Nonlinear Dynamics of Rotating Multi-Component Pair Plasmas and e-p-i Plasmas*)

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The propagation of small amplitude stationary profile nonlinear electrostatic excitations in a pair plasma is investigated, mainly drawing inspiration from experiments on fullerene pair-ion plasmas. Two distinct pair ion species are considered of opposite polarity and same mass, in addition to a massive charged background species, which is assumed to be stationary, given the frequency scale of interest. In the pair-ion context, the third species is thought of as a background defect (e.g. charged dust) component. On the other hand, the model also applies formally to electron-positron-ion (*e-p-i*) plasmas, if one neglects electron-positron annihilation. A two-fluid plasma model is employed, incorporating both Lorentz and Coriolis forces, thus taking into account the interplay between the gyroscopic (Larmor) frequency ω_c and the (intrinsic) plasma rotation frequency Ω_0. By employing a multi-dimensional reductive perturbation technique, a Zakharov-Kuznetsov (ZK) type equation is derived for the evolution of the electric potential perturbation. Assuming an arbitrary direction of propagation, with respect to the magnetic field, we derive the exact form of nonlinear solutions, and study their characteristics. A parametric analysis is carried out, as regards the effect of the dusty plasma composition (background number density), species temperature(s) and the relative strength of rotation to Larmor frequencies. