddot{u}_0 + \left(I_1 - c_1 I_3\right)\ddot{\phi}_x - c_1 I_3 \frac{\partial \ddot{w}_0}{\partial x} \qquad (10a)$$

$$A_{66}\frac{\partial^2 v_0}{\partial x^2} + A_{22}\frac{\partial^2 v_0}{\partial y^2} + \left(A_{21} + A_{66}\right)\frac{\partial^2 u_0}{\partial x \partial y}$$
$$+\left(B_{66} + c_1 E_{66}\right)\frac{\partial^2 \phi_y}{\partial x^2} + \left(B_{22} + c_1 E_{22}\right)\frac{\partial^2 \phi_y}{\partial y^2}$$
$$+\left(B_{21} - c_1 E_{21} + B_{66} - c_1 E_{66}\right)\frac{\partial^2 \phi_x}{\partial x \partial y} + A_{66}\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial x^2}$$
$$+A_{22}\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y^2} + \left(A_{21} + A_{66}\right)\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x \partial y}$$
$$-c_1 E_{22}\frac{\partial^3 w_0}{\partial y^3} - c_1\left(E_{12} + 2E_{66}\right)\frac{\partial^3 w_0}{\partial x^2 \partial y} - N_{yy1,y}^T$$
$$= I_0 \ddot{v}_0 + \left(I_1 - c_1 I_3\right)\ddot{\phi}_y - c_1 I_3 \frac{\partial \ddot{w}_0}{\partial y} \qquad (10b)$$

$$A_{21}\frac{\partial u_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2} + A_{11}\frac{\partial u_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2} + 2A_{66}\frac{\partial u_0}{\partial y}\frac{\partial^2 w_0}{\partial x \partial y}$$
$$+\left(A_{21} + A_{66}\right)\frac{\partial^2 u_0}{\partial x \partial y}\frac{\partial w_0}{\partial y} + A_{11}\frac{\partial^2 u_0}{\partial x^2}\frac{\partial w_0}{\partial x}$$
$$+A_{66}\frac{\partial^2 u_0}{\partial y^2}\frac{\partial w_0}{\partial x} + c_1 E_{11}\frac{\partial^3 u_0}{\partial x^3} + c_1\left(E_{21} + 2E_{66}\right)\frac{\partial^3 u_0}{\partial x \partial y^2}$$
$$+2A_{66}\frac{\partial v_0}{\partial x}\frac{\partial^2 w_0}{\partial x \partial y} + A_{22}\frac{\partial v_0}{\partial y}\frac{\partial^2 w_0}{\partial y^2} + \left(A_{12} + A_{66}\right)\frac{\partial^2 v_0}{\partial x \partial y}\frac{\partial w_0}{\partial x}$$
$$+A_{12}\frac{\partial v_0}{\partial y}\frac{\partial^2 w_0}{\partial x^2} + A_{22}\frac{\partial^2 v_0}{\partial y^2}\frac{\partial w_0}{\partial y} + A_{66}\frac{\partial^2 v_0}{\partial x^2}\frac{\partial w_0}{\partial y}$$
$$+c_1 E_{22}\frac{\partial^3 v_0}{\partial y^3} + c_1\left(E_{12} + 2E_{66}\right)\frac{\partial^3 v_0}{\partial y \partial x^2} + 2c_1 E_{66}\frac{\partial^2 w_0}{\partial x \partial y}\frac{\partial^2 w_0}{\partial y^2}$$
$$+\left(A_{55} + c_2^2 F_{55} - 2c_2 D_{55}\right)\frac{\partial^2 w_0}{\partial x^2} + \left(c_1 F_{11} - c_1^2 H_{11}\right)\frac{\partial^3 \phi_x}{\partial x^3}$$
$$+\left(A_{44} + c_2^2 F_{44} - 2c_2 D_{44}\right)\frac{\partial^2 w_0}{\partial y^2} - c_1^2 H_{22}\frac{\partial^4 w_0}{\partial y^4}$$
$$+2c_1\left(E_{66} - E_{12}\right)\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2} + 2c_1\left(E_{21} - 2E_{66}\right)\left(\frac{\partial^2 w_0}{\partial x \partial y}\right)^2$$
$$+2\left(A_{21} + 2A_{66}\right)\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x \partial y}\frac{\partial w_0}{\partial y} + \frac{3}{2}A_{22}\left(\frac{\partial w_0}{\partial y}\right)^2\frac{\partial^2 w_0}{\partial y^2}$$
$$+\left(\frac{1}{2}A_{21} + A_{66}\right)\left(\frac{\partial w_0}{\partial x}\right)^2\frac{\partial^2 w_0}{\partial y^2} - c_1^2 H_{11}\frac{\partial^4 w_0}{\partial x^4}$$
$$+\left(A_{66} + \frac{1}{2}A_{21}\right)\left(\frac{\partial w_0}{\partial y}\right)^2\frac{\partial^2 w_0}{\partial x^2} + \frac{3}{2}A_{11}\left(\frac{\partial w_0}{\partial x}\right)^2\frac{\partial^2 w_0}{\partial x^2}$$
$$-2c_1^2\left(H_{21} - 2H_{66}\right)\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + \left(B_{21} - c_1 E_{21}\right)\frac{\partial \phi_x}{\partial x}\frac{\partial^2 w_0}{\partial y^2}$$
$$+\left(-2c_2 D_{55} + c_2^2 F_{55} + A_{55}\right)\frac{\partial \phi_x}{\partial x} + \left(c_1 F_{22} - c_1^2 H_{22}\right)\frac{\partial^3 \phi_y}{\partial y^3}$$
$$+\left(B_{21} + B_{66} - c_1 E_{21} - c_1 E_{66}\right)\frac{\partial^2 \phi_x}{\partial x \partial y}\frac{\partial w_0}{\partial y}$$
$$+2\left(B_{66} - c_1 E_{66}\right)\frac{\partial \phi_x}{\partial y}\frac{\partial^2 w_0}{\partial x \partial y} + \left(B_{11} - c_1 E_{11}\right)\frac{\partial^2 \phi_x}{\partial x^2}\frac{\partial w_0}{\partial x}$$
$$+\left(B_{11} - c_1 E_{11}\right)\frac{\partial \phi_x}{\partial x}\frac{\partial^2 w_0}{\partial x^2} + \left(B_{66} - c_1 E_{66}\right)\frac{\partial^2 \phi_x}{\partial y^2}\frac{\partial w_0}{\partial x}$$
$$+c_1\left(F_{21} + 2F_{66} - c_1 H_{21} - 2c_1 H_{66}\right)\frac{\partial^3 \phi_x}{\partial x \partial y^2}$$
$$+\left(-2c_2 D_{44} + A_{44} + c_2^2 F_{44}\right)\frac{\partial \phi_y}{\partial y} + \left(B_{22} - c_1 E_{22}\right)\frac{\partial \phi_y}{\partial y}\frac{\partial^2 w_0}{\partial y^2}$$
$$+2\left(B_{66} - c_1 E_{66}\right)\frac{\partial \phi_y}{\partial x}\frac{\partial^2 w_0}{\partial x \partial y} + \left(B_{22} - c_1 E_{22}\right)\frac{\partial^2 \phi_y}{\partial y^2}\frac{\partial w_0}{\partial y}$$
$$+\left(B_{66} - c_1 E_{66}\right)\frac{\partial^2 \phi_y}{\partial x^2}\frac{\partial w_0}{\partial y} + \left(B_{12} - c_1 E_{12}\right)\frac{\partial \phi_y}{\partial y}\frac{\partial^2 w_0}{\partial x^2}$$
$$+\left(B_{12} + B_{66} - c_1 E_{12} - c_1 E_{66}\right)\frac{\partial^2 \phi_y}{\partial x \partial y}\frac{\partial w_0}{\partial x}$$
$$+c_1\left(F_{12} + 2F_{66} - c_1 H_{12} - 2c_1 H_{66}\right)\frac{\partial^3 \phi_y}{\partial x^2 \partial y} - N_{xx}^T\frac{\partial^2 w_0}{\partial x^2}$$
$$-N_{yy}^T\frac{\partial^2 w_0}{\partial y^2} + F\cos\Omega_1 t - N_{yy1,y}^T\frac{\partial w_0}{\partial y} - N_{yy1}^T\frac{\partial^2 w_0}{\partial y^2}$$
$$-N_{xx1,x}^T\frac{\partial w_0}{\partial x} - N_{xx1}^T\frac{\partial^2 w_0}{\partial x^2} - c_1\left(P_{xx1,xx}^T + P_{yy1,yy}^T\right) - \gamma\frac{\partial w_0}{\partial t}$$

$$= I_0\ddot{w}_0 - c_1^2 I_6\left(\frac{\partial^2\ddot{w}_0}{\partial x^2}+\frac{\partial^2\ddot{w}_0}{\partial y^2}\right) + c_1 I_3\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y}\right)$$

$$+c_1\left(I_4 - c_1 I_6\right)\left(\frac{\partial\ddot{\phi}_x}{\partial x}+\frac{\partial\ddot{\phi}_y}{\partial y}\right) \quad (10c)$$

$$+\left(B_{66}-c_1E_{66}\right)\frac{\partial^2 u_0}{\partial y^2}+\left(B_{12}+B_{66}-c_1E_{12}-c_1E_{66}\right)\frac{\partial^2 v_0}{\partial x\partial y}$$

$$+\left(B_{11}-c_1E_{11}\right)\frac{\partial^2 u_0}{\partial x^2}+\left(B_{66}-c_1E_{66}\right)\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2}$$

$$+\left(B_{11}-c_1E_{11}\right)\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2}+\left(-c_1F_{11}+c_1^2H_{11}\right)\frac{\partial^3 w_0}{\partial x^3}$$

$$+\left(B_{12}+B_{66}-c_1E_{12}-c_1E_{66}\right)\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial x\partial y}$$

$$+c_1\left(-F_{12}-2F_{66}+c_1H_{12}+2c_1H_{66}\right)\frac{\partial^3 w_0}{\partial x\partial y^2}$$

$$-\left(A_{55}-2c_2D_{55}+c_2^2F_{55}\right)\frac{\partial w_0}{\partial x}$$

$$+\left(D_{11}-2c_1F_{11}+c_1^2H_{11}\right)\frac{\partial^2\phi_x}{\partial x^2}-M_{xx1,x}^T-c_1P_{xx1,x}^T$$

$$+\left(D_{66}-2c_1F_{66}+c_1^2H_{66}\right)\frac{\partial^2\phi_x}{\partial y^2}-\left(A_{55}-2c_2D_{55}+c_2^2F_{55}\right)\phi_x$$

$$+\left(D_{12}-2c_1F_{12}-2c_1F_{66}+D_{66}+c_1^2H_{12}+c_1^2H_{66}\right)\frac{\partial^2\phi_y}{\partial x\partial y}$$

$$=\left(I_1-c_1I_3\right)\ddot{u}_0+\left(I_2-2c_1I_4+c_1I_6\right)\ddot{\phi}_x-c_1\left(I_4-c_1I_6\right)\frac{\partial\ddot{w}_0}{\partial x} \quad (10d)$$

$$\left(B_{66}-c_1E_{66}\right)\frac{\partial^2 v_0}{\partial x^2}+\left(B_{21}+B_{66}-c_1E_{21}-c_1E_{66}\right)\frac{\partial^2 u_0}{\partial x\partial y}$$

$$+\left(B_{22}-c_1E_{22}\right)\frac{\partial^2 v_0}{\partial y^2}+\left(B_{66}-c_1E_{66}\right)\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial x^2}$$

$$+\left(B_{21}+B_{66}-c_1E_{21}-c_1E_{66}\right)\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial y}$$

$$+\left(B_{22}-c_1E_{22}\right)\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y^2}+\left(-c_1F_{22}+c_1^2H_{22}\right)\frac{\partial^3 w_0}{\partial y^3}$$

$$+c_1\left(-F_{21}-2F_{66}+c_1H_{21}+2c_1H_{66}\right)\frac{\partial^3 w_0}{\partial x^2\partial y}$$

$$+\left(D_{66}-2c_1F_{66}+c_1^2H_{66}\right)\frac{\partial^2\phi_y}{\partial x^2}$$

$$-\left(A_{44}-2c_2D_{44}+c_2^2F_{44}\right)\frac{\partial w_0}{\partial y}$$

$$+\left(D_{21}-2c_1F_{21}-2c_1F_{66}+D_{66}+c_1^2H_{21}+c_1^2H_{66}\right)\frac{\partial^2\phi_x}{\partial x\partial y}$$

$$+\left(D_{22}-2c_1F_{22}+c_1^2H_{22}\right)\frac{\partial^2\phi_y}{\partial y^2}$$

$$-\left(A_{44}-2c_2D_{44}+c_2^2F_{44}\right)\phi_y-M_{xx1,x}^T-c_1P_{xx1,x}^T\,,$$

$$=\left(I_1-c_1I_3\right)\ddot{v}_0+\left(I_2-2c_1I_4+c_1I_6\right)\ddot{\phi}_y-c_1\left(I_4-c_1I_6\right)\frac{\partial\ddot{w}_0}{\partial y} \quad (10e)$$

where γ is the damping coefficient, A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, and H_{ij} respectively are the stiffness elements of the FGM plate, which are denoted as

$$\left(A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},H_{ij}\right)=\int_{-h/2}^{h/2}Q_{ij}\left(1,z,z^2,z^3,z^4,z^6\right)dz\,,$$

$$\left(i,j=1,2,6\right) \quad (11)$$

$$\left(A_{ij},D_{ij},F_{ij}\right)=\int_{-h/2}^{h/2}Q_{ij}\left(1,z^2,z^4\right)dz\,,\ \left(i,j=4,5\right). \quad (12)$$

All kinds of inertias in Eq. (10) are calculated by

$$I_i=\int_{-h/2}^{h/2}z^i\rho(z)dz\,,\ \left(i=0,1,2,3,4,6\right) \quad (13)$$

$$\begin{Bmatrix}N_{xx1}^T\\N_{yy1}^T\\N_{xy1}^T\end{Bmatrix}=T_1\left(x,y\right)\cos\Omega_3t\int_{-h/2}^{h/2}\begin{bmatrix}Q_{11}&Q_{12}&0\\Q_{21}&Q_{22}&0\\0&0&Q_{66}\end{bmatrix}\left[-\begin{Bmatrix}\alpha\\\alpha\\0\end{Bmatrix}\right]dz$$

$$=T_1\left(x,y\right)\cos\Omega_3t\begin{bmatrix}\tilde{L}_{xx1}\\\tilde{L}_{yy1}\\\tilde{L}_{xy1}\end{bmatrix},$$

$$\begin{Bmatrix}M_{xx1}^T\\M_{yy1}^T\\M_{xy1}^T\end{Bmatrix}=T_1\left(x,y\right)\int_{-h/2}^{h/2}\begin{bmatrix}Q_{11}&Q_{12}&0\\Q_{21}&Q_{22}&0\\0&0&Q_{66}\end{bmatrix}\left[-\begin{Bmatrix}\alpha\\\alpha\\0\end{Bmatrix}z\right]dz$$

$$=T_1\left(x,y\right)\cos\Omega_3t\begin{bmatrix}\widehat{L}_{xx1}\\\widehat{L}_{yy1}\\\widehat{L}_{xy1}\end{bmatrix},$$

$$\begin{Bmatrix}P_{xx1}^T\\P_{yy1}^T\\P_{xy1}^T\end{Bmatrix}=T_1\left(x,y\right)\cos\Omega_3t\int_{-h/2}^{h/2}\begin{bmatrix}Q_{11}&Q_{12}&0\\Q_{21}&Q_{22}&0\\0&0&Q_{66}\end{bmatrix}\left[-\begin{Bmatrix}\alpha\\\alpha\\0\end{Bmatrix}z^3\right]dz$$

$$=T_1\left(x,y\right)\cos\Omega_3t\begin{bmatrix}\overline{L}_{xx1}\\\overline{L}_{yy1}\\\overline{L}_{xy1}\end{bmatrix}. \quad (14)$$

The simply supported boundary conditions can be expressed as

$$\text{at } x=0 \text{ and } x=a\,,\ w=\phi_y=M_{xx}=N_{xy}=0 \quad (15a)$$

$$\text{at } y=0 \text{ and } y=b\,,\ w=\phi_x=M_{yy}=N_{xy}=0 \quad (15b)$$

$$\int_0^b N_{xx}\Big|_{x=0,\,a}dy=\int_0^b\left(P_0+P_1\cos\Omega_2t\right)\Big|_{x=0,\,a}dy. \quad (15c)$$

In order to obtain the dimensionless equations, we introduce the transformations of the variables and parameters

$$\bar{u}=\frac{u_0}{a},\ \bar{v}=\frac{v_0}{b},\ \bar{w}=\frac{w_0}{h},\ \bar{\phi}_x=\phi_x,\ \bar{\phi}_y=\phi_y,\ \bar{x}=\frac{x}{a},$$

$$\bar{y}=\frac{y}{b},\ \bar{P}_1=\frac{b^2}{Eh^3}P_1,\ \bar{F}=\frac{(ab)^{7/2}}{\pi^4 Eh^7}F,\ \bar{\gamma}=\frac{(ab)^2}{\pi^2 h^4}\left(\frac{1}{\rho E}\right)^{1/2}\gamma,$$

$$\bar{P}_0=\frac{b^2}{Eh^3}P_0,\ \bar{T}_1=\frac{T_1}{T_0},\ \bar{\Omega}_i=\frac{1}{\pi^2}\left(\frac{ab\rho}{E}\right)^{1/2}\Omega_i\ (i=1,2),$$

$$\bar{t}=\pi^2\left(\frac{E}{ab\rho}\right)^{1/2}t,\ \bar{A}=\frac{(ab)^{1/2}}{Eh^2}A,\ \bar{B}=\frac{(ab)^{1/2}}{Eh^3}B,$$

$$\bar{D}=\frac{(ab)^{1/2}}{Eh^4}D,\ \bar{E}=\frac{(ab)^{1/2}}{Eh^5}E,\ \bar{F}=\frac{(ab)^{1/2}}{Eh^6}F,$$

$$\bar{H}=\frac{(ab)^{1/2}}{Eh^8}H,\ \bar{I}_i=\frac{1}{(ab)^{(i+1)/2}\rho}I_i. \tag{16}$$

We mainly consider transverse nonlinear oscillations of the FGM rectangular plate in the first two modes. It is our desirable to choose a suitable mode function to satisfy the first two modes of transverse nonlinear oscillations and the boundary conditions for the FGM rectangular plates. Thus, we write the w as follows:

$$\bar{w}=\bar{w}_1\sin\frac{\pi\bar{x}}{a}\sin\frac{3\pi\bar{y}}{b}+\bar{w}_2\sin\frac{3\pi\bar{x}}{a}\sin\frac{\pi\bar{y}}{b} \tag{17}$$

where $\bar{w}_1$ and $\bar{w}_2$ are the amplitudes of two modes, respectively.

The transverse excitation can be represented as

$$\bar{F}(\bar{x},\bar{y})=\bar{F}_1\sin\frac{\pi\bar{x}}{a}\sin\frac{3\pi\bar{y}}{b}+\bar{F}_2\sin\frac{3\pi\bar{x}}{a}\sin\frac{\pi\bar{y}}{b} \tag{18}$$

where $\bar{F}_1$ and $\bar{F}_2$ represent the amplitude of the transverse forcing excitation .

The time dependent temperature field is defined as

$$\bar{T}_1(\bar{x},\bar{y})=\bar{T}_{11}\sin\frac{\pi\bar{x}}{a}\sin\frac{3\pi\bar{y}}{b}+\bar{T}_{12}\sin\frac{3\pi\bar{x}}{a}\sin\frac{\pi\bar{y}}{b} \tag{19}$$

where $\bar{T}_{11}$ and $\bar{T}_{12}$ represent the amplitude of the temperature field .

For simplicity we drop the overbars in the following analysis. Based on research given in Refs. [31, 32], neglecting all inertia terms on u, v, ϕ_x and ϕ_y in Eq. (10) and the term of time dependent temperature stress in Eqs. (16a), (16b), (16d) and (16e), substituting Eq. (17) into Eqs. (10a), (10b), (10d) and (10e), we obtain the displacements u, v, ϕ_x and ϕ_y with respect to w. Substituting Eqs. (17), (18) and (19) into Eq. (10c) and applying the Galerkin procedure yield the governing differential equation of transverse motion of the FGM rectangular plate for the dimensionless as follows:

$$\begin{aligned}&\ddot{w}_1+a_0w_1+a_1\dot{w}_1+a_2w_1\cos\Omega_2t+a_3w_1^2+a_4w_2^2\\&+a_5w_1w_2^2+a_6w_1^3+a_7w_1w_2\\&+\left[(a_8T_{11}+a_9T_{12})w_1+(a_{10}T_{11}+a_{11}T_{12})w_2+a_{12}T_{11}\right]\cos\Omega_3t\\&=f_1\cos\Omega_1t\end{aligned} \tag{20a}$$

$$\begin{aligned}&\ddot{w}_2+b_0w_2+b_1\dot{w}_2+b_2w_2\cos\Omega_2t+b_3w_1w_2+b_4w_1^2\\&+b_5w_2^2+b_6w_2w_1^2+b_7w_2^3\\&+\left[(b_8T_{11}+b_9T_{12})w_1+(b_{10}T_{11}+b_{11}T_{12})w_2+b_{12}T_{12}\right]\cos\Omega_3t\\&=f_2\cos\Omega_1t.\end{aligned} \tag{20b}$$

All coefficients aforementioned in Eq. (20) are too long to be listed out in the paper for abbreviation.

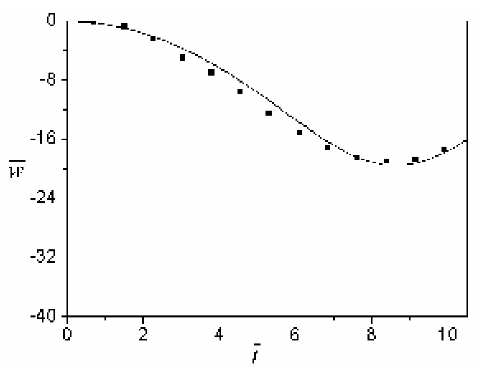

Fig. 2. Comparison of temporal evolution of center transverse deflection obtained by present results(—) and that published in Ref. [36] (■ read from graph).

3. Numerical simulations of periodic and chaotic motions

Before proceeding to the nonlinear vibration analysis of the FGM plates, a comparison example is solved to validate the present analysis firstly. The transient response results are compared in Fig. 2 with the finite element solutions provided by Reddy [33], where the temporal evolution curves of center deflection are presented for a simply supported intact aluminum-zirconia FGM square plate ($a=b=0.2m$, $h=0.01m$) under a suddenly applied uniform load of intensity of $q_0=1MPa$.

The material composition is assumed to follow a simple power-law distribution through the thickness direction such that the plate is 100% zirconia ($E_t=151GPa$, $\rho_t=3000kg/m^3$) at the top surface and 100% aluminum ($E_b=70GPa$, $\rho_b=2707kg/m^3$) at the bottom surface. The power-law exponent and the Poisson's ratio are taken as $n=0.2$ and $\nu=0.3$, respectively. The dimensionless center deflection and dimensionless time are defined as $\bar{w}=w_cE_bh/(q_0a^2)$ and $\bar{t}=t\sqrt{E_b/(\rho ba^2)}$, respectively. Our results agree well with the finite element results.

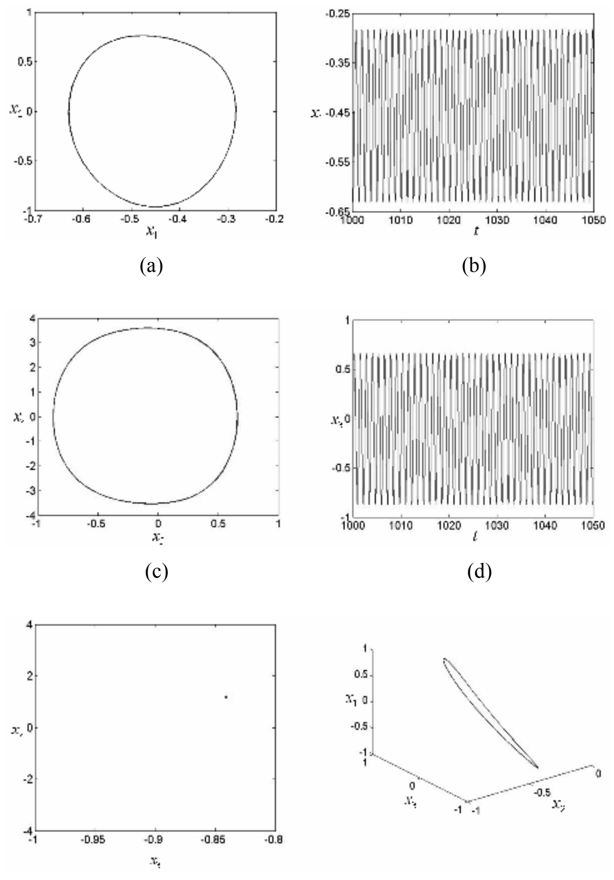

Fig. 3. The periodic motion of the FGMs rectangular plate exists when $T_{11}=0$, $T_{12}=0$.

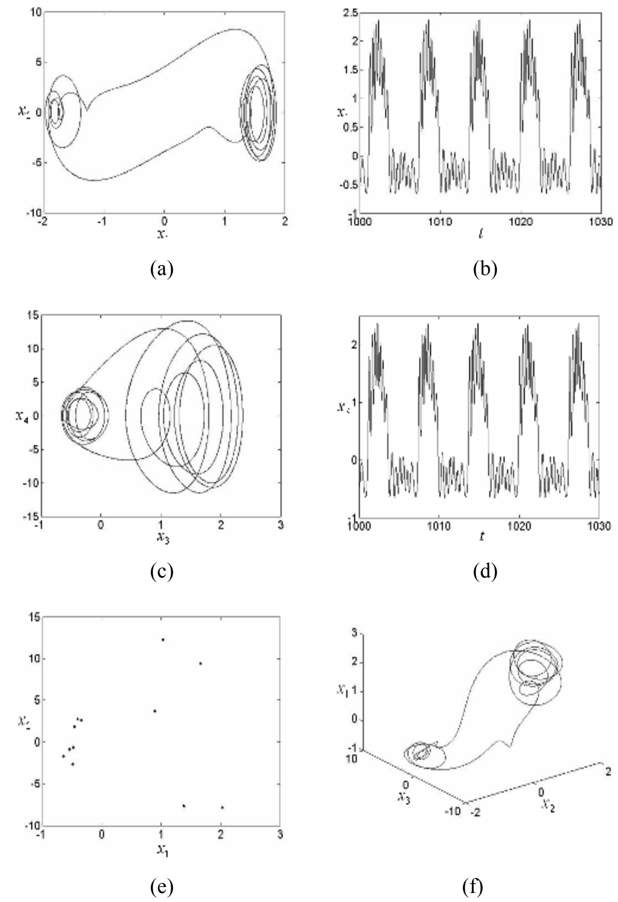

Fig. 4. The periodic motion of the FGMs rectangular plate exists when $T_{11}=11.8$, $T_{12}=11.8$.

In the following investigation, the Runge-Kutta algorithm [34] is utilized to numerically analyze the periodic and chaotic motions of the FGMs rectangular plate subjected to time dependent thermal and mechanical loads. We consider the Eq. (20) to do numerical simulation. To study the thermal effect on nonlinear dynamic response, we choose the time dependent temperature T_{11} and T_{12} as the controlling parameters when the periodic and chaotic responses of the FGM rectangular plate are investigated. At the same time, we consider the governing equations of motion for the FGM plate without the terms of the time dependent temperature to do numerical simulation too as a comparison studies. The zirconia and titanium alloy are selected for the two constituent materials of the plate in the present examples, referred to as ZrO2/Ti-6Al-4V shown in Ref. [35]. The two-dimensional phase portrait, waveform, three-dimensional phase portrait are plotted to demonstrate the nonlinear dynamic behaviors of the FGMs rectangular plate.

Fig. 3 illustrates the existence of the periodic motion for the FGMs rectangular plate when the governing equations of motion for the FGM plate don't include the terms of the time dependent temperature. Obviously, Fig. 3 illustrates that the periodic response of the FGM rectangular plate occurs if we don't consider the effect of the time dependent temperature. The parameters and the initial conditions are respectively chosen as $a_0=0.29$, $a_1=3.014$, $a_2=1.8$, $a_3=5.03$, $a_4=8.69$, $a_5=12.37$, $a_6=16.7$, $b_7=0.93$, $a_8=0.2$, $a_9=0.93$, $a_{10}=0.93$, $a_{11}=0.93$ $a_{12}=10.0$, $b_0=37.8$, $b_1=0.14$, $b_2=15.49$, $b_3=37.7$, $b_4=10.1$, $b_5=14.3$, $b_6=16.6$, $b_7=25.7$, $b_8=3.08$, $b_9=3.08$, $b_{10}=3.08$, $b_{11}=9.02$, $b_{12}=12.0$, $f_1=1.86$, $f_2=8.79$, $\Omega_1=5$, $\Omega_2=5$, $\Omega_3=1$, $x_{10}=0.21$, $x_{20}=0.21$, $x_{30}=0.38$, $x_{40}=0.16$.

Figs. 3(a) and 3(c) represent the phase portraits on the planes (x_1, x_2) and (x_3, x_4), respectively. Figs. 3(b) and 3(d) respectively denote the waveforms on the planes (t, x_1) and (t, x_3). Figs. 3(e) and 3(f) represent the three-dimensional phase portrait in space (x_1, x_2, x_3) and the Poincare map on plane (x_3, x_4), respectively. Here x_1, x_2, x_3 and x_4 can be expressed as $x_1=w_1$ $x_2=\dot{w}_1$, $x_3=w_2$, $x_4=\dot{w}_2$ respectively.

