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Nonlinear dynamical analysis on four semi-active dynamic vibration absorbers with time delay

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Abstract. In this paper four semi-active dynamic vibration absorbers (DVAs) are analytically studied, where the time delay induced by measurement and execution in control procedure is included in the system. The first-order approximate analytical solutions of the four semi-active DVAs are established by the averaging method, based on the illustrated phase difference of the motion parameters. The comparisons between the analytical and the numerical solutions are carried out, which verify the correctness and satisfactory precision of the approximate analytical solutions. Then the effects of the time delay on the dynamical responses are analyzed, and it is found that the stability conditions for the steady-state responses of the primary systems are all periodic functions of time delay, with the same period as the excitation one. At last the effects of time delay on control performance are discussed.

Keywords: Dynamic vibration absorber, averaging method, time delay, nonlinear dynamics

1. Introduction

Dynamic vibration absorber (DVA), also called as tuned vibration absorber (TVA), is a subsystem attached to the primary system and designed to cancel or mitigate the unexpectedly transmitted force or motion to the primary system [1,2]. After its invention [3], many design methods for the system parameters of DVA had been investigated and summarized in the famous monograph by Den Hartog [4], especially when there was no damping force in the primary system. In recent thirty years, the researchers studied the design method for DVA when there was damping force in the primary system, and some approximate design methods had been presented [5,6]. Nishihara and Asami divided the design purposes for DVA parameters into three kinds, named as H_2 optimization to minimize the maximum amplitude of the primary system, H_∞ optimization to minimize the total energy of the primary system in overall frequency, and stability maximization to attenuate the transient response of the primary system as soon as possible [5]. Moreover, they gave perturbation analysis on the DVA parameters for the primary system with viscous damping force [6]. Generally speaking, the design methods may be more complicated due to the presence of damping force in the primary system.

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According to the realization manners of the force between the subsystem and the primary system, DVA could be divided into three kinds, named as passive, semi-active and active DVA [7]. As we all know, the traditional DVA is passive, where the DVA parameters could not be changed when it works. In many practical applications, the inevitable variations of the parameters of the primary system, called as off-tuning in some circumstance, maybe cause the passive DVA to be ineffective. Even in some cases, the off-tuning phenomenon would magnify the vibration levels of the primary system, which may lead to unexpected damage and deteriorate the performance of vibration control. The active DVA could present excellent vibration control performance because the controlled force could be momentarily changed according to the different design rules, which needs much additional energy [7–10]. Although it could present much better control performance than the passive one, the active DVA had not been used widely due to higher energy consumption, more complicated structure and lower reliability, etc.

The semi-active DVA is a compromise between the passive and active one, which may attain the perfect combination of performance improvement and implementation simplicity. For example, Jalili [11] gave a review on the basic theoretical concepts of the design and implementation methods for semi-active vibration control system, and surveyed the developments and control techniques for these kinds of systems. Casciati et al. [12] presented a comprehensively summarization on the control law, control devices, implementation issues, and experimental verification of semi-active DVA. Hrovat et al. [13] used a semi-active DVA to control wind-induced vibrations in tall buildings, and found that the semi-active DVA was comparable to the active one. Patten et al. [14] presented a semi-active hydraulic bridge vibration absorber to retrofit an existing bridge, and the experimental results indicated the perfect control performance. Abe and Igusa [15] presented analytical research on optimal control algorithms for semi-active absorbers, and found that tit were suitable to control the impulse response of the structure. Hidaka et al. [16] applied electro-rheological (ER) fluid in a semi-active DVA for a three-story structural model, and found that the single semi-active DVA could simultaneously reduce the vibration amplitude at several frequencies. Pinkaew and Fujino [17] studied the optimal control theory and found that the semi-active DVA could suppress the vibration of the primary system in both transient and steady-state responses. Walsh and Lamancusa [18] designed a stiffness-variable DVA to suppress the transient vibrations of rotating machines during startup and shutdown conditions. Hu et al. [19,20] presented a semi-active vibration absorber with piecewise linear elastic components, which could present adjustable working frequency to follow the large variation of the excitation one. Koo and Ahmadian [21,22] studied two kinds of semi-active on-off DVAs numerically, and then substituted the magneto-rheological (MR) damper for the traditional one and verified the improved control performance experimentally. After that Koo et al. [23] investigated the effects of parameters variation of two semi-active DVAs on the control performance, and found that the increase of off-state damping ratio may decrease the effectiveness of the semi-active DVAs in reducing maximum vibration levels. Shen et al. [24] analytically studied nonlinear dynamics of four semi-active on-off DVAs, and presented the optimization method for system parameters based on the obtained approximate algebra equations.

There is inevitable time delay in the control procedure, which arises from the procedure of measuring system variables, judging and computing the control laws, and realizing the control force. The effects of time delay on control performance are important and could not be neglected in vibration control. Hu and Wang [25] studied the nonlinear dynamics in vibration control systems with time delay in detail, and obtained some important criteria for stability analysis of the controlled system with delayed feedback. Olgac and Hansen [26] presented an active DVA considering time delay, and found that delayed DVA may own more excellent features than the traditional one. After that Hosek et al. [27] applied delayed DVA to the vibration control for multi-degree-of-freedom torsional oscillations in rotating mechanical structures, and discussed the stability condition deduced from time delay. Jalili and Olgac [28] extended the above results to an active DVA using partial state feedback with controlled time delay, and discussed the parameters range for stable motion and the sensitivity analysis. From these references about the effects of time delay on vibration control, one could find that inappropriate time delay could affect the stability of the controlled system and may enlarge the response of the primary system, which will deteriorate the control performance of DVA.

According to the authors' knowledge, there is little analytical research on multi-degree-of-freedom semi-active DVA especially with time delay, although some works about analytical study on single degree-of-freedom (SDOF) semi-active oscillators existed [24,29,30]. In this paper, we present the analytical research on four semi-active delayed DVAs, which may be the first analytical study on 2-DOF semi-active system with time delay. In Section 2, the analytical study on four semi-active DVAs with time delay are fulfilled, and the first-order approximate analytical solutions are established by averaging method. Comparisons of the analytical solutions with the numerical ones are carried out in Section 3, and the good agreements between them verify the correctness and satisfactory precision of the analytical solutions. Section 4 presents the study on stability analysis of semi-active DVAs with time delay, and illustrates the periodicity of the stability condition. At last, the results are summarized in conclusion.

2. Approximate analytical solution for semi-active DVAs with time delay

The dynamical model is shown in Fig. 1, where m_1, k_1, c_1 are the mass, stiffness, and the viscous damping coefficient of the primary system, m_2, k_2, c_{SA} are the mass, stiffness, and the adjustable viscous damping coefficient of the subsystem (DVA), and x_{in} is the external displacement excitation with the form as $x_{in} = b\cos(\omega t)$ respectively.

Based on Newtonian second law, one could establish the differential equation of motion

$$\begin{cases} m_2\ddot{x}_2 + c_{SA}(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = 0 \\ m_1\ddot{x}_1 + c_1(\dot{x}_1 - \dot{x}_{in}) + c_{SA}(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_{in}) + k_2(x_1 - x_2) = 0 \end{cases}. \tag{1}$$

Considering the time delay τ_0 induced by measurment and execution in the control procedure, Eq. (1) could be transformed into

$$\begin{cases} m_2\ddot{x}_2 + c_{SA}[\dot{x}_2(t-\tau_0) - \dot{x}_1(t-\tau_0)] + k_2(x_2 - x_1) = 0 \\ m_1\ddot{x}_1 + c_1(\dot{x}_1 - \dot{x}_{in}) + c_{SA}[\dot{x}_1(t-\tau_0) - \dot{x}_2(t-\tau_0)] + k_1(x_1 - x_{in}) + k_2(x_1 - x_2) = 0 \end{cases} \tag{2}$$

Introducing the transformation of coordinate

$$\delta = \omega t, \quad \omega_1 = \sqrt{\frac{k_1}{m_1}}, \quad \omega_2 = \sqrt{\frac{k_2}{m_2}}, \quad x_r = x_1 - x_2, \quad r_1 = \frac{\omega}{\omega_1}, \quad r_2 = \frac{\omega}{\omega_2}, \quad p = \frac{\omega_1}{\omega_2}, \quad q = \frac{m_2}{m_1},$$
$$c_1 = 2m_1\zeta_1\omega_1, \quad c_{SA} = 2m_2\zeta_{SA}\omega_2, \quad \tau = \omega\tau_0,$$

Equation (2) could be

$$\begin{cases} r_2^2\left[\frac{d^2x_1}{d\delta^2} - \frac{d^2x_r}{d\delta^2}\right] - x_r - 2r_2\zeta_{SA}\frac{dx_r(\delta-\tau)}{d\delta} = 0 \\ r_1r_2\frac{d^2x_1}{d\delta^2} + px_1 + \frac{q}{p}x_r + 2r_2\zeta_1\frac{dx_1}{d\delta} + 2qr_1\zeta_{SA}\frac{dx_r(\delta-\tau)}{d\delta} = pb\cos\delta - 2br_2\zeta_1\sin\delta \end{cases}. \tag{3}$$

Supposing Eq. (3) has the solution as

$$\begin{cases} x_1 = a_1(\delta)\cos\phi_1(\delta) \\ \frac{dx_1}{d\delta} = -a_1(\delta)\sin\phi_1(\delta) \end{cases}, \tag{4}$$

$$\begin{cases} x_r = a_r(\delta)\cos\phi_r(\delta) \\ \frac{dx_r}{d\delta} = -a_r(\delta)\sin\phi_r(\delta) \end{cases}, \tag{5}$$

where $\phi_1(\delta) = \delta + \theta_1(\delta)$, $\phi_r(\delta) = \delta + \theta_r(\delta)$, and differentiating Eqs (4) and (5) with respect to δ, one could obtain

$$\begin{cases} \frac{dx_1}{d\delta} = \frac{da_1}{d\delta}\cos\phi_1(\delta) - a_1(\delta)\sin\phi_1(\delta)\left(1 + \frac{d\theta_1}{d\delta}\right) \\ \frac{d^2x_1}{d\delta^2} = -\left[\frac{da_1}{d\delta}\sin\phi_1(\delta) + a_1(\delta)\cos\phi_1(\delta)\left(1 + \frac{d\theta_1}{d\delta}\right)\right] \end{cases}, \tag{6}$$

$$\begin{cases} \frac{dx_r}{d\delta} = \frac{da_r}{d\delta}\cos\phi_r(\delta) - a_r(\delta)\sin\phi_r(\delta)\left(1 + \frac{d\theta_r}{d\delta}\right) \\ \frac{d^2x_r}{d\delta^2} = -\left[\frac{da_r}{d\delta}\sin\phi_r(\delta) + a_r(\delta)\cos\phi_r(\delta)\left(1 + \frac{d\theta_r}{d\delta}\right)\right] \end{cases}. \tag{7}$$

From Eq. (3) to Eq. (7) and after some symbolic deductions, the next system of equations with $\frac{da_1}{d\delta}$, $\frac{d\theta_1}{d\delta}$, $\frac{da_r}{d\delta}$, $\frac{d\theta_r}{d\delta}$ as unknown variables could be obtained

$$\frac{da_1}{d\delta}\cos\phi_1 - a_1\sin\phi_1\frac{d\theta_1}{d\delta} = 0, \tag{8a}$$

$$\frac{da_r}{d\delta}\cos\phi_r - a_r\sin\phi_r\frac{d\theta_r}{d\delta} = 0, \tag{8b}$$

$$\begin{aligned}\frac{da_1}{d\delta}\sin\phi_1 + a_1\cos\phi_1\frac{d\theta_1}{d\delta} = &-\frac{1}{r_1r_2}\Big[pb\cos\delta - 2br_2\zeta_1\sin\delta + 2a_1r_2\zeta_1\sin\phi_1 \\ &+ 2a_rr_1q\zeta_{SA}\sin(\phi_r-\tau) - pa_1\cos\phi_1 - \frac{qa_r}{p}\cos\phi_r\Big] - a_1\cos\phi_1\end{aligned}, \tag{8c}$$

$$\begin{aligned}\frac{da_r}{d\delta}\sin\phi_r + a_r\cos\phi_r\frac{d\theta_r}{d\delta} = &-\frac{1}{r_1r_2}\Big[pb\cos\delta - 2br_2\zeta_1\sin\delta + 2a_1r_2\zeta_1\sin\phi_1 \\ &+ 2a_rr_1q\zeta_{SA}\sin(\phi_r-\tau) - pa_1\cos\phi_1 - \frac{qa_r}{p}\cos\phi_r\Big] - a_r\cos\phi_r \\ &- \frac{2a_r\zeta_{SA}}{r_2}\sin(\phi_r-\tau) + \frac{a_r}{r_2^2}\cos\phi_r\end{aligned}. \tag{8d}$$

Moreover, one could obtain the four unknown variables from Eq. (8)

$$\begin{aligned}\frac{da_1}{d\delta} = &-\frac{\sin\phi_1}{r_1r_2}\Big[pb\cos(\phi_1-\theta_1) - 2br_2\zeta_1\sin(\phi_1-\theta_1) + 2a_1r_2\zeta_1\sin\phi_1 - pa_1\cos\phi_1 \\ &+ 2a_rr_1q\zeta_{SA}\sin(\phi_1-\theta_1+\theta_r-\tau) - \frac{qa_r}{p}\cos(\phi_1-\theta_1+\theta_r)\Big] - \frac{a_1}{2}\sin 2\phi_1\end{aligned}, \tag{9a}$$

$$\begin{aligned}\frac{d\theta_1}{d\delta} = &-\frac{\cos\phi_1}{a_1r_1r_2}\Big[pb\cos(\phi_1-\theta_1) - 2br_2\zeta_1\sin(\phi_1-\theta_1) + 2a_1r_2\zeta_1\sin\phi_1 - pa_1\cos\phi_1 \\ &+ 2a_rr_1q\zeta_{SA}\sin(\phi_1-\theta_1+\theta_r-\tau) - \frac{qa_r}{p}\cos(\phi_1-\theta_1+\theta_r)\Big] - \cos^2\phi_1\end{aligned}, \tag{9b}$$

$$\begin{aligned}\frac{da_r}{d\delta} = &-\frac{\sin\phi_r}{r_1r_2}\Big[pb\cos(\phi_r-\theta_r) - 2br_2\zeta_1\sin(\phi_r-\theta_r) + 2a_1r_2\zeta_1\sin(\phi_r-\theta_r+\theta_1) \\ &+ 2a_rr_1q\zeta_{SA}\sin(\phi_r-\tau) - pa_1\cos(\phi_r-\theta_r+\theta_1) - \frac{qa_r}{p}\cos\phi_r\Big] - \frac{a_r}{2}\sin 2\phi_r, \\ &- \frac{2a_r\zeta_{SA}}{r_2}\sin\phi_r\sin(\phi_r-\tau) + \frac{a_r}{2r_2^2}\sin 2\phi_r\end{aligned} \tag{9c}$$

$$\begin{aligned}\frac{d\theta_r}{d\delta} = &-\frac{\cos\phi_r}{a_rr_1r_2}\Big[pb\cos(\phi_r-\theta_r) - 2br_2\zeta_1\sin(\phi_r-\theta_r) + 2a_1r_2\zeta_1\sin(\phi_r-\theta_r+\theta_1) \\ &+ 2a_rr_1q\zeta_{SA}\sin(\phi_r-\tau) - pa_1\cos(\phi_r-\theta_r+\theta_1) - \frac{qa_r}{p}\cos\phi_r\Big] - \cos^2\phi_r \\ &- \frac{2\zeta_{SA}}{r_2}\cos\phi_r\sin(\phi_r-\tau) + \frac{1}{r_2^2}\cos^2\phi_r\end{aligned}, \tag{9d}$$

where the relationships $\delta = \phi_1 - \theta_1$ and $\delta = \phi_r - \theta_r$ are used in the deduction procedure. Equation (9) are usually called as standard equations of the amplitude and phase in the averaging method [31,32].

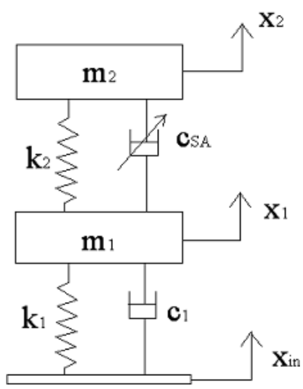


Fig. 1. Dynamical model.

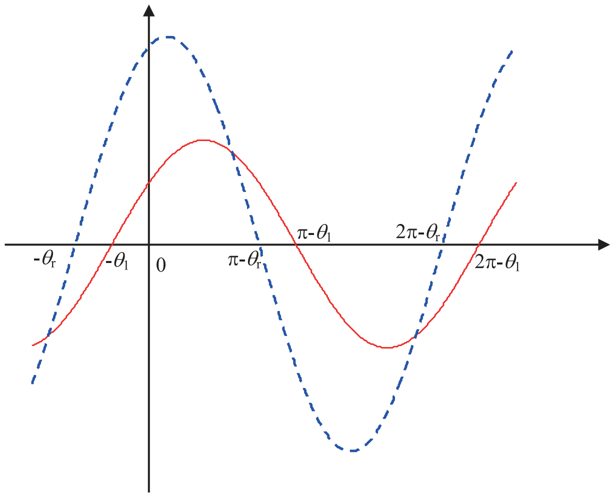


Fig. 2. A typical phase difference between $\dot{x}_1$ and $\dot{x}_r$, where the X-coordinates and Y-coordinates are the phases and amplitudes of $\dot{x}_1$ (the solid line) and $\dot{x}_r$ (the dashed line) respectively.

Then one should obtain the approximate forms for the derivatives of the two amplitudes and phases, by integrating the right-hand sides of Eq. (9) in one period, i.e. $\phi_i = [0 \;\; 2\pi]$, $i = 1, r$.

$$\begin{aligned}\frac{da_1}{d\delta} = &-\frac{1}{2\pi r_1 r_2}\int_0^{2\pi}\left\{\left[pb\cos(\phi_1-\theta_1) - 2br_2\zeta_1\sin(\phi_1-\theta_1) - pa_1\cos\phi_1\right.\right.\\ &+ 2a_1\zeta_1 r_2\sin\phi_1 + 2a_r r_1 q\zeta_{SA}\sin(\phi_1-\theta_1+\theta_r-\tau)\\ &\left.\left.-\frac{qa_r}{p}\cos(\phi_1-\theta_1+\theta_r)\right]\sin\phi_1 + \frac{a_1 r_1 r_2}{2}\sin 2\phi_1\right\}d\phi_1\end{aligned}, \tag{10a}$$

$$\begin{aligned}\frac{d\theta_1}{d\delta} = &-\frac{1}{2\pi a_1 r_1 r_2}\int_0^{2\pi}\left\{\left[pb\cos(\phi_1-\theta_1) - 2br_2\zeta_1\sin(\phi_1-\theta_1) - pa_1\cos\phi_1\right.\right.\\ &+ 2a_1\zeta_1 r_2\sin\phi_1 + 2a_r r_1 q\zeta_{SA}\sin(\phi_1-\theta_1+\theta_r-\tau)\\ &\left.\left.-\frac{qa_r}{p}\cos(\phi_1-\theta_1+\theta_r)\right]\cos\phi_1 + a_1 r_1 r_2\cos^2\phi_1\right\}d\phi_1\end{aligned}, \tag{10b}$$

$$\begin{aligned}\frac{da_r}{d\delta} = &-\frac{1}{2\pi r_1 r_2}\int_0^{2\pi}\left\{\left[pb\cos(\phi_r-\theta_r) + 2a_1\zeta_1 r_2\sin(\phi_r-\theta_r+\theta_1) - \frac{qa_r}{p}\cos\phi_r\right.\right.\\ &\left.+ 2a_r r_1 q\zeta_{SA}\sin(\phi_r-\tau) - pa_1\cos(\phi_r-\theta_r+\theta_1) - 2br_2\zeta_1\sin(\phi_r-\theta_r)\right]\sin\phi_r,\\ &\left.+\frac{a_r r_1 r_2}{2}\sin 2\phi_r + 2a_r r_1\zeta_{SA}\sin\phi_r\sin(\phi_r-\tau) - \frac{a_r r_1}{2r_2}\sin 2\phi_r\right\}d\phi_r\end{aligned} \tag{10c}$$

$$\begin{aligned}\frac{d\theta_r}{d\delta} = &-\frac{1}{2\pi a_r r_1 r_2}\int_0^{2\pi}\left\{\left[pb\cos(\phi_r-\theta_r) + 2a_1\zeta_1 r_2\sin(\phi_r-\theta_r+\theta_1) - \frac{qa_r}{p}\cos\phi_r\right.\right.\\ &\left.+ 2a_r r_1 q\zeta_{SA}\sin(\phi_r-\tau) - pa_1\cos(\phi_r-\theta_r+\theta_1) - 2br_2\zeta_1\sin(\phi_r-\theta_r)\right]\cos\phi_r,\\ &\left.+ a_r r_1 r_2\cos^2\phi_r + 2a_r r_1\zeta_{SA}\cos\phi_r\sin(\phi_r-\tau) - \frac{a_r r_1}{r_2}\cos^2\phi_r\right\}d\phi_r\end{aligned} \tag{10d}$$

Because the adjustable damping ratio ζ_{SA} of the subsystem is changeable based on the semi-active control laws, the integrating procedure should be divided into several time intervals [31,32]. Hence, we should integrate Eq. (10) based on the different semi-active control laws.

2.1. Approximate solution of VVBG DVA

The first semi-active on-off DVA by Koo and Ahmadian [21–23] is called as VVBG (velocity-velocity based ground-hook) DVA, which means the controllable damping ratio is adjusted based on the absolute velocity of the primary system and the relative velocity of the primary system to the subsystem. It reads

$$\zeta_{SA} = \begin{cases} \zeta_{\max} & \dot{x}_1\dot{x}_r \geqslant 0 \\ \zeta_{\min} & \dot{x}_1\dot{x}_r < 0 \end{cases}, \tag{11}$$

where $\zeta_{\max}$ is the maximum viscous ratio corresponding to the on state, and $\zeta_{\min}$ is the minimum one corresponding to the off state.

Then one should find the relationship between the phases of $\dot{x}_1$ and $\dot{x}_r$. Due to $x_1 = a_1\cos(\delta+\theta_1)$ and $x_2 = a_2\cos(\delta+\theta_2)$, one could get

$$x_r = x_1 - x_2 = a_r\cos(\delta+\theta_r), \tag{12}$$

where

$$a_r = \sqrt{(a_1\sin\theta_1 - a_2\sin\theta_2)^2 + (a_1\cos\theta_1 - a_2\cos\theta_2)^2},$$
$$\theta_r = \arctan\frac{a_1\sin\theta_1 - a_2\sin\theta_2}{a_1\cos\theta_1 - a_2\cos\theta_2}. \tag{13}$$

For most DVAs, it should be satisfied

$$a_2 \gg a_1 \text{ and } \theta_2 > \theta_1,$$

which leads to

$$\theta_r \approx \theta_2 > \theta_1. \tag{14}$$

Due to $\dot{x}_1 = -a_1\sin(\delta+\theta_1)$ and $\dot{x}_r = -a_r\sin(\delta+\theta_r)$, and from the general time history of $\dot{x}_1$ and $\dot{x}_r$ shown in Fig. 2, Eq. (11) could be changed into

$$\zeta_{SA} = \begin{cases} \zeta_{\max} & -\theta_1 < \phi \leqslant \pi - \theta_r \\ \xi_{\min} & \pi - \theta_r < \phi \leqslant \pi - \theta_1 \\ \zeta_{\max} & \pi - \theta_1 < \phi \leqslant 2\pi - \theta_r \\ \xi_{\min} & 2\pi - \theta_r < \phi \leqslant 2\pi - \theta_1 \end{cases}. \tag{15}$$

According to the averaging method, the approximate solution of semi-active on-off VVBG DVA could be obtained from Eq. (10). Based on Eqs (10) and (15), after some symbolic deduction one could obtain

$$\begin{aligned}\frac{da_1}{d\delta} =& -\frac{1}{2\pi r_1 r_2}\{\pi[2a_1r_2\zeta_1 - 2br_2\zeta_1\cos\theta_1 + bp\sin\theta_1 - a_rq\sin(\theta_1-\theta_r)/p] \\ &+ 2a_rqr_1[\zeta_{\max}(\pi+\theta_1-\theta_r)\cos(\tau+\theta_1-\theta_r) + \zeta_{\min}(\theta_r-\theta_1)\cos(\tau+\theta_1-\theta_r)\text{'} \\ &+ (\zeta_{\min}-\zeta_{\max})\cos(\tau+2\theta_1)\sin(\theta_1-\theta_r)]\}\end{aligned} \tag{16a}$$

$$\begin{aligned}\frac{d\theta_1}{d\delta} =& -\frac{1}{2\pi a_1 r_1 r_2}\{\pi[-a_1p + a_1r_1r_2 + 2br_2\zeta_1\sin\theta_1 - a_rq\cos(\theta_1-\theta_r)/p \\ &+ bp\cos\theta_1] + a_rqr_1(\zeta_{\max}-\zeta_{\min})[\cos(\tau+3\theta_1-\theta_r) - \cos(\tau+\theta_1+\theta_r)]\text{'} \\ &- 2a_rqr_1\sin(\tau+\theta_1-\theta_r)[\zeta_{\max}(\pi+\theta_1-\theta_r) - \zeta_{\min}(\theta_1-\theta_r)]\}\end{aligned} \tag{16b}$$

$$\begin{aligned}\frac{da_r}{d\delta} =& -\frac{1}{2\pi r_1 r_2}\{\pi[2a_1r_2\zeta_1\cos(\theta_1-\theta_r) - 2br_2\zeta_1\cos\theta_r + a_1p\sin(\theta_1-\theta_r) \\ &+ bp\sin\theta_r] + 2a_rr_1(1+q)[\zeta_{\max}(\pi+\theta_1-\theta_r)\cos\tau + \zeta_{\min}(\theta_r-\theta_1)\cos\tau \\ &+ (\zeta_{\min}-\zeta_{\max})\cos(\tau+\theta_1-\theta_r)\sin(\theta_1-\theta_r)]\}\end{aligned} \tag{16c}$$

$$\begin{aligned}\frac{d\theta_r}{d\delta} =& -\frac{1}{2\pi a_r r_1 r_2}\left\{\pi\left[a_rr_1r_2 - \frac{qa_r}{p} - \frac{a_rr_1}{r_2} - pa_1\cos(\theta_1-\theta_r)\right.\right. \\ &\left. + bp\cos\theta_r + 2a_1r_2\zeta_1\sin(\theta_1-\theta_r) + 2br_2\zeta_1\sin\theta_r\right] \\ &+ a_rr_1(1+q)(\zeta_{\max}-\zeta_{\min})[\cos(\tau+2\theta_1) - \cos(\tau+2\theta_r)] \\ &\left. - 2a_rr_1(1+q)\sin\tau[\zeta_{\max}(\pi+\theta_1-\theta_r) - \zeta_{\min}(\theta_1-\theta_r)]\right\}\end{aligned}. \tag{16d}$$

In the field of vibration control engineering, the steady-state system response is more important and meaningful. By letting $\frac{da_1}{d\delta}=0$, $\frac{d\theta_1}{d\delta}=0$, $\frac{da_r}{d\delta}=0$ and $\frac{d\theta_r}{d\delta}=0$, one could obtain a system of four nonlinear algebra equations with four unknowns, i.e. steady-state amplitudes and phases $\bar{a}_1$, $\bar{a}_r$, $\bar{\theta}_1$, $\bar{\theta}_r$. Based on some numerical methods for system of nonlinear algebra equations [33,34], one could obtain the steady-state solution of semi-active VVBG DVA.

2.2. *Approximate solution of DVBG DVA*

The second semi-active on-off DVA by Koo and Ahmadian [21–23] is called as DVBG (displacement-velocity based ground-hook) DVA, which means the controllable damping ratio is adjusted based on the absolute displacement of the primary system and the relative velocity of the primary system to the subsystem. It is

$$\zeta_{SA} = \begin{cases}\zeta_{\max} & x_1\dot{x}_r \geqslant 0 \\ \zeta_{\min} & x_1\dot{x}_r < 0\end{cases}, \tag{17}$$

where $\zeta_{\max}$ is the maximum viscous ratio corresponding to the on state and $\zeta_{\min}$ is the minimum one corresponding to the off state.

According to the similar procedure for the relationship of phase difference between x_1 and $\dot{x}_r$, Eq. (17) could be transformed into

$$\zeta_{SA} = \begin{cases}\zeta_{\min} & -\theta_r < \phi \leqslant \frac{\pi}{2} - \theta_1 \\ \xi_{\max} & \frac{\pi}{2} - \theta_1 < \phi \leqslant \pi - \theta_r \\ \zeta_{\min} & \pi - \theta_r < \phi \leqslant \frac{3\pi}{2} - \theta_1 \\ \xi_{\max} & \frac{3\pi}{2} - \theta_1 < \phi \leqslant 2\pi - \theta_r\end{cases}. \tag{18}$$

From Eqs (10) and (18), one could obtain the first-order differential equations for the steady-state amplitudes and phases as

$$\begin{aligned}\frac{da_1}{d\delta} =& -\frac{1}{2\pi r_1 r_2}\{\pi[2a_1r_2\zeta_1 - 2br_2\zeta_1\cos\theta_1 + bp\sin\theta_1 - a_rq\sin(\theta_1-\theta_r)/p] \\ &+ a_rqr_1\cos(\tau+\theta_1-\theta_r)[\zeta_{\max}(\pi+2\theta_1-2\theta_r) + \zeta_{\min}(\pi+2\theta_r-2\theta_1)] \\ &+ 2a_rqr_1(\zeta_{\max}-\zeta_{\min})\sin(\tau+2\theta_1)\cos(\theta_1-\theta_r)\}\end{aligned} \tag{19a}$$

$$\begin{aligned}\frac{d\theta_1}{d\delta} =& -\frac{1}{2\pi a_1 r_1 r_2}\{\pi[-a_1 p + a_1 r_1 r_2 + 2br_2\zeta_1 \sin\theta_1 - a_r q\cos(\theta_1-\theta_r)/p \\ &+ bp\cos\theta_1] + a_r q r_1(\zeta_{\min}-\zeta_{\max})[\cos(\tau+3\theta_1-\theta_r)+\cos(\tau+\theta_1+\theta_r)] \\ &- 2a_r q r_1 \sin(\tau+\theta_1-\theta_r)[\zeta_{\max}(\pi+2\theta_1-2\theta_r)+\zeta_{\min}(\pi-2\theta_1+2\theta_r)]\}\end{aligned} \quad (19b)$$

$$\begin{aligned}\frac{da_r}{d\delta} =& -\frac{1}{2\pi r_1 r_2}\{\pi[2a_1 r_2\zeta_1\cos(\theta_1-\theta_r) - 2br_2\zeta_1\cos\theta_r + a_1 p\sin(\theta_1-\theta_r) \\ &+ bp\sin\theta_r] + 2a_r r_1(1+q)(\zeta_{\max}-\zeta_{\min})\sin(\tau+\theta_1+\theta_r)\cos(\theta_1-\theta_r) \\ &+ a_r r_1(1+q)\cos\tau[\zeta_{\max}(\pi+2\theta_1-2\theta_r)+\zeta_{\min}(\pi+2\theta_r-2\theta_1)]\}\end{aligned} \quad (19c)$$

$$\begin{aligned}\frac{d\theta_r}{d\delta} =& -\frac{1}{2\pi a_r r_1 r_2}\left\{\pi\left[a_r r_1 r_2 - \frac{qa_r}{p} - \frac{a_r r_1}{r_2} - pa_1\cos(\theta_1-\theta_r)\right.\right. \\ &\left. + bp\cos\theta_r + 2a_1 r_2\zeta_1\sin(\theta_1-\theta_r) + 2br_2\zeta_1\sin\theta_r\right] \\ &- a_r r_1(1+q)(\zeta_{\max}-\zeta_{\min})[\cos(\tau+2\theta_1)+\cos(\tau+2\theta_r)] \\ &\left. - a_r r_1(1+q)\sin\tau[\zeta_{\max}(\pi+2\theta_1-2\theta_r)+\zeta_{\min}(\pi-2\theta_1+2\theta_r)]\right\}\end{aligned} . \quad (19d)$$

Similarly, one could obtain a system of four nonlinear algebra equations with four unknowns by letting $\frac{da_1}{d\delta} = 0$, $\frac{d\theta_1}{d\delta} = 0$, $\frac{da_r}{d\delta} = 0$ and $\frac{d\theta_r}{d\delta} = 0$. Moreover, one could obtain the steady-state solution of semi-active DVBG DVA based on some numerical methods for system of nonlinear algebra equations.

2.3. Approximate solution of VDBG DVA

The two aforementioned semi-active on-off DVAs, namely VVBG and DVBG, are presented by Koo et al. [21–23], where the absolute velocity and the absolute displacement of the primary system, associated with the relative velocity of the primary system to the sub-system are used. Intuitionally, the relative displacement maybe substitutes for the relative velocity in the two on-off control strategies, so that two other new semi-active on-off DVAs could be established [24].

The third semi-active on-off DVA by Shen et al. [24] is called as VDBG (velocity-displacement based groundhook control) DVA, and the control strategy of VDBG DVA is shown as

$$\zeta_{SA} = \begin{cases}\zeta_{\max} & \dot{x}_1 x_r \geqslant 0 \\ \zeta_{\min} & \dot{x}_1 x_r < 0\end{cases}. \quad (20)$$

Based on the form of $\dot{x}_1$ and x_r, one can also obtain

$$\zeta_{SA} = \begin{cases}\zeta_{\min} & -\theta_1 < \phi \leqslant \frac{\pi}{2} - \theta_r \\ \xi_{\max} & \frac{\pi}{2} - \theta_r < \phi \leqslant \pi - \theta_1 \\ \zeta_{\min} & \pi - \theta_1 < \phi \leqslant \frac{3\pi}{2} - \theta_r \\ \xi_{\max} & \frac{3\pi}{2} - \theta_r < \phi \leqslant 2\pi - \theta_1\end{cases}. \quad (21)$$

The approximate solution could also be obtained by averaging method based on Eqs (10) and (21). After some similar simplifying procedure, the obtained results are

$$\begin{aligned}\frac{da_1}{d\delta} =& -\frac{1}{2\pi r_1 r_2}\{\pi[2a_1 r_2\zeta_1 - 2br_2\zeta_1\cos\theta_1 + bp\sin\theta_1 - a_r q\sin(\theta_1-\theta_r)/p] \\ &+ a_r q r_1\cos(\tau+\theta_1-\theta_r)[\zeta_{\min}(\pi+2\theta_1-2\theta_r)+\zeta_{\max}(\pi+2\theta_r-2\theta_1)] \\ &+ 2a_r q r_1(\zeta_{\max}-\zeta_{\min})\sin(\tau+2\theta_1)\cos(\theta_1-\theta_r)\}\end{aligned} \quad (22a)$$

$$\begin{aligned}\frac{d\theta_1}{d\delta} =& -\frac{1}{2\pi a_1 r_1 r_2}\{\pi[-a_1p + a_1r_1r_2 + 2br_2\zeta_1\sin\theta_1 - a_rq\cos(\theta_1-\theta_r)/p \\ &+ bp\cos\theta_1] + a_rqr_1(\zeta_{\min}-\zeta_{\max})[\cos(\tau+3\theta_1-\theta_r)+\cos(\tau+\theta_1+\theta_r)] \\ &- a_rqr_1\sin(\tau+\theta_1-\theta_r)[\zeta_{\max}(\pi-2\theta_1+2\theta_r)+\zeta_{\min}(\pi+2\theta_1-2\theta_r)]\}\end{aligned} \tag{22b}$$

$$\begin{aligned}\frac{da_r}{d\delta} =& -\frac{1}{2\pi r_1 r_2}\{\pi[2a_1r_2\zeta_1\cos(\theta_1-\theta_r) - 2br_2\zeta_1\cos\theta_r + a_1p\sin(\theta_1-\theta_r) \\ &+ bp\sin\theta_r] + 2a_rr_1(1+q)(\zeta_{\max}-\zeta_{\min})\sin(\tau+\theta_1+\theta_r)\cos(\theta_1-\theta_r) \\ &+ a_rr_1(1+q)\cos\tau[\zeta_{\min}(\pi+2\theta_1-2\theta_r)+\zeta_{\max}(\pi+2\theta_r-2\theta_1)]\}\end{aligned} \tag{22c}$$

$$\begin{aligned}\frac{d\theta_r}{d\delta} =& -\frac{1}{2\pi a_r r_1 r_2}\Big\{\pi\Big[a_rr_1r_2 - \frac{qa_r}{p} - \frac{a_rr_1}{r_2} - pa_1\cos(\theta_1-\theta_r) \\ &+ bp\cos\theta_r + 2a_1r_2\zeta_1\sin(\theta_1-\theta_r) + 2br_2\zeta_1\sin\theta_r\Big] \\ &- a_rr_1(1+q)(\zeta_{\max}-\zeta_{\min})[\cos(\tau+2\theta_1)+\cos(\tau+2\theta_r)] \\ &- a_rr_1(1+q)\sin\tau[\zeta_{\min}(\pi+2\theta_1-2\theta_r)+\zeta_{\max}(\pi-2\theta_1+2\theta_r)]\Big\}\end{aligned} \tag{22d}$$

By letting $\frac{da_1}{d\delta} = 0$, $\frac{d\theta_1}{d\delta} = 0$, $\frac{da_r}{d\delta} = 0$ and $\frac{d\theta_r}{d\delta} = 0$, one could also obtain a system of four nonlinear algebra equations with four unknowns. Based on numerical methods for system of nonlinear algebra equations, one could obtain the steady-state solution of semi-active VDBG DVA.

2.4. Approximate solution of DDBG DVA

The fourth semi-active on-off DVA by Shen et al. [24] is called as DDBG (displacement-displacement based ground-hook control) DVA, and the control strategy of DDBG DVA is shown as

$$\zeta_{SA} = \begin{cases}\zeta_{\max} & x_1x_r \geqslant 0 \\ \zeta_{\min} & x_1x_r < 0\end{cases}. \tag{23}$$

Based on the form of x_1 and x_r, one can also obtain

$$\zeta_{SA} = \begin{cases}\zeta_{\max} & -\theta_1 < \phi \leqslant \frac{\pi}{2}-\theta_r \\ \xi_{\min} & \frac{\pi}{2}-\theta_r < \phi \leqslant \frac{\pi}{2}-\theta_1 \\ \zeta_{\max} & \frac{\pi}{2}-\theta_1 < \phi \leqslant \frac{3\pi}{2}-\theta_r \\ \zeta_{\min} & \frac{3\pi}{2}-\theta_r < \phi \leqslant \frac{3\pi}{2}-\theta_1 \\ \xi_{\max} & \frac{3\pi}{2}-\theta_1 < \phi \leqslant 2\pi-\theta_1\end{cases}. \tag{24}$$

The approximate solution could be obtained by averaging method based on Eqs (10) and (24). After some similar simplifying procedure, the results are

$$\begin{aligned}\frac{da_1}{d\delta} =& -\frac{1}{2\pi r_1 r_2}\{\pi[2a_1r_2\zeta_1 - 2br_2\zeta_1\cos\theta_1 + bp\sin\theta_1 - a_rq\sin(\theta_1-\theta_r)/p] \\ &+ 2a_rqr_1\cos(\tau+\theta_1-\theta_r)[\zeta_{\max}(\pi+\theta_1-\theta_r)+\zeta_{\min}(\theta_r-\theta_1)] \\ &+ 2a_rqr_1(\zeta_{\max}-\zeta_{\min})\cos(\tau+2\theta_1)\sin(\theta_1-\theta_r)\}\end{aligned}, \tag{25a}$$

Table 1
Basic system parameters

Parameters	Values
Mass of primary system (m_1)	261.05 kg
DVA mass (m_2)	14.528 kg
Stiffness of primary system (k_1)	334075 N/m
Damping ratio of primary system (ζ_1)	0.03
DVA stiffness (k_2)	16500 N/m
On-state damping ratio ($\zeta_{\max}$)	0.11
Off-state damping ratio ($\zeta_{\min}$)	0.095
Time delay (τ_0)	0.002 s

$$\begin{aligned}\frac{d\theta_1}{d\delta} =& -\frac{1}{2\pi a_1 r_1 r_2}\{\pi[-a_1 p + a_1 r_1 r_2 + 2br_2\zeta_1 \sin\theta_1 - a_r q\cos(\theta_1-\theta_r)/p \\ &+ bp\cos\theta_1] + a_r q r_1(\zeta_{\min}-\zeta_{\max})[\cos(\tau+3\theta_1-\theta_r)-\cos(\tau+\theta_1+\theta_r)] \\ &- 2a_r q r_1\sin(\tau+\theta_1-\theta_r)[\zeta_{\max}(\pi+\theta_1-\theta_r)+\zeta_{\min}(\theta_r-\theta_1)]\}\end{aligned}, \tag{25b}$$

$$\begin{aligned}\frac{da_r}{d\delta} =& -\frac{1}{2\pi r_1 r_2}\{\pi[2a_1 r_2\zeta_1\cos(\theta_1-\theta_r) - 2br_2\zeta_1\cos\theta_r + a_1 p\sin(\theta_1-\theta_r) \\ &+ bp\sin\theta_r] + 2a_r r_1(1+q)(\zeta_{\max}-\zeta_{\min})\cos(\tau+\theta_1+\theta_r)\sin(\theta_1-\theta_r) \\ &+ 2a_r r_1(1+q)\cos\tau[\zeta_{\max}(\pi+\theta_1-\theta_r)+\zeta_{\min}(\theta_r-\theta_1)]\}\end{aligned}, \tag{25c}$$

$$\begin{aligned}\frac{d\theta_r}{d\delta} =& -\frac{1}{2\pi a_r r_1 r_2}\Bigg\{\pi\Bigg[a_r r_1 r_2 - \frac{q a_r}{p} - \frac{a_r r_1}{r_2} - pa_1\cos(\theta_1-\theta_r) \\ &+ bp\cos\theta_r + 2a_1 r_2\zeta_1\sin(\theta_1-\theta_r) + 2br_2\zeta_1\sin\theta_r\Bigg] \\ &- a_r r_1(1+q)(\zeta_{\max}-\zeta_{\min})[\cos(\tau+2\theta_1)-\cos(\tau+2\theta_r)] \\ &- 2a_r r_1(1+q)\sin\tau[\zeta_{\max}(\pi+\theta_1-\theta_r)+\zeta_{\min}(\theta_r-\theta_1)]\Bigg\}\end{aligned}. \tag{25d}$$

Similarly, one could obtain a system of four nonlinear algebra equations with four unknowns by letting $\frac{da_1}{d\delta}=0$, $\frac{d\theta_1}{d\delta}=0$, $\frac{da_r}{d\delta}=0$ and $\frac{d\theta_r}{d\delta}=0$. One could also obtain the steady-state solution of semi-active DDBG DVA based on numerical methods for system of nonlinear algebra equations.

3. Comparisons of analytical solutions with numerical ones

In order to verify the correctness and precision of the first-order approximate analytical solutions, the system parameters are selected as Table 1 [21–24]. Although the value of time delay is influenced by the system characteristics [25–28,35], we omit this effect and consider it as a constant value for simplicity. Here the value for time delay is selected as 0.002 s or 2 ms because the response time for many controllable devices in vibration control system is on the order of millisecond. Based on Eqs (16), (19), (22) and (25) one could obtain the amplitude-frequency curves shown in Figs 3–6, where the line with circles is for numerical results and the solid line is for the analytical solutions.

The numerical method used in this paper is the classic four-stage, fourth-order explicit Runge-Kutta method for delay differential equation (DDE) in MatLab software. The code and integrator options could be found in [36]. The correctness and precision of the code have been verified by some DDE examples with analytical solutions.

From the observation of these figures, it could be found that the first-order approximate analytical solutions agree well with the numerical ones. Accordingly, the first-order approximate solutions are correct and have satisfactory precision.

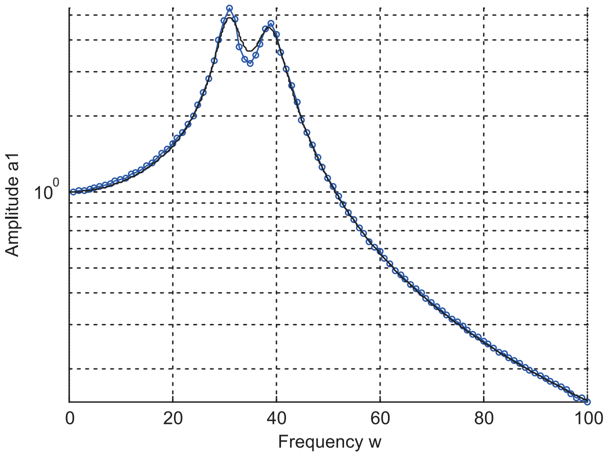


Fig. 3. Comparison of the amplitude-frequency curves of VVBG DVA by the analytical method (solid line) and the numerical one (circle line).

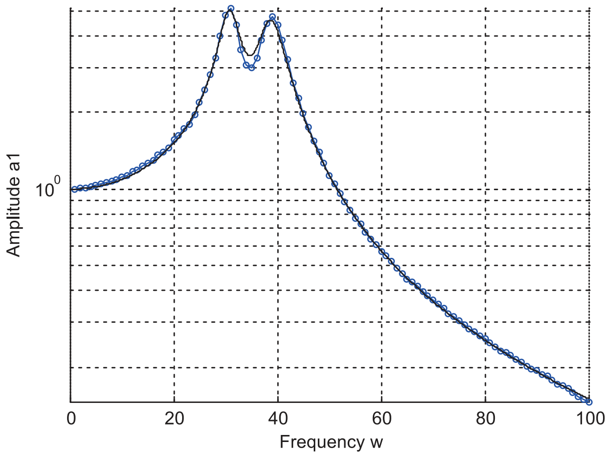


Fig. 4. Comparison of the amplitude-frequency curves of DVBG DVA by the analytical method (solid line) and the numerical one (circle line).

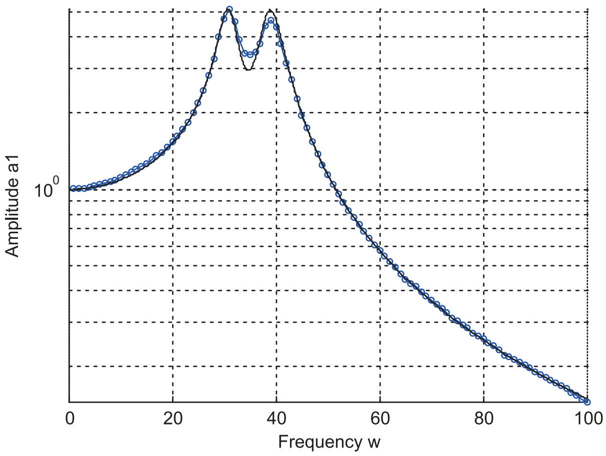


Fig. 5. Comparison of the amplitude-frequency curves of VDBG DVA by the analytical method (solid line) and the numerical one (circle line).

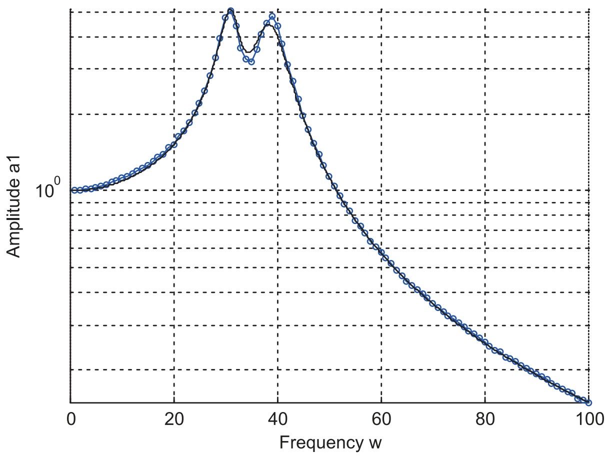


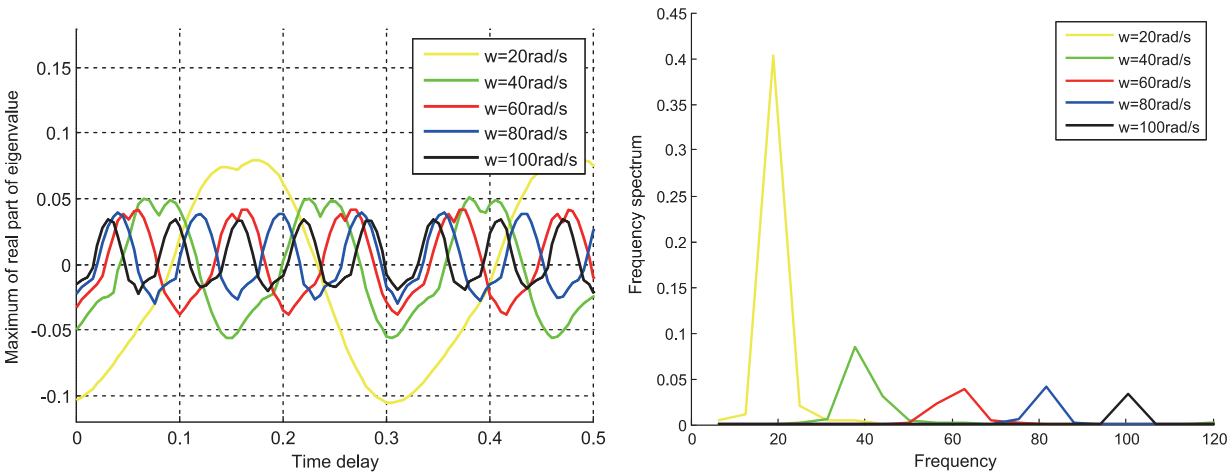
Fig. 6. Comparison of the amplitude-frequency curves of DDBG DVA by the analytical method (solid line) and the numerical one (circle line).

4. Stability analysis for the approximate solution

As illustrated in many literatures [25–28,35], time delay will affect the dynamical response significantly, especially the system stability. Here we would analyze the stability for the steady-state responses of the four semi-active DVAs.

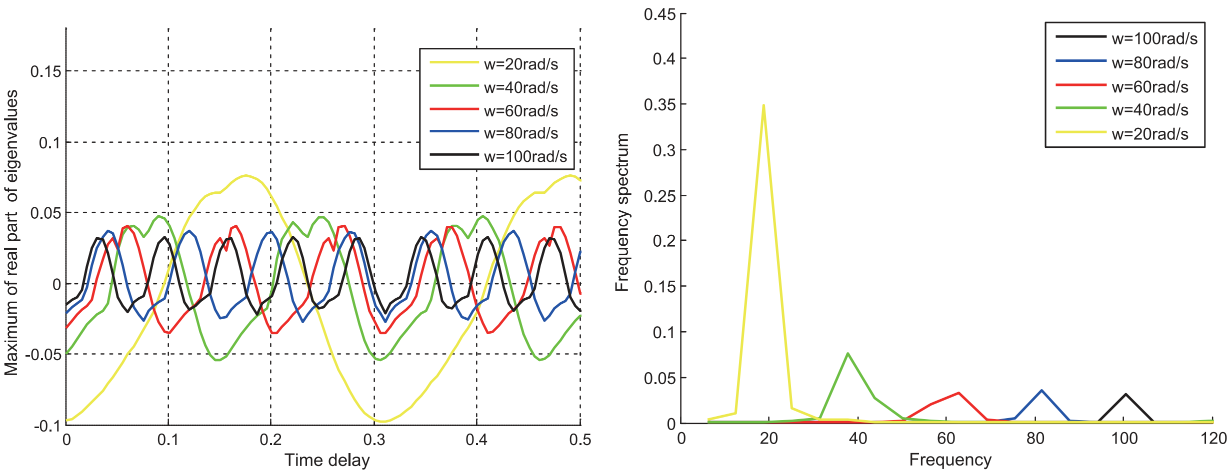
Introducing $a_1 = \bar{a}_1 + \Delta a_1$, $\theta_1 = \bar{\theta}_1 + \Delta\theta_1$, $a_r = \bar{a}_r + \Delta a_r$, $\theta_r = \bar{\theta}_r + \Delta\theta_r$ in Eqs (16), (19), (22) and (25), expanding all the terms and omitting the higher-order terms, one could get

$$\begin{pmatrix} \dfrac{d\Delta a_1}{d\delta} \\ \dfrac{d\Delta\theta_1}{d\delta} \\ \dfrac{d\Delta a_r}{d\delta} \\ \dfrac{d\Delta\theta_r}{d\delta} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{pmatrix} \begin{pmatrix} \Delta a_1 \\ \Delta\theta_1 \\ \Delta a_r \\ \Delta\theta_r \end{pmatrix}. \tag{26}$$



(a) (b)

Fig. 7. Effect of time delay on the stability of VVBG DVA, (a) the largest values of the real parts of characteristic roots; (b) frequency spectrum.



(a) (b)

Fig. 8. Effect of time delay on the stability of DVBG DVA, (a) the largest values of the real parts of characteristic roots; (b) frequency spectrum.

For the four semi-active DVAs, the first-order perturbations of the steady-state solutions have the same forms as Eq. (26), where the differences are based on the entries A_{ij} ($i = 1, \ldots, 4, j = 1, \ldots, 4$). Due to the complicated computation, symbolic software such as Mathematica is used in this procedure. Because the entries A_{ij} are huge in forms, we do not show it herein.

According to Lyapunov theory, the steady-state solutions of the four semi-active DVAs would be stable, if and only if the real parts of all the characteristic roots of matrix A are negative. Accordingly, we could determine the stability of the steady-state responses by detecting the largest value of the real parts for the four characteristic roots in the four semi-active DVAs. The results are shown in Figs 7–10.

From these figures, it could be found that the largest value of the real parts of the characteristic roots is periodic functions of time delay, which is similar to the effects of time delay on the continuous dynamical system. Moreover, the spectrums show that the period of stability condition is the same as the excitation one.

Although these figures look similar or almost the same, they belong to different kinds of semi-active control DVAs. The similar results verify the four semi-active DVAs have similar control performance, so that engineers could select the appropriate control manner according to the existing control conditions they own.

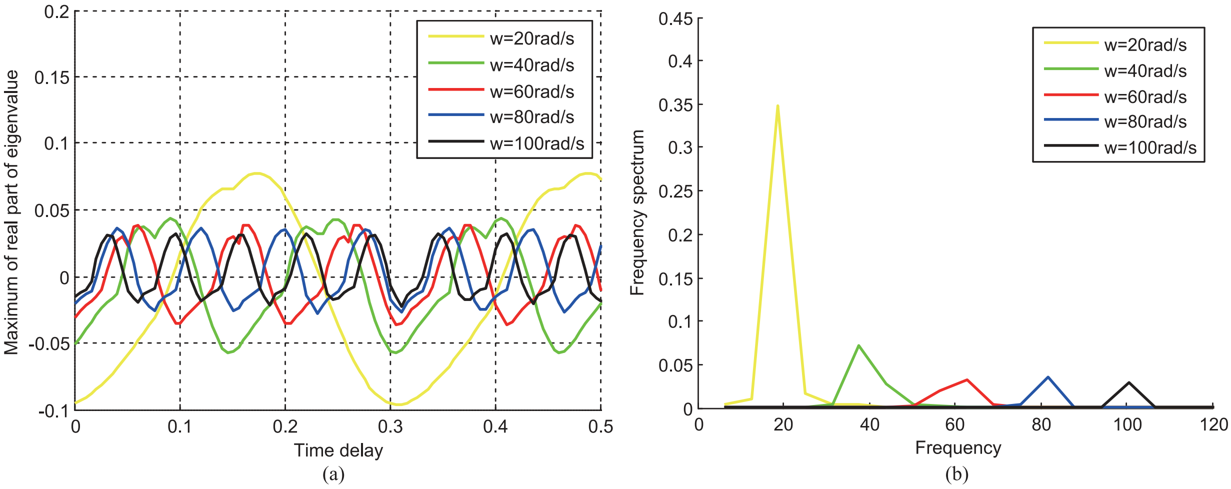


Fig. 9. Effect of time delay on the stability of VDBG DVA, (a) the largest values of the real parts of characteristic roots; (b) frequency spectrum.

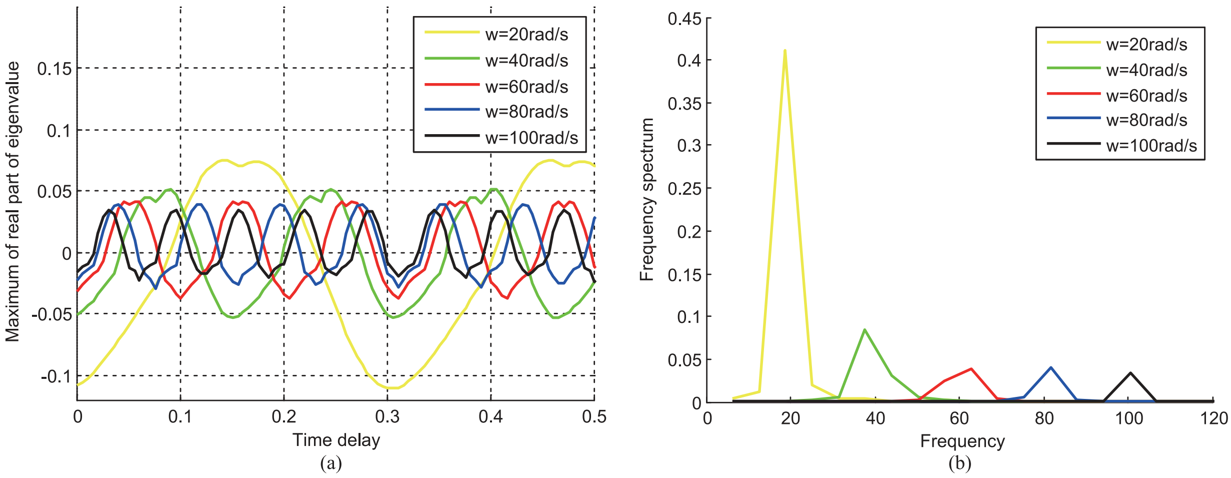


Fig. 10. Effect of time delay on the stability of DDBG DVA, (a) the largest values of the real parts of characteristic roots; (b) frequency spectrum.

5. Conclusion

In this paper, the first-order approximate solutions of four semi-active on-off DVAs are studied analytically by averaging method. The comparisons between the analytical and the numerical solutions verify the correctness and satisfactory precision of the analytical results. The stability analysis of the steady-state solution for the four semi-active DVAs is also fulfilled. The analytical and illustrated results show that the amplitude and stability condition for the steady-state solutions are all periodic functions of time delay, which is important to design and/or retrofit vibration control system. Moreover, the analytical study makes the selection for system parameters more convenient than the direct numerical integration.

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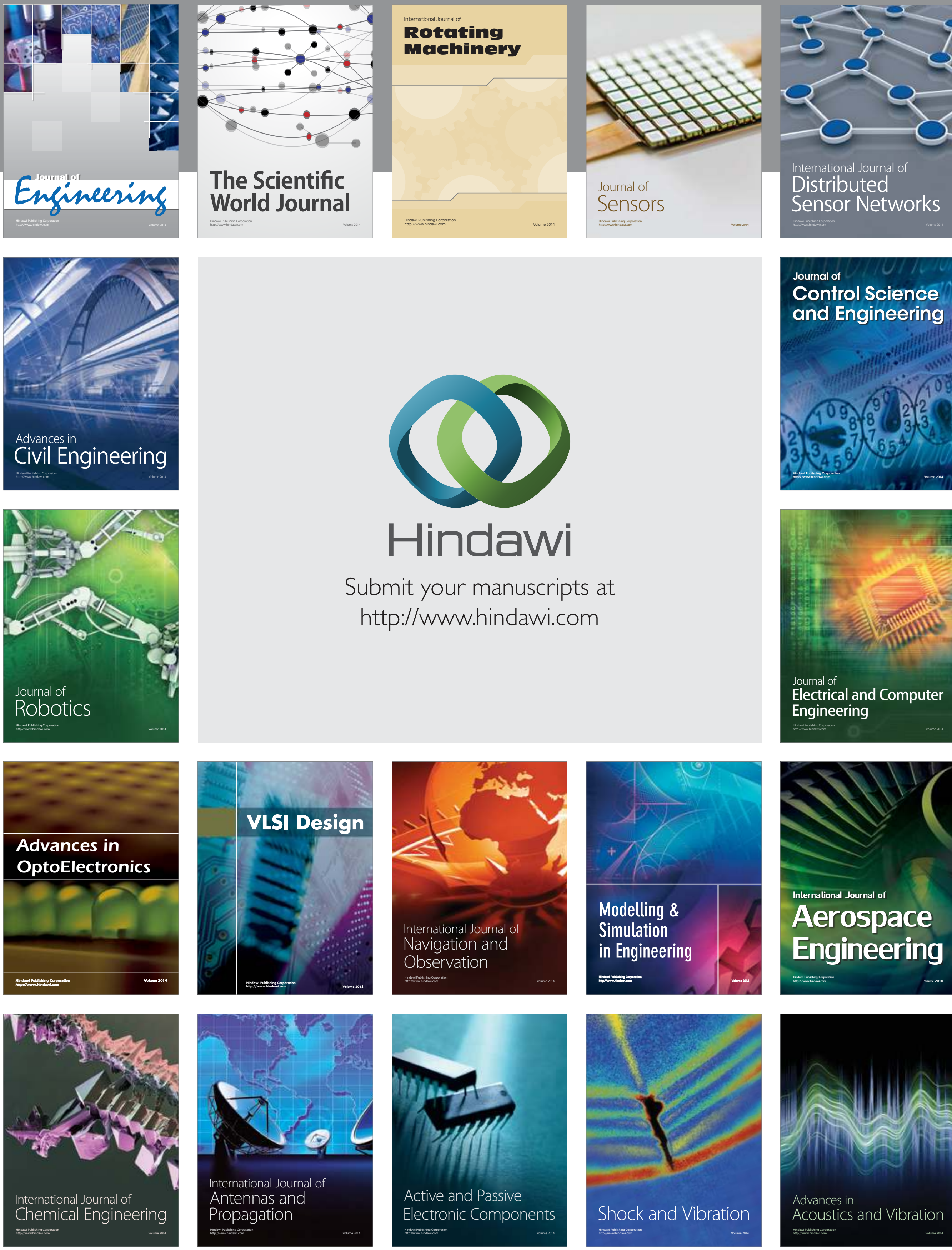
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