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Abstract

A model of a stack/wire system, wind-induced vibration of the stack based on an unsteady-flow theory, and nonlinear dynamics of the stack's heavy elastic suspended cables was developed in this study. The response characteristics of the stack and cables are presented for different conditions. The dominant excitation mechanisms are lock-in resonance of the stack by vortex shedding and parametric resonance of suspended cables by stack motion at their support ends.

INTRODUCTION

A 100-m high stack supported by guy wires at four levels (see Fig. 1) was susceptible to large-amplitude oscillations, and some of the guy wires at the lower two levels had been damaged when wind speed exceeded 15 m/s (54 km/h) for a period of time. The excitation mechanism was identified through scoping calculations, analytical prediction with a finite-element code, and observation of the stack/wire response [1-2].

The dimensional parameters and material properties of the system shown in Fig. 1 [1], referred to as the original system, are summarized in Table 1. The damping ratios for the stack and wires are assumed to be a few percent. The Young's modulus of helically woven guy wire is assumed to be 6.2×10^{10} - 10.3×10^{10} N/m^2 [3].

The objective of this paper is to derive a coupled nonlinear dynamic model of the wind-induced vibration of the stack based on an unsteady-flow theory, and the heavy elastic suspended cable whose upper end is subject to bending vibration of the stack. Numerical analysis of the coupled system presents the effect of fluid/structure interaction and cable parameters on parametric and external resonances of cables.

LOCK-IN RESONANCE OF STACK

Vortex shedding across a bluff body has been studied for more than 100 years. Many reviews on this subject are available [4-8]. A fluid/structure system with wind-induced vibration of a stack can be described by an unsteady-flow theory [9]. Once fluid excitation forces and motion-dependent fluid forces are known, the response of the stack with vortex-shedding-induced vibration can be predicted. The stack is subjected to a crossflow wind uniformly along its length ℓ_s. The equation of motion in the lift direction is

$$EI\frac{\partial^4 w(z,t)}{\partial z^4} + C\frac{\partial w(z,t)}{\partial t} + m\frac{\partial^2 w(z,t)}{\partial t^2} + \alpha\frac{\partial^2 w(z,t)}{\partial t^2} - \frac{\rho U^2}{\omega}\alpha'\frac{\partial w(z,t)}{\partial t}$$
$$-\rho U^2 \alpha'' w(z,t) = \frac{1}{2}\rho U^2 D C_L' \cos(\omega_s t), \tag{1}$$

where w(z,t) is the displacement of the stack in the lift direction, D is the stack diameter, U is the wind speed, ρ is the air density, EI is the flexural rigidity, C is the stack damping coefficient, m_s is the stack mass per unit length, C_L' is the fluctuating lift coefficient, and ω_s (= $2\pi SU/D$) is the circular frequency of vortex shedding. α, α', and α'' are the added-mass, fluid-damping, and fluid-stiffness coefficients, respectively. All of the fluid force coefficients are based on experimental data [10-11].

Let

$$w(z,t) = \sum_{n=1}^{\infty} D a_n(t) \psi_n(z), \qquad U_r = \frac{U}{fD}, \qquad \gamma = \frac{\rho \pi D^2}{4m}, \tag{2}$$

where $\psi_n(z)$ is the n-th normal mode, f is the oscillation frequency, and U_r is reduced wind speed. Substituting Eqs. 2 in Eq. 1, yields

$$\frac{d^2 a_n}{dt^2} + 2\zeta\omega \frac{da_n}{dt} + \omega^2 a_n = \frac{1}{2(1+\gamma\alpha)} \left(\frac{\rho U^2 C_L' D}{m} \right) \cos(\omega_s t), \tag{3}$$

where

$$\omega = \omega_v \left(1 + \gamma C_M\right)^{-0.5}, \qquad \zeta = \frac{\zeta_v}{1+\gamma\alpha} \left[\left(1 + \gamma C_M\right)^{0.5} - \frac{\gamma U_r^2 \alpha'}{2\zeta_v \pi^3} \right],$$

$$C_M = \alpha + \frac{U_r^2 \alpha''}{\pi^3}, \qquad C_n = \frac{1}{\ell_s} \int_0^{\ell_s} \psi_n(z) dz. \tag{4}$$

Note that ω and ζ are the circular frequency and modal damping ratio, respectively, for the stack in crosswind flow. C_M is called an added mass coefficient; when $U_r = 0$, it is equal to α. When $U_r \neq 0$, C_M depends on both U_r and α", which in turn, depends on U_r and the stack oscillation amplitude. ω_v and ζ_v are the in-vacuum natural frequency and modal damping ratio, respectively [10].

When guy wires were modeled as springs and the stack was modeled as a Bernoulli-Euler beam with the lower end fixed and the top end free, the first four modes of all models of the stack were analyzed with the finite-element code MSC-PAL, because the previous study [1] showed that coupling between the stack and guy wires is important only for low-frequency modes. The natural frequencies and natural modes of the stack are given in Fig. 2 for the first four modes.

Consider a stack with the mass ratio γ of 0.2 and damping ratio ξ_v of 2%, a Strouhal number S of 0.175, and a fluctuating lift coefficient C_L of 0.5. The root mean square (RMS) values of the four-point stack motions (corresponding to four guy-wire ends) in the lift direction were plotted as the function of wind speed in Fig. 3.

It is noted that for the third mode, lock-in resonance occurs at $U \approx 15$ m/s, and point 2 (45.7 m) associated with the second-level guy wire has the largest oscillating amplitude. From this observation, the vibration mode was about the same as the mode shape of the third mode (Fig. 2c). Because the upper portion of the stack had spoilers and the lower portion did not, this particular mode was vulnerable to vortex-shedding-induced resonance due to a large participation factor associated with vortex shedding [1]. From the calculation (Fig. 3) and the observation, we concluded that the stack vibration was excited by vortex shedding at the lower portion associated with the third mode of the stack.

PARAMETRIC RESONANCE OF GUY WIRES

Forced vibration of elastic suspended cables has been studied by many investigators [12-16]. However, very little has appeared in the literature that reports work on parametric and external resonances of suspended cables. Perkins [17] derived a nonlinear model of a suspended elastic cable under parametric and external excitation. Cai and Chen [2] derived a nonlinear model of in-plane motion of a heavy elastic cable in a tilted configuration. This model included a pulsating excitation at the support at the upper end of the cable due to lock-in resonance of the stack. Axial motion at the support leads to parametric excitation, whereas transverse motion contributes to both parametric and external excitations. Therefore, the angle of the cable, which determines the ratio of axial and transverse motions, is very important to the parametric and external resonances [2].

Consider the original system in Fig. 1: the guy wires can be described as heavy elastic cables suspended between two supports in an angled configuration (see Fig. 4). The lower end of the cable is fixed to the ground and the upper end is pin-supported and movable horizontally to simulate the bending motion of the stack due to vortex shedding. Thus, the motion components at the support at the upper end in both axial and transverse directions of the cable u and v are

$$u(\ell_k,t) = -w(z_k,t)\cos\theta, \tag{5}$$

$$v(\ell_k,t) = w(z_k,t)\sin\theta, \tag{6}$$

where ℓ is the distance between the two supports of the cables and α is the angle between the cable and the ground; w is the stack motions at the upper end of cables and can be calculated from Eqs. 2 and 3. Subscript k represents the levels of wires.

The initial static equilibrium configuration C^I in Fig. 4 lies in the OXY plane and is represented by the function y(s), s being a curvilinear abscissa. Let E, H, A, and m_c be the elastic modulus, the tension, the cross-sectional area, and the mass per unit length of the cable, respectively. The varied configuration C^v can be described by the displacement coordinate u(x,t) and v(x,t) [2].

The dimensionless static equilibrium configuration of the cable can be derived easily:

$$\frac{dy}{dx} = \text{ctg}\theta\left[\frac{\beta\cdot\sin\theta}{(1+\beta\cdot\sin\theta\cdot x)\ell n(1+\beta\cdot\sin\theta)} - 1\right] \quad (7)$$

where $\beta = \frac{m_c g \ell}{H}$.

Let

$$\mu = \frac{EA}{H}, \quad (8)$$

and

$$\omega_n = \frac{n\pi}{\ell}\sqrt{\frac{H}{m_c}}, \qquad \zeta_n = \frac{c}{2m_c\omega_n}, \qquad n = 1,2,3,..., \quad (9)$$

where ω_n is the frequency and ζ_n is the damping ratio of the cable in the n-th mode. Let

$$v(x,t) = v_1 + v_2 = \sum_{n=1}^{\infty} q_n(t)\phi_n(x) + xw, \quad (10)$$

where $x/\ell = 1$, v_2 [= xw] corresponds to the boundary condition at the support at the upper end (see Eq. 3), $q_n(t)$ is the generalized transverse displacement of the cable, and $\phi_n(x)$ is the modal function and can be described simply as

$$\phi_n(x) = \sqrt{2}\sin n\pi x \quad (11)$$

and

$$\int_0^\ell \phi_n^2(x)dx = 1. \quad (12)$$

Then, by applying the Galerkin method, a system of ordinary differential equations is obtained [2].

$$\ddot{q}_n + 2\zeta_n\left(\frac{\omega_n}{\omega}\right)\dot{q}_n + \left(\frac{\omega_n}{\omega}\right)^2\left(\left\{1+\mu\left[w\cos\theta + I_o w\sin\alpha + \frac{1}{2}(w\sin\theta)^2\right]\right\}q_n\right.$$

$$-\frac{\mu}{n^2\pi^2}(I_{1n}w\sin\theta + I_{5n})(I_{2n}w\sin\theta + I_{7n})\sum_{n=1}^{\infty}q_n + \mu\left[I_{6n} - \frac{1}{n\pi}(I_{2n}w\sin\theta + I_{7n})\right]q_n\sum_{n=1}^{\infty}q_n$$

$$\left.+\mu I_{3n}q_n\sum_{n=1}^{\infty}q_n^2\right) = -I_{4n}\sin\theta\left[\ddot{w} + 2\zeta_n\left(\frac{\omega_n}{\omega}\right)\dot{w}\right] \qquad n = 1,2,3,\ldots, \qquad (13)$$

where I_0, I_{jn} ($j = 1,2,3,\ldots7$) are defined in Ref. 2.

Dynamic response of cables can be numerically calculated from Eqs. 3 and 13 together as a function of wind speed. Figure 5 gives nondimensional RMS displacements of the cable at $x/\ell_k = 0.500$ for the first four modes of the stack as a function of wind speed of U, with S = 0.175, $C_L' = 0.5$, $H = 2.85 \times 10^4$ N, $\zeta = 0.02$. Regardless of the values of the excitation amplitudes, the cable was subjected to external resonance at lock-in resonances of the stack. When the vortex-shedding frequency is the of cable's natural frequency, parametric resonance exists corresponding to the first primary parametric instability frequencies, i.e., primary parametric instabilities occur near $\omega_s = 2\omega_1$. These phenomena can be viewed clearly in Fig. 5. In Fig. 5a, which corresponds to mode 1 of the stack (f = 1.27 Hz), the first primary parametric resonance occurs in the level-four cable, whose natural frequency is 0.73 Hz at U = 11 m/s. In Fig. 5b, which corresponds to mode 2 of the stack (f = 1.66 Hz), the first primary parametric resonances occur in the level-four cable, whose natural frequency is 0.73 Hz at U = 11 m/s, and in the level-three cable, whose natural frequency is 0.90 Hz at U = 13.5 m/s. In Fig. 5c, which corresponds to mode 3 of the stack (f = 2.20 Hz), the second primary parametric resonances occur in the level-four cable, whose natural frequency is 0.73 Hz at U = 11 m/s, and the first primary parametric resonances occur in the level-two cable, whose natural frequency is 1.1 Hz at U = 14.5 to 18.5 m/s. In Fig. 5d, which corresponds to mode 4 of the stack (f = 2.56 Hz), no parametric resonances occur under selected stack and cable parameters over the entire range of wind speed.

Figure 6 shows power spectral densities of cable displacement at $x/\ell = 0.5$, corresponding to Fig. 5c, at U = 17.5 m/s. At this wind speed, the vortex-shedding frequency is 2.2 Hz, the third stack-mode frequency is 2.2 Hz, and the second-level-cable natural frequency is 1.1 Hz. The figure clearly shows external resonances for all four cable levels at lock-in resonance frequency. However, at the second-level cable, very strong parametric resonance is demonstrated (see first peak in Fig. 6 for the level-2 cable).

To better understand the effects of cable tension and damping on parametric resonances, RMS displacements of the second-level cable with stack excitation mode 3 were calculated for (a) $\zeta = 0.02$, and $H = 1.85 \times 10^4$ N, 2.85×10^4 N, 4.25×10^4 N, and 6.50×10^4 N, where the parametric resonance effect is reduced as the cable tension increases; and (b) $H = 2.85 \times 10^4$ N, and $\zeta = 0.01$, 0.02, 0.03, and 0.04, where the parametric resonance windows subject to wind speed narrowed as cable damping increases.

In the original stack/wire system, the dominant frequency of the stack oscillation was the third mode [1]. It was about twice the natural frequency of the wires when wire tensions were within certain ranges; this made parametric resonance possible. Corresponding to the third mode of the stack, the bending displacement amplitudes are very small at the two upper levels (Fig. 1). At the two lower levels, the displacement amplitudes reach ≈25-50 mm. From Fig. 5c, at U = 14.5 to 18.5 m/s, parametric resonance will definitely dominate cable motion. Moreover, it is evident that small dampings and certain tension values are more likely to cause parametric resonance. From these calculation results, it is not surprising that some of the wires, only at the lower two levels, were damaged by large-amplitude oscillations [1].

CONCLUSIONS

A coupled model of wind-induced vibration of a stack, based on an unsteady-flow theory and nonlinear dynamics of heavy elastic suspended cables, was developed in this study. Numerical analysis of the coupled system results in good agreement with observations of the original stack/wire response. The excitation mechanisms of the fluid/structure system were identified as (a) lock-in resonance of the stack by vortex shedding and (b) parametric resonance of the suspended cables by stack motion at the cable support ends. Wind speed, the fluctuating lift coefficient, cable tension, and damping are key to parametric resonance of the cables. Adjusting cable tension to certain values, which will change the natural frequency of the cables, may eliminate parametric resonance of the cables. Even if resonance is not completely eliminated, however, the vibration amplitudes of the wires are expected to be much smaller. Installation of damping ropes on the wires will further reduce wire vibration.

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Table 1. Dimensional parameters and material properties of system shown in Fig. 1

Parameter/ property	Stack	Guy wire
Cross-sectional area, cm^2	366.46	1.54
Density, g/cm^3	1.66	7.83
Inside diam., m	1.219	0.0
Outside diam., m	1.238	0.016
Mass/unit length, kg/m	61.08	1.20
Young's modulus, kg/cm^2	8.09×10^4	$6.2 \approx 10.3 \times 10^{10}$

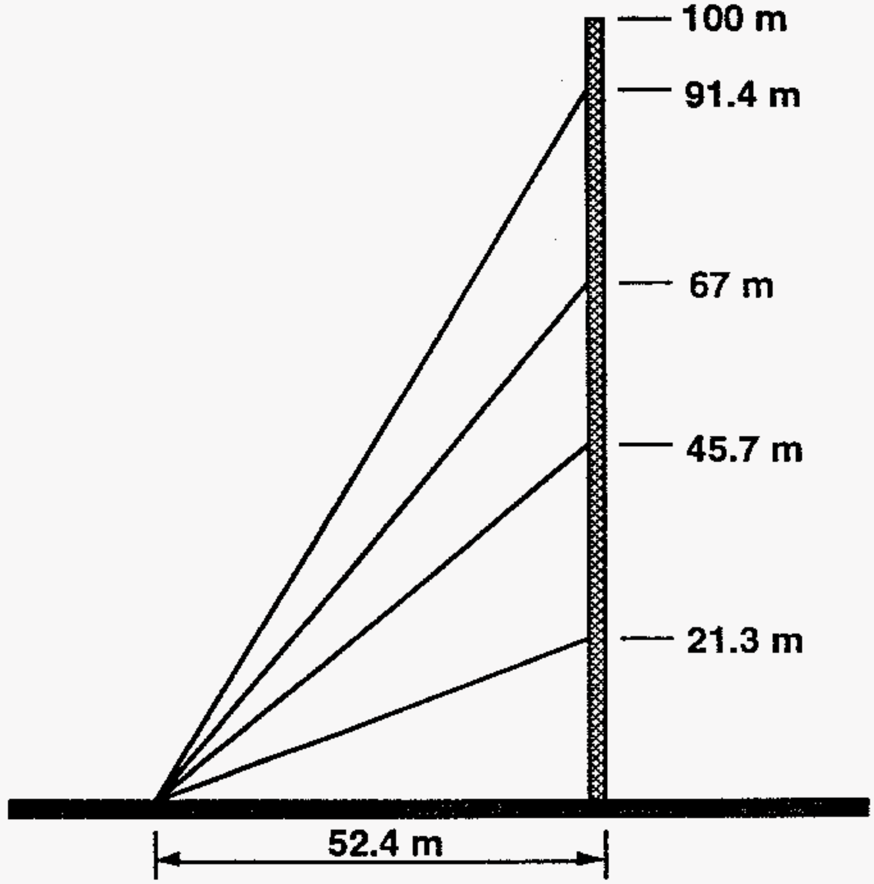

Fig. 1. Stack supported by guy wires at four levels

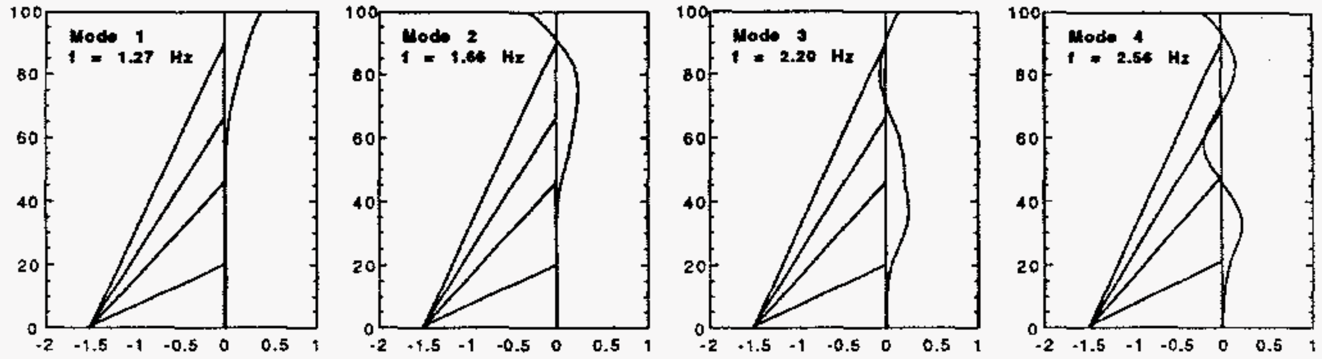

Fig. 2. Frequencies and mode shapes of first four natural modes of stack

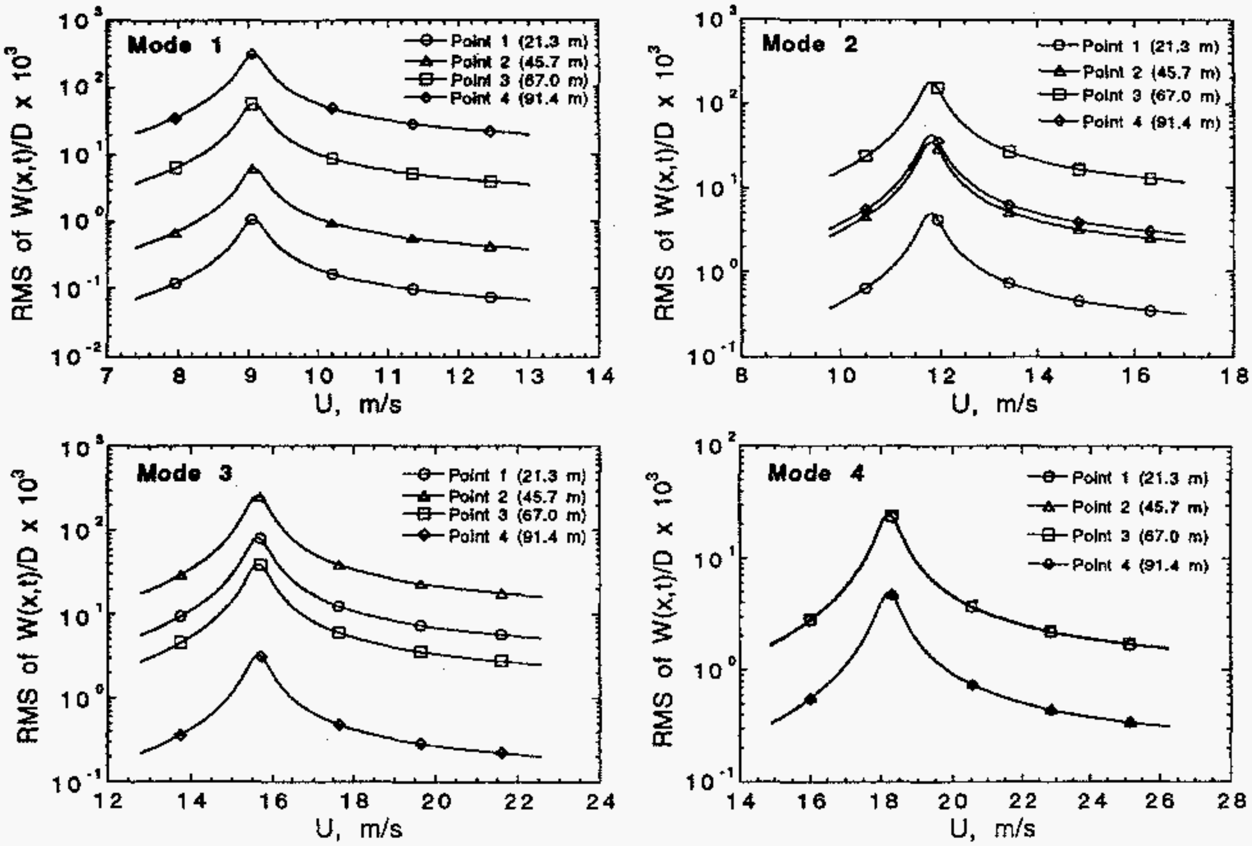

Fig. 3. RMS values of stack motions as a function of wind speed

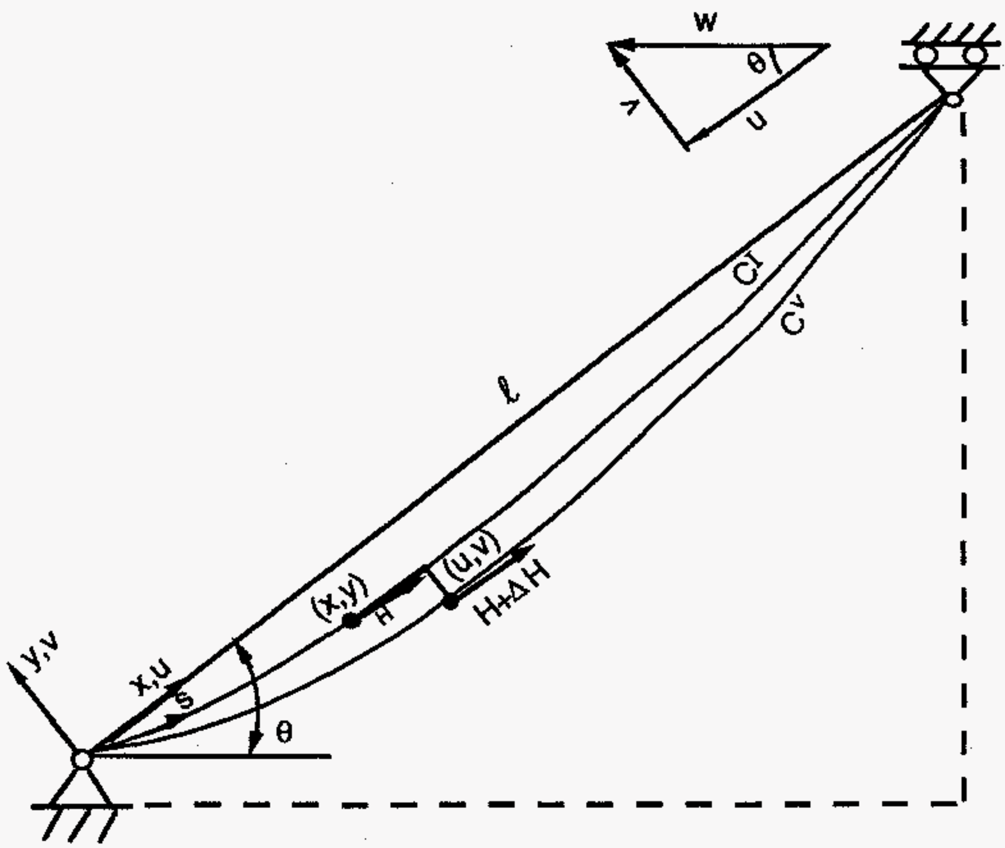

Fig. 4. Cable supported in angled configuration

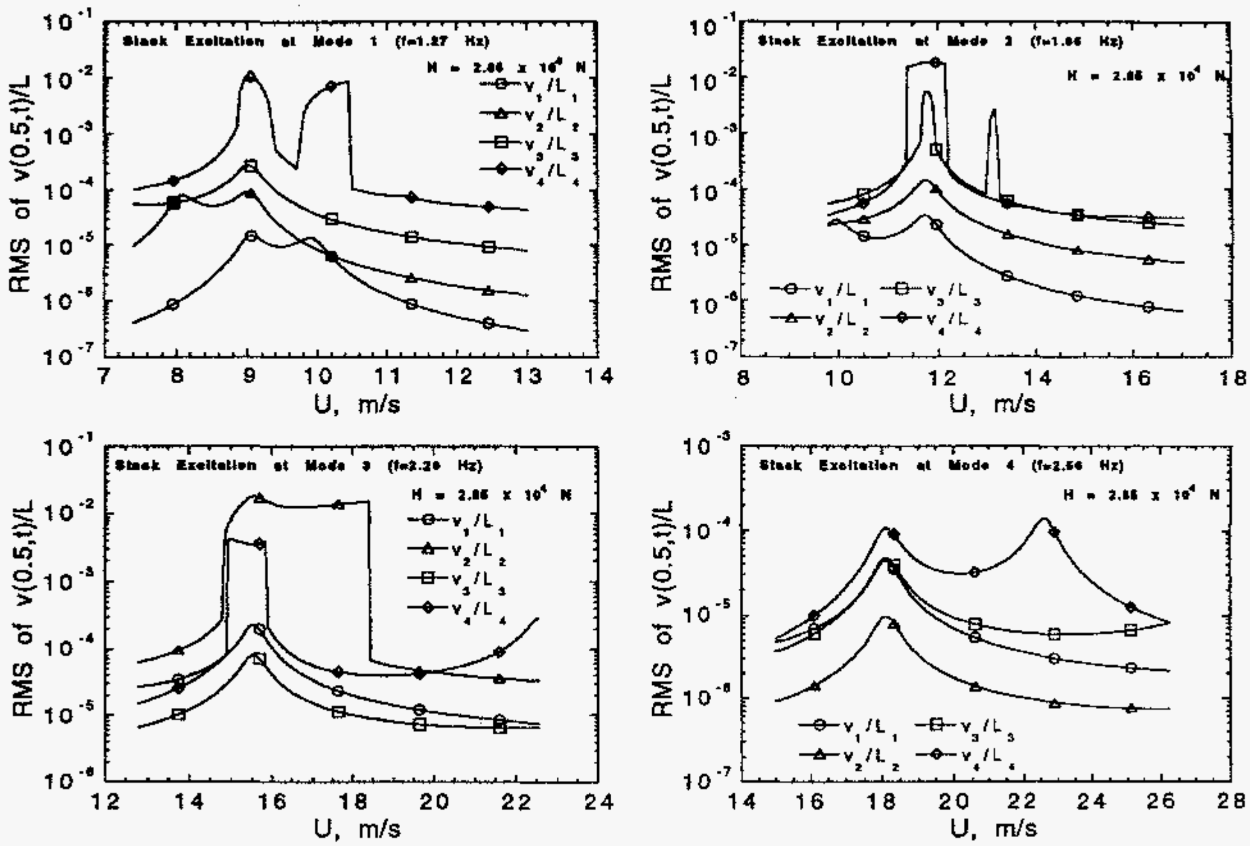

Fig. 5. Nondimensional RMS displacements of cable at $x/\ell = 0.5$