It can be shown from Fig. 3 that the amplitude of the second order mode is larger than one of the first order mode. With the increasing of the controlling parameters T_{11} and T_{12}, multi-periodic occurs. Fig. 4 shows that the multi-periodic motion occurs when the temperature increased to $T_{11}=11.8$ and $T_{12}=11.8$. Until the temperature is increased to $T_{11}=50.8$

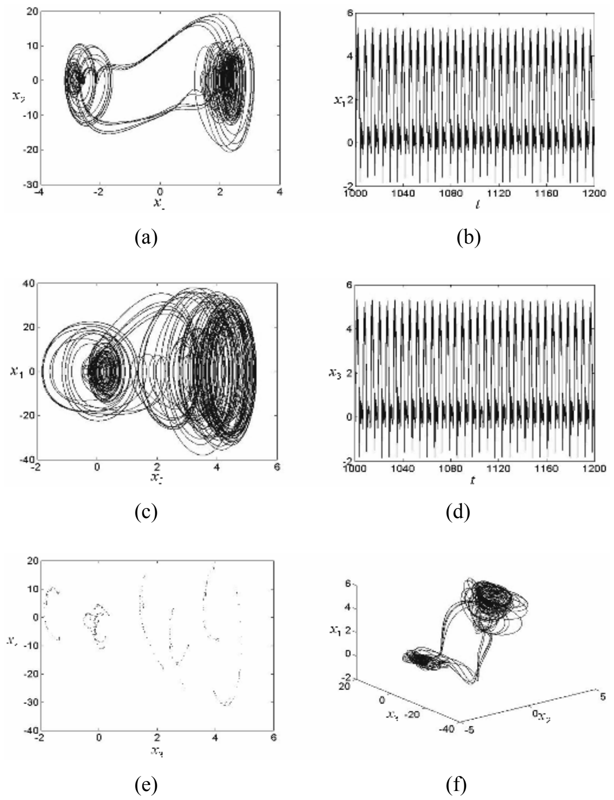

Fig. 5. The chaotic motion of the FGMs rectangular plate exists when $T_{11} = 50.8$, $T_{12} = 50.8$.

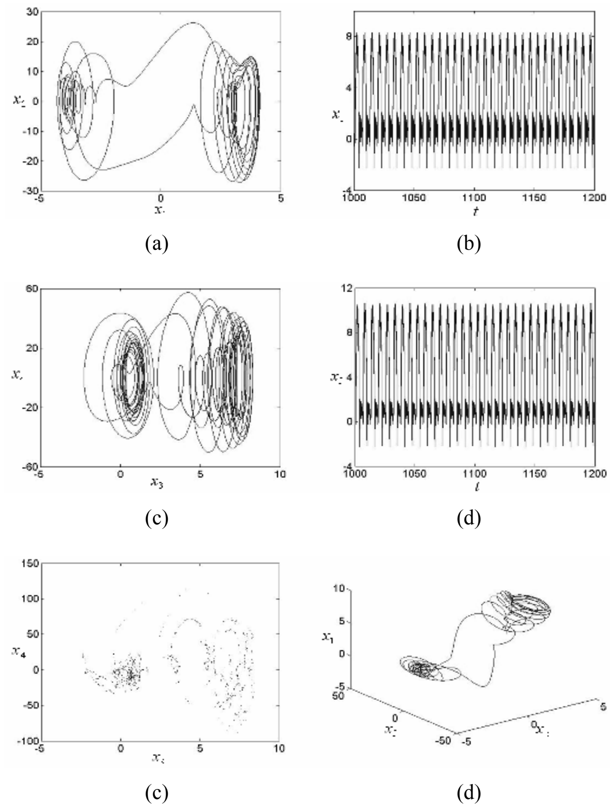

Fig. 6. The chaotic motion of the FGMs rectangular plate exists when $T_{11} = 138$, $T_{12} = 138$.

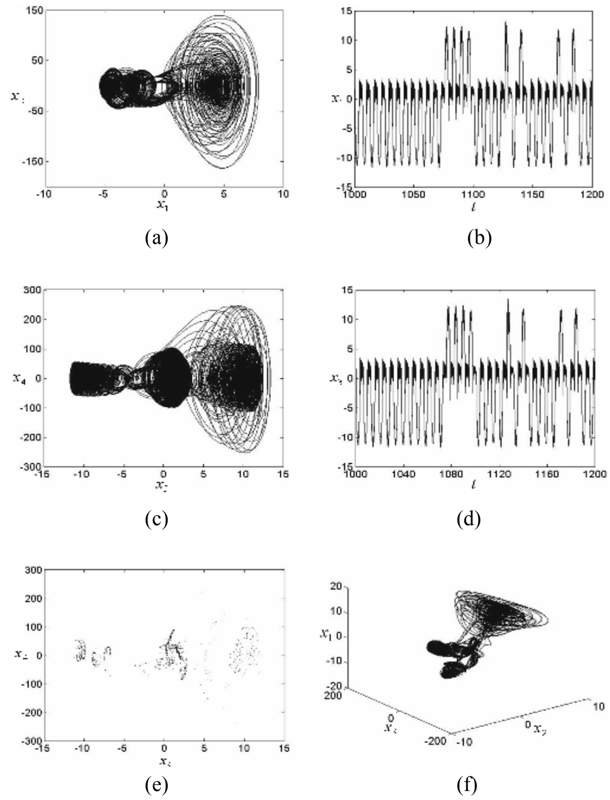

Fig. 7. The chaotic motion of the FGMs rectangular plate exists when $T_{11} = 300$, $T_{12} = 285$.

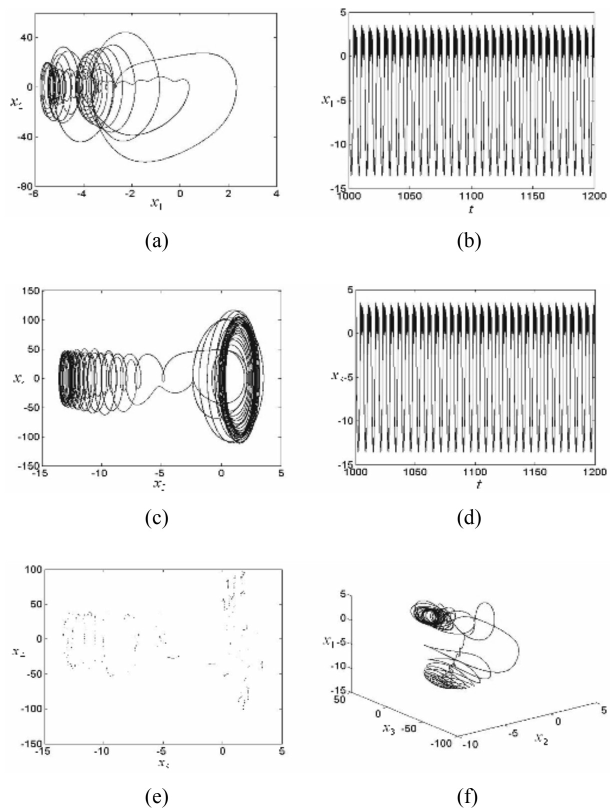

Fig. 8. The chaotic motion of the FGMs rectangular plate exists when $T_{11} = 420$, $T_{12} = 433$.

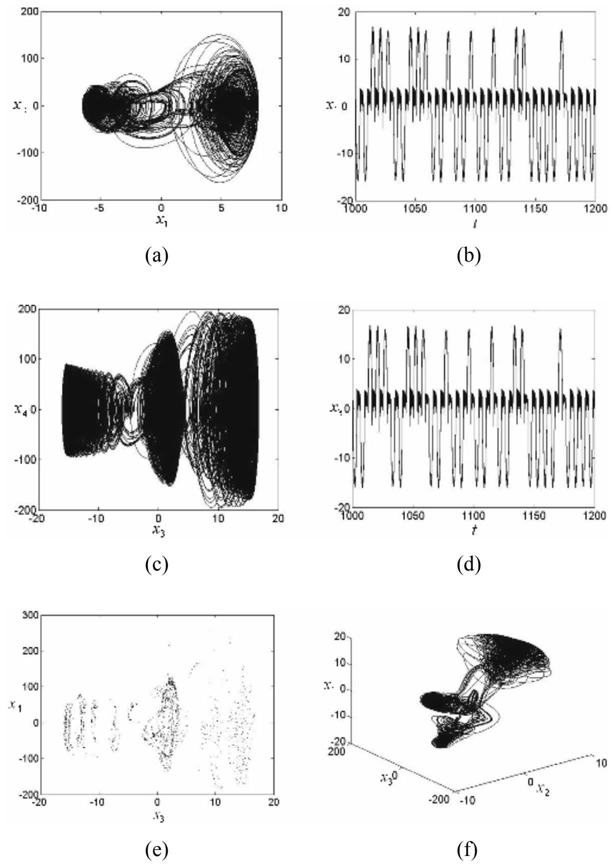

Fig. 9. The chaotic motion of the FGMs rectangular plate exists when $T_{11} = 536$, $T_{12} = 518$.

and $T_{12} = 50.8$, the response of the FGMs rectangular plate is the chaotic motion, as shown in Fig. 5. From Fig. 6 which the temperature is $T_{11} = 138$ and $T_{12} = 136$ to Fig. 9 which the temperature is $T_{11} = 832$ and $T_{12} = 808$ it can illustrate that the chaotic response of the FGM rectangular plate exists. In the fact, until the temperature increases to $T_{11} = 2800$ and $T_{12} = 2800$, the FGM plate is in the conditions of the chaotic motion. Because of the limit of the page number, we don't give other figures.

From Figs. 3-9 it can be shown that the process of change for the motions of the FGMs rectangular plate is as follows: the periodic motion $\rightarrow$ the multi-periodic motion $\rightarrow$ the chaotic motion.

4. Conclusions

The nonlinear oscillations and chaotic dynamics of the FGMs rectangular plate under combined the transverse and in-plane excitations in the time dependent thermal environment are investigated for the first time. The materials properties are assumed to be temperature-dependent. The geometrical nonlinearity using Von Karman's assumption is introduced. Based on the Reddy's third-order plate theory, the governing equations of motion for the FGM rectangular plate are derived by using the Hamilton's principle. Only transverse nonlinear oscillations of the FGM plate are considered, then, Galerkin's approach is utilized to discretize the governing equations of motion to a two-degree-of-freedom nonlinear system including the quadratic and cubic nonlinear terms. Using numerical method, the control equations are analyzed to obtain the response curves. Under certain conditions the chaotic motions of the FGM plates are found. It is found that because of the existence of the temperature which relate to the time the motions of the FGM plate show the great difference. A period motion can be changed into the chaotic motions which are affected by the time dependent temperature.

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