

Nonlinear Dynamics of a Laser Diode With Optical Feedback Systems Subject to Modulation

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Abstract—The nonlinear dynamic behavior of a direct frequency-modulated diode laser with strong optical feedback is examined and compared to a laser diode subject to electro-optically modulated, strong optical feedback. Direct modulation is achieved by sinusoidal modulation of the diode laser injection current. Electro-optic modulation is achieved by applying a sinusoidal voltage to an intracavity phase modulating element. The output state (characterized by the output power versus time, the intensity noise spectrum and the optical frequency spectrum) for both types of modulation is dependent on the ratio of the modulation frequency to the external cavity resonant frequency, and the modulation power. A number of distinct states are observed: conventional amplitude modulation (with FM spectra); multimode, low-noise amplitude modulation; multimode, high-noise amplitude modulation; periodic limit-cycle operation; quasi-periodicity; chaos; low-frequency fluctuations; and mode-locking. There are significant differences between the direct and electro-optic frequency-modulation cases. The onset of the dynamic instability is characterized as a noisy period-one oscillation for direct modulation and a low-frequency fluctuation for intracavity electro-optic modulation. Phase portraits produced experimentally with the use of a digital phosphor oscilloscope are shown to agree well with those constructed from output power versus time data. This represents an experimental method for examining the dynamics phase portraits in real-time.

Index Terms—Laser diode, modulated laser diode, nonlinear dynamics, optical feedback.

I. INTRODUCTION

DIRECT modulation of the injection current and/or the introduction of optical feedback into diode lasers have been shown (both theoretically and experimentally) to induce various dynamic instabilities in the output power of these devices, when the levels of modulation and/or feedback have certain ranges of values. These dynamic instabilities influence many of the operating parameters, such as the optical frequency spectrum and the intensity noise spectrum. Limit-cycle periodicity, quasi-periodicity, and chaotic output have been observed in directly modulated diode lasers, dependent on the modulation frequency, injection current, and modulation power [1]–[8]. Similarly, the introduction of optical feedback into an (unmodulated) semiconductor laser can induce a range of dynamic behaviors dependent on the feedback level. Weak (regime III) or strong (regime V) optical feedback (into a diode laser with an anti-reflection coated front facet) results in reduced output noise, increased side-mode suppression, and increased laser stability [9]. Moderate (regime IV) feedback levels result in an induced dynamic

instability, the output power exhibiting periodic or chaotic fluctuations [10].

The output characteristics of a diode laser that is subject to both optical feedback and either direct current modulation or intracavity phase modulation [using an electro-optic modulator (EOM)] has attracted less attention. Previous studies have demonstrated that introducing strong optical feedback into a frequency-modulated (direct or intracavity EOM) diode laser can enhance the modulation bandwidth [11]–[14], induce modal instabilities [14]–[17], produce mode-locked output pulses [18]–[23], frequency-locked states [24], low-frequency fluctuations [25], [26], and quasi-periodic [24] and chaotic output power [27], depending on the system operating parameters. No single study has considered the full range of parameters over which all of these states are observed.

Due to the strong coupling of the refractive index to the carrier density in semiconductor diode lasers, a direct modulation of the injection current results in a modulation of both the output power and the output frequency. The frequency modulation results in an optical frequency spectrum that consists of the center lasing carrier frequency and a series of sidebands separated by the modulation frequency. The relative amplitudes of the sideband pairs are given by a Bessel function [28] that depends on the modulation power (amplitude of the driving modulation). All frequencies are not present simultaneously; the output frequency sweeps through the bandwidth of the comb of modes at the modulation frequency [28], [29]. The accompanying amplitude (power) modulation manifests itself as an asymmetry in the sideband pairs [30]. The ratio of the frequency modulation to the intensity modulation is dependent on the modulation frequency, the linewidth enhancement factor, and the relaxation oscillation frequency (hence injection current) of the diode laser [31].

Generally, low-modulation powers result in period-1 sinusoidal oscillation. However, under certain circumstances, higher order limit cycle behavior, quasi-periodicity, and chaos are possible [1]–[8]. These states depend on the ratio of the modulation frequency to dominant resonant frequencies of the laser. Resonant frequencies of solitary diode lasers include the relaxation oscillation frequency [3], the pulsing frequency (for self-pulsing diode lasers [32]), and the longitudinal-mode frequency. As the longitudinal mode spacing is typically 100 GHz for diode lasers, the effects of this resonance are negligible at modulation frequencies lower than the maximum possible (~ 20 GHz, as determined from the relaxation oscillation frequency damping [31]).

Introducing strong optical feedback from an external mirror introduces an additional resonant frequency, that of the external cavity, to the system. This can be made comparable to the modulation frequency. Such a system is an analog of the

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FM laser [14], [28]. Additionally, in an external cavity diode laser system, modulation can be applied using an intracavity phase-modulating element, which is frequency matched to the external cavity. Strong feedback allows multiple passes through the modulating element. Combined with a close, but not exact, match of the modulation frequency to the external cavity resonant frequency, this leads to an enhanced modulation index. Only a small modulation index on the diode laser output is achievable by a single pass through the phase modulator (with no external mirror). The result of such phase-modulated feedback is a frequency-modulated output, and such a system is termed an FM laser [14], [28].

For both direct and electro-optic modulation, mode-locking is possible (at low injection currents) if the modulation frequency matches the external cavity resonant frequency (or its harmonics) [21], [22]. At higher injection currents, the modulation index (bandwidth of the optical frequency spectrum) is enhanced for modulation frequencies close to the cavity resonant frequency [11]–[14]. However, as the modulation power is increased, there is a transition into a multimode state that has previously been considered a dynamic instability similar to the regime-IV coherence collapsed state for diode lasers with optical feedback [14]–[17]. For the case of direct modulation, operating at modulation frequencies further from the cavity resonant frequency has been shown to induce a frequency-locked state (periodic output pulses) at integer fractions of the ratio between the two frequencies [24], and a quasi-periodic state for noninteger fractions of the two frequencies [27].

The current paper reports a systematic experimental investigation into the output state of a directly modulated quantum-well diode laser subject to strong external optical feedback as compared to the same laser subject to strong optical feedback which is phase modulated by an electro-optic modulator in the external cavity. The system without modulation operates in regime V. The output state is characterized by its spectral properties (optical frequency spectrum) and temporal properties (intensity noise spectrum and output power versus time). The output state is examined as a function of the modulation power and modulation frequency for a fixed injection current and external cavity length.

It is found that the modulation-induced multimode instability (observed for modulation frequencies close to the external cavity resonant frequency for either direct or electro-optic) is similar spectrally to the coherence-collapsed state observed for diode lasers with moderate feedback levels (regime IV). However, it has temporal characteristics, which are distinct from the regime-IV state. It does not represent a chaotic nor a quasi-periodic trajectory. Additionally, this unstable state is dynamically distinct for the cases of direct and electro-optic modulation. The direct modulation instability is a limit cycle (AM) with significantly increased noise, and the electro-optic modulation instability is a type of low-frequency fluctuation. The latter is closer dynamically to the coherence-collapse state induced by optical feedback alone.

As the modulation frequency (for direct modulation) is detuned from the external cavity resonant frequency, the output switches to a new state that is chaotic or of high periodicity. Limit 1 behavior is observed for modulation frequencies close

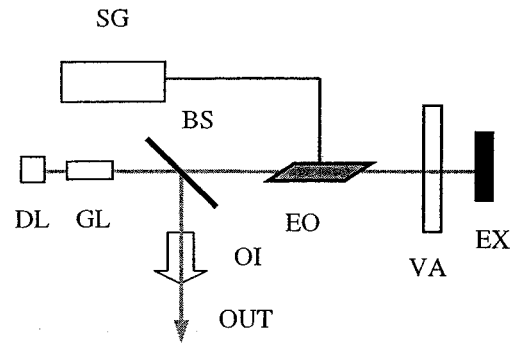


Fig. 1. Experimental setup for indirect modulation of a diode laser with feedback. DL: diode laser. GL: GRIN rod lens. BS: intracavity beamsplitter. OI: optical isolator. EO: electro-optic modulator. VA: variable attenuator. EX: external mirror. SG: microwave signal generator. OUT: output to diagnostics. For direct modulation, the signal generator is connected to the diode laser input current with a bias tee.

to multiples of the external cavity resonant frequency. Also observed are windows of periodic orbits. The spectral output is distinct for each of these states. The presence of chaos and instability are inferred from a combination of phase diagrams and estimates of the time series correlation dimension as predicted by a Grassberger algorithm [33], [34]. It is unknown whether the same behavior would be observed for electro-optic modulation at higher modulation powers, as the use of a tuned *LC*-lumped modulator means that the single-pass modulation index is a strong function of frequency and efficient electro-optic modulation is only achievable for a band of frequencies close to the external cavity resonant frequency.

II. EXPERIMENT

The experimental setup (for electro-optic modulation) is shown in Fig. 1. The solitary diode laser is a 50-mW SDL quantum-well index-guided 850-nm device, which is mounted on a heat sink and temperature controller. The output is collimated with a GRIN rod lens. Feedback is provided from an external plane mirror of 95% reflectivity. The external cavity length is fixed at ~ 317 mm. An uncoated intracavity beamsplitter (10 mm thickness) is used to monitor the system output and the feedback fraction is fixed at 65%. The coupling coefficient of this optical feedback to the laser diode is 0.22. This leads to system operation well within the strong feedback, single-mode region (regime V) for this type of diode laser [35], [36]. The free-running threshold injection current is approximately 25 mA, which is reduced to 20 mA with feedback. The injection current is fixed at 35 mA.

Indirect (electro-optic) frequency modulation is achieved with a Brewster cut lithium niobate crystal (a capacitor, *C*), coupled to an inductor *L*, to form an *LC*-lumped modulator. The center frequency of the resonance of this modulator is set close to the external cavity resonant frequency. The lumped modulator is driven by a sinusoidal signal from a Rhode & Schwarz signal generator. Direct frequency modulation is achieved by applying a radio frequency signal (from the signal generator) to the dc bias of the diode injection current, with a bias Tee.

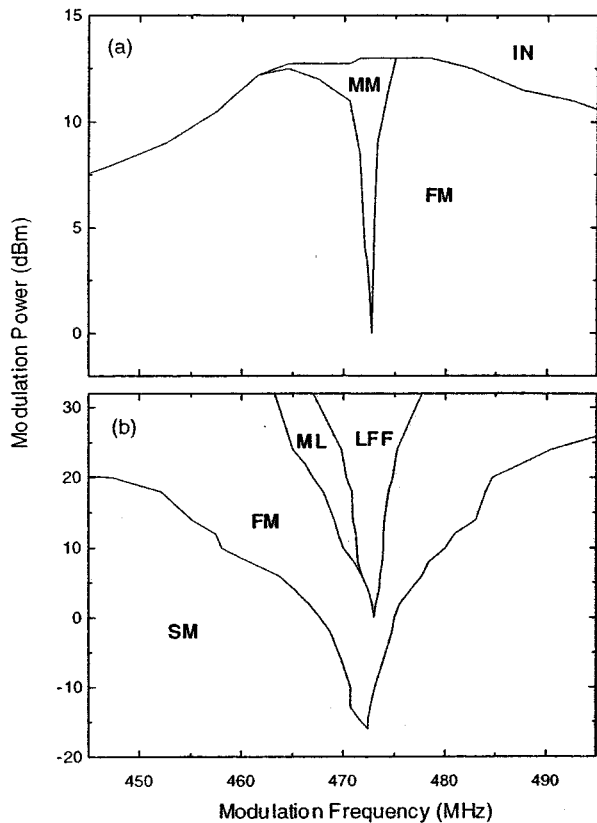


Fig. 2. Output map of dynamic states close to the external cavity resonant frequency (473 MHz) in the parameter space of modulation power and modulation frequency, for: (a) direct modulation and (b) electro-optic modulation. IN: temporal instability. MM: multimode instability. FM: frequency modulation. LFF: low-frequency fluctuation. ML: mode-locking. SM: single-mode small-signal modulation.

The output power is monitored with an Ophir (Nova Display) power meter, and with a fast photodiode (bandwidth ~ 3 GHz) connected to a 2-GHz digital phosphor oscilloscope (DPO). The oscilloscope can be used as a digital storage oscilloscope (DSO) to study real-time transients or in DPO mode. In DPO mode, the real-time signal and color-coded average distribution of the signal over time are displayed against time. This allows many of the dynamics to be observed by reducing the effect of the output noise. Additionally, the output power can be viewed against itself delayed in time using the X-Y mode DSO mode so that a phase portrait is produced in real-time. Changing the length of cable varies the delay time. The optical spectrum is recorded with a 10- or 1000-GHz FSR Fabry-Perot interferometer. The output intensity noise spectrum is recorded with a radio-frequency spectrum analyzer.

III. RESULTS AND DISCUSSION

A. Modulation Close to the External Cavity Frequency

The output power of the directly modulated solitary diode laser (with no optical feedback) is a limit-1 cycle, representing a sinusoidal modulation of the injection current at the modulation frequency transferred directly to the output power (ie amplitude modulation). The depth of this modulation increases with modulation power, until for large values (>20 dBm) either

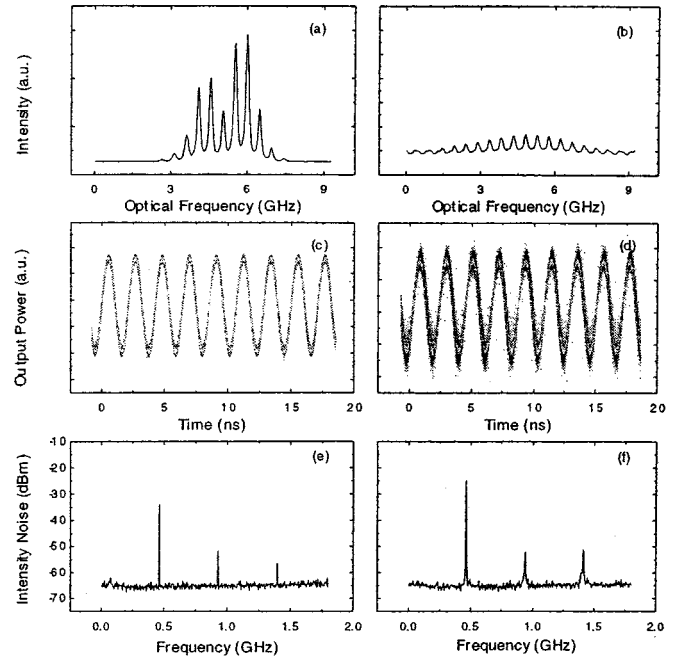


Fig. 3. Direct frequency-modulated diode laser: (a), (b) optical frequency spectrum; (c), (d) output power versus time; and (e), (f) intensity noise spectrum. (a), (c), (e) FM state. (b), (d), (f) State of multimode (MM) instability.

gain-switching, or period-doubling behavior is observed dependent on the ratio of the modulation frequency to the relaxation oscillation frequency (injection current). The former is observed at low injection currents when the modulation can bring the drive current below threshold for some fraction of the modulation cycle.

For modulation powers below 20 dBm, the modulation transfer function, which represents the ratio of the intensity modulation index (determined from the depth of the output power modulation) to the frequency-modulation index (determined from the Bessel function relative amplitudes of the carrier and first sidebands in the optical frequency spectrum) for the solitary diode laser is relatively flat over the modulation frequency interval investigated (10 MHz to 2 GHz). There is a small peak at the relaxation oscillation frequency of the solitary diode (~ 2 GHz for $I = 35$ mA).

Initially, the behavior is examined for a modulation frequency close to the external cavity resonant frequency. For direct modulation a map of the observed states in the parameter space of modulation power and modulation frequency is shown in Fig. 2(a) for an external cavity ~ 371 mm in length (determined from the measured external cavity resonant frequency 473 MHz [37]). For low-modulation powers (<0 dBm), the output is typical of an FM laser. The optical frequency spectrum [Fig. 3(a)] shows sidebands with Bessel function amplitudes, and the output power [Fig. 3(c)] is a period-1 sinusoidal oscillation, which manifests as strong peaks at multiples of the modulation frequency in the noise spectrum [Fig. 3(e)]. The modulation depth on the laser diode output power and the modulation index scales with the modulation power. As the modulation frequency is tuned close to the external cavity resonant frequency, the output power modulation depth remains

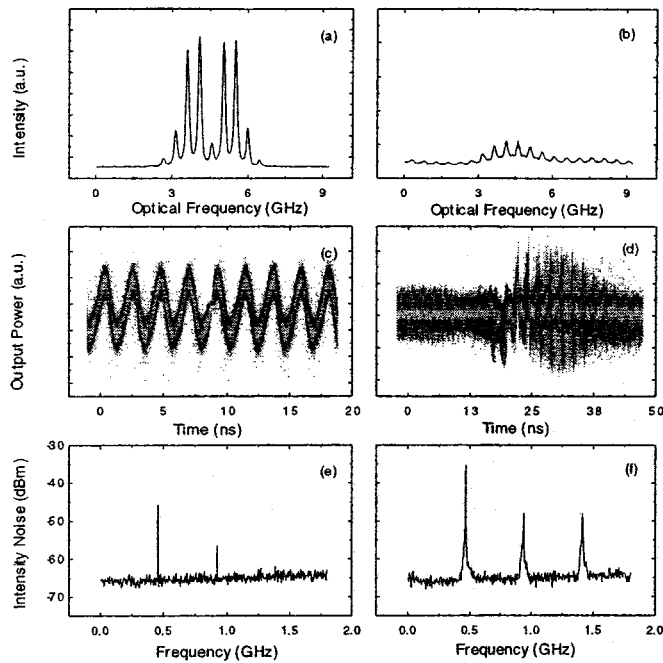


Fig. 4. Electro-optic frequency-modulated diode laser: (a), (b) optical frequency spectrum; (c), (d) output power versus time; and (e), (f) intensity noise spectrum. (a), (c), (e) FM state. (b), (d), (f) Low-frequency fluctuation state.

constant but the frequency-modulation index increases. This enhancement occurs at a resonant frequency near the external cavity resonant frequency, dependent on the feedback level and the external cavity length [11], [12].

For increased modulation power, there is a region of multimode output (labeled MM in Fig. 2) centered at a detuning of -2 MHz from the external cavity resonant frequency. This region, as observed previously [16], [17], is characterized by an oscillation on multiple external cavity modes [Fig. 3(b)]. Such an optical state is similar to the coherence collapsed state observed in diode lasers operated with feedback appropriate to regime IV, which represents the development of optical chaos. However, the abrupt and distinct transition into this new modal state is not accompanied by a drastic change in the output power. The output power remains in an amplitude modulation state (similar to the FM lasing state) but with a small increase in the noise [Fig. 3(d)]. This increased noise also manifests itself as a broadening of the peaks in the intensity noise spectrum [Fig. 3(f)].

When electro-optic modulation is applied, the system behaves somewhat differently. A map of the output states is shown in Fig. 2(b). There is no measured correspondence between the modulation power used for the direct modulation and modulation power applied to the EOM. However, it is clear that the enhanced modulation index for Figs. 3(a) and 4(a) are similar by virtue of the total bandwidth of the optical frequency spectrum being close. At modulation frequencies near to the external cavity resonance, four distinct states are observed. For low modulation powers, or large detuning, the output is a single frequency (with no resolvable sidebands) and the output power does not show oscillations above the noise. This represents very-small-signal modulation (labeled SM in Fig. 2). As the

modulation power is increased or the detuning from the external cavity resonant frequency is decreased, an FM lasing region is observed with an output spectrum consisting of a Bessel function mode envelope [Fig. 4(a)], with an oscillating output power [Fig. 4(c)]. The amplitude modulation on the diode laser output power is less than 5% of that observed in the direct modulation case for the same FM bandwidth. This results in lower asymmetry in the FM sideband pairs. Because the smaller amplitude fluctuations are closer in amplitude to the diode laser output noise, the sinusoidal oscillation appears noisier in Fig. 4(c) and the intensity noise peaks are lower [Fig. 4(e)]. A further increase in the modulation power (or decrease in the frequency detuning) results (as for direct modulation) in a multimode instability [Fig. 4(b)] and a widening of the intensity noise peaks [Fig. 4(f)]. However, dynamically this instability is an intermittent state similar to the low-frequency fluctuation (LFF) state observed in diode lasers with optical feedback at low injection currents. On a time scale of several hundred nanoseconds, it comprises rapid power drop-outs, followed by resonant recovery at combinations of the external cavity and relaxation oscillation frequencies [Fig. 4(d)].

For the electro-optic case, the instability close to the external cavity resonant frequency is predicted to occur theoretically using an iterative model based on a perturbation approach. It is attributed to mode competition between external cavity modes and coupled cavity modes [38]. This is in partial agreement with the experimental observations. The theory predicts a dynamic state consisting of large quasi-periodic amplitude modulation with irregular (intermittent) behavior over longer time scales, similar to the coherence collapse state in diode lasers with optical feedback but with greater periodic structure than is generally observed for the case of optical feedback alone [38]. To the authors' knowledge, no current theoretical treatment describes the dynamic (MM) instability observed for the direct modulation case.

Close to the transition between the LFF and FM states, on the low-frequency side, a small region of mode-locked type operation is observed (ML). In this state the output power consists of "pulsed" oscillations with increased amplitude modulation relative to the FM amplitude modulation. In general, the amplitude of the pulses increases (and the width decreases) as the injection current is decreased toward threshold. The mode-locked state is accompanied by an optical frequency spectrum that has the appearance of a highly asymmetric FM spectrum, not the gaussian mode envelope that is expected for true mode-locking.

For direct and EO modulation, the multimode instability and LFF, respectively, is centered at ~ 2 -MHz lower than the external cavity resonant frequency. This indicates that, although the instabilities are distinct states, they are probably driven by the same mechanisms. The reason for the offset of the external cavity resonant frequency from the center of the instability region is unclear.

The external cavity resonant frequency is determined by increasing the injection current in the unmodulated diode laser with strong feedback. Beyond a certain threshold value, the diode laser output is found to operate multimode. This multimode state has been attributed to spatial and/or spectral hole-burning effects in this type of diode laser [37]. Current

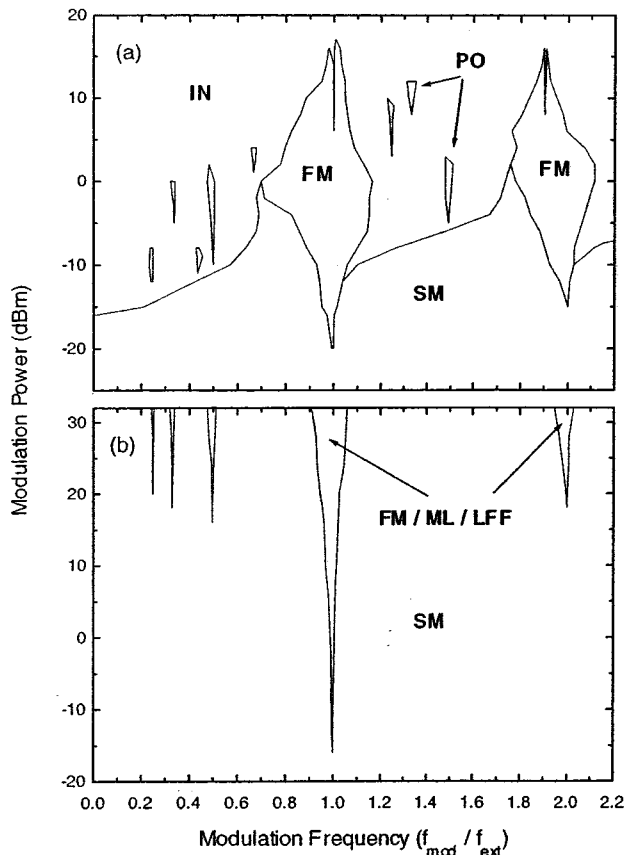


Fig. 5. Output map of dynamic states from 50 MHz to 1 GHz ($0.1\text{--}2 f_{\text{ext}}$) for: (a) direct modulation and (b) electro-optic modulation. PO: periodic orbit. Other states are as referred to in Fig. 2.

observations indicate that this state is dynamically a low-frequency fluctuation-type instability. Within this multimode region, a strong peak is observed in the output intensity noise spectrum (which is independent of feedback or injection current) indicating the external cavity resonant frequency. For the direct modulation case, the offset does not change with external cavity length (for cavities from 400 MHz to 2 GHz examined). For electro-optic modulation resonant modulation is not possible for shorter cavity lengths with the lumped EOM currently used. The offset may be due to competition between the external cavity frequency, and the FM resonant frequency. The latter is the frequency of maximum FM bandwidth for a fixed modulation power. For both electro-optic and direct injection current modulation, this FM resonant frequency occurs at a frequency offset from the external cavity resonant frequency, which is determined from the feedback fraction and the external cavity length (higher feedback fractions and shorter cavity lengths result in lower offset frequencies and higher modulation bandwidths) [11], [12].

B. Modulation Over an Extended Range of Frequencies

In Fig. 5, the output states are examined for a larger range of modulation frequencies relative to the external cavity resonant frequency. For the electro-optic case [Fig. 5(b)], the dominant state is the single-mode (SM) very-small-signal modulation due to the small modulation index that is achieved over most of the

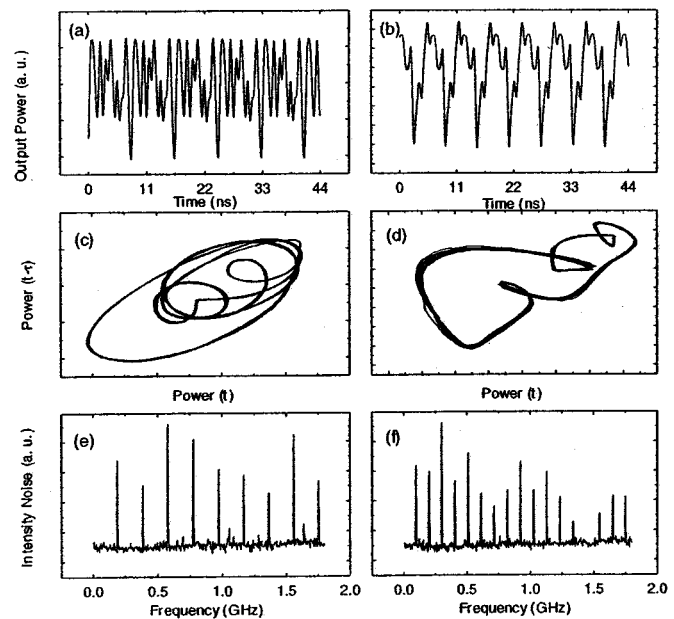


Fig. 6. Direct frequency-modulated diode laser in a periodic state: (a), (b) output power versus time; (c), (d) reconstructed phase space portrait, and (e), (f) intensity noise spectrum. Modulation frequency: (a), (c), (e) 705 MHz ($3/2 f_{\text{ext}}$) and (b), (d), (f) 158 MHz ($1/3 f_{\text{ext}}$).

modulation frequency range due to the EO modulator being a lumped LC -tuned circuit. FM lasing, mode-locking, and low frequency fluctuations are observed for small regions centered around sub-harmonics and multiples of the external cavity resonant frequency. These occur due to the narrow resonances of the electro-optic crystal when the modulation index is enhanced by the Q of the lumped modulator.

For direct modulation [Fig. 5(a)], several more distinct dynamic output states are observed. Low modulation powers result in a nearly single-mode spectrum with significant amplitude modulation. Large regions of FM and multimode output are observed at modulation frequencies close to multiples of the external cavity resonant frequency. The dominant state is a temporal instability. Within this region of temporal instability, there are distinct regions of periodic oscillation. These states occur at integer fraction ratios of the modulation frequency to the external cavity resonant frequency. The output power for two examples of such periodic oscillation is given in Fig. 6(a) and 6(b), showing period 14 and period 10 behavior, respectively. The phase portraits [Figs. 6(c) and 6(d)] are reconstructed by plotting the output power $P(t)$ versus the delayed output power $P(t + \tau)$ following the method of [39]. In each case, the intensity noise spectrum shows strong peaks at some multiple of the modulation frequency. No relationship between the periodicity of the periodic states and the modulation parameters has so far been determined.

The observed limit cycle oscillations are generic to nonlinear systems characterized by the presence of two competing frequencies [40]. In this case, the competing frequencies are the modulation frequency and the external cavity resonant frequency. At modulation frequencies that are integral multiples or submultiples of the external cavity frequency, frequency-locking occurs. Similar results were demonstrated

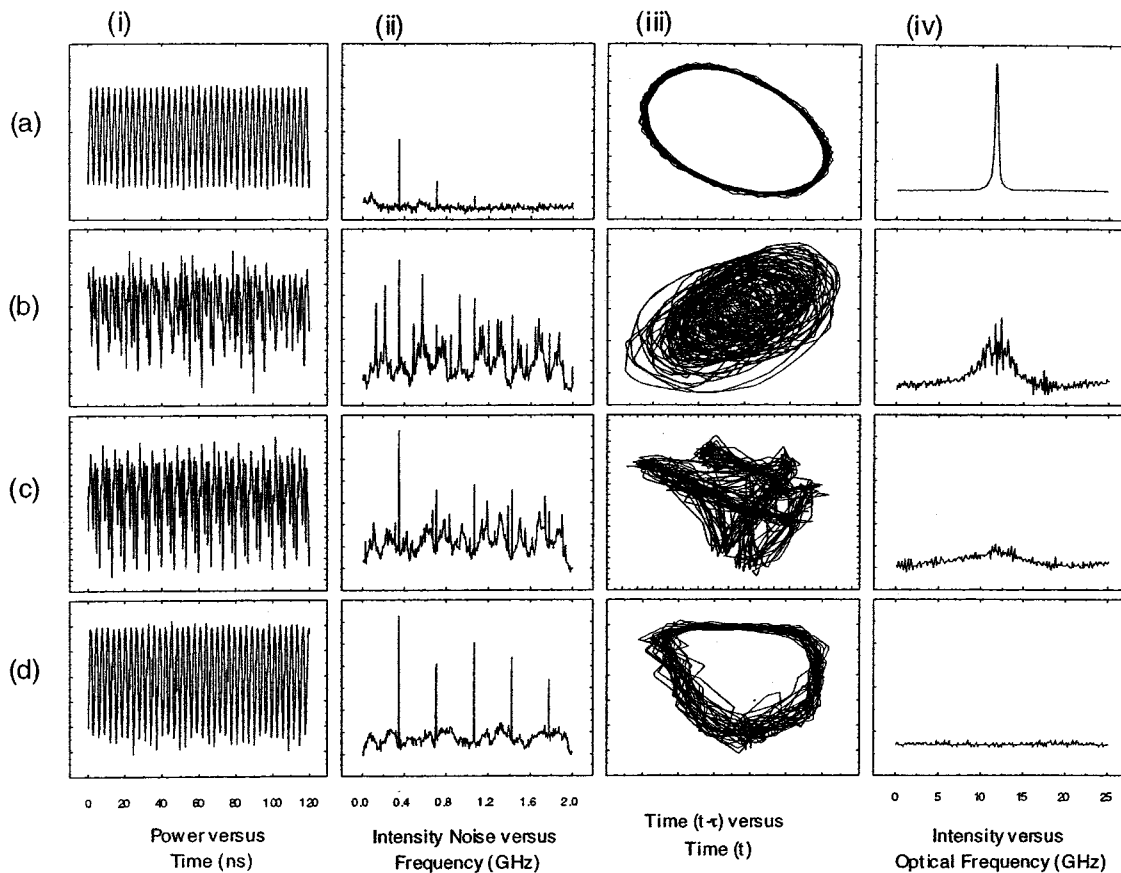


Fig. 7. (i) Output power versus time; (ii) intensity noise spectrum; (iii) reconstructed phase space portrait; and (iv) optical frequency spectrum for the direct frequency-modulated diode laser at a modulation frequency of 292 MHz, and a modulation power of: (a) -5 dBm; (b) 0 dBm; (c) 5 dBm; and (d) 13 dBm.

in [24]. However, in [24], the frequency-locking regions were characterized by pulses with a repetitive sequence of amplitudes, rather than the periodic power variations observed here. Similar periodic power variations have also been observed in self-pulsing diode lasers modulated with frequencies that are fractions of the pulsing frequency [32].

Within the region corresponding to temporal instability, a number of output states with distinct characteristics are observed. Fig. 7 shows four such states which are characterized by the output power, reconstructed phase portrait, intensity noise spectrum, and optical frequency spectrum observed at increasing modulation powers for a fixed modulation frequency (292 MHz). The ratio of the modulation frequency to the external cavity frequency is 0.618 . This represents the golden mean value $(\sqrt{5} - 1)/2$, which is the hardest irrational number to approximate by rational fractions. Thus, this modulation frequency is as far from any periodic oscillation as is possible.

As the modulation power is increased, the output switches from the period 1 (single mode) amplitude modulation state [Fig. 7(a)] to a multimode state with more noise [Fig. 7(b)] and no apparent structure in the phase space portrait. As the modulation index is increased further, the optical frequency spectrum broadens, the intensity noise increases further, and structure is evident in the phase space portrait [Fig. 7(c)]. At high modulation powers, a state with very noisy oscillating power is observed [Fig. 7(d)]. This state is common to all modulation frequencies at high modulation powers.

As a more rigorous analysis of the dynamic states observed in Fig. 7, an estimation of the correlation dimension for each of these output states has been performed. The correlation dimension is a measure of the information or fractal dimension of a time series. A standard Grassberger algorithm is used. This involves embedding the time series in an n -dimensional space and counting the distance between points. The correlation dimension is then obtained by examining regions of the slope of the correlation integral that merge to a common value as the embedding dimension is increased (see [33], [34], and [40] for more details). Data sets of $4 \cdot 10^5$ data points have been used. As expected, the correlation integral for the time series in Fig. 7(a) is 1, representing a periodic oscillation. Correlation dimension estimation indicates that the time series in Fig. 7(b) is quasi-periodic, and the time series in Fig. 7(c) is chaotic, with a dimension of 2.45 . This agrees with theoretical predictions of periodicity being bounded by quasi-periodic oscillations that develop into chaotic fluctuations when the driving force is increased, for such two-frequency systems [40]. For the high modulation power [Fig. 7(d)] state, however, the correlation dimension does not converge for embedding dimensions lower than 16 and, therefore, cannot be defined. This indicates that this state is either a very noisy state, or high dimensional chaos.

Often from the time trace of the oscilloscope, the exact output dynamic state is difficult to ascertain in real time. In Fig. 8, two examples of real-time-phase space portraits are shown. Fig. 8(a) shows the output of the direct frequency-modulated diode laser

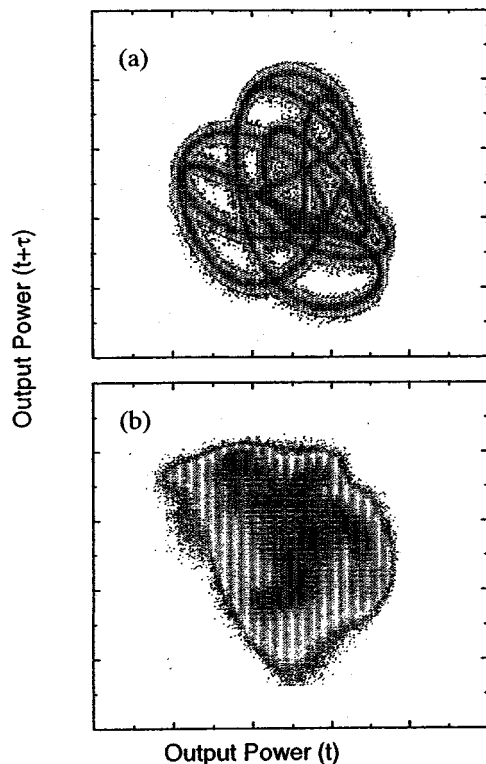


Fig. 8. Real-time experimental phase space portrait for the diode laser in a periodic state (modulation frequency 315 MHz ($2/3 f_{\text{ext}}$), modulation power 2 dBm), and in a chaotic state (modulation frequency 292 MHz ($0.618 f_{\text{ext}}$), modulation power 5 dBm).

in a periodic state [similar to that in Fig. 6(a)], and Fig. 8(b) shows the output in a chaotic state [similar to that shown in Fig. 7(c)]. This diagnostic mode allows for real-time experimental analysis of time series and, in these experiments, has been demonstrated to give good agreement with phase portraits generated from the output power versus time data.

IV. CONCLUSION

The dynamic and spectral output states of a frequency-modulated diode laser with optical feedback have been compared for direct injection current and electro-optic modulation. A large number of different dynamic states have been observed, dependent on the ratio of the external cavity frequency to the modulation frequency. Generally, a richer array of states is observed for the direct modulated system, as in the current experiments the modulation index can be increased to much higher values over an extended frequency range in this case. The narrow resonance and limited maximum modulation frequency of the lumped phase modulating crystal used for electro-optic modulation prevent similar modulation indices being achieved for all but the frequencies near resonance in the electro-optic modulation case.

The modulation instability, as it first occurs, when induced by increased modulation at a frequency close to the external cavity resonant frequency, is shown to be dynamically distinct for direct modulation (a “noisy” period 1 oscillation) compared to electro-optic modulation (a low-frequency fluctuation). This

indicates that the instability in the electro-optic modulation case is more akin to those generated by optical feedback alone.

The results presented here reconfirm the complexity in such diode laser systems. There are a large number of variable parameters in this system. Varying two of these parameters, the modulation power, and frequency, eight distinct dynamic states have been observed. The role of the external cavity length and the injection current have yet to be ascertained in a complete systematic way. Variation of the injection current is expected to further complicate the system due to the variation of the relaxation oscillation frequency, which can become harmonically resonant with either the external cavity frequency or the modulation frequency. Such factors are difficult to predict theoretically due to the number of variable parameters and the number of unknown or imprecisely known physical characteristics of the diode laser.

A useful method for real-time measurement of the phase space portrait of the system has been demonstrated. The resulting phase portraits are in good agreement with those generated from output power versus time data recorded with a 0.5-ps time resolution. Using this method enables the different dynamic states to be readily identified for more careful study.

There is every reason to believe a similarly diverse map of dynamic behaviors would be generated for electro-optic modulation, but in the current experiments, high modulation indices are achieved in a narrow range of frequencies about the resonance of the lumped modulator. Thus, it is these frequencies where high modulation index behavior has been studied. Small modulation index behavior is seen over an extended frequency range with narrow regions of frequency modulation, mode-locking, and low-frequency fluctuations being seen at harmonics and sub-harmonics of the resonant frequency of the EOM, which is also tuned to be very close to the external cavity frequency.

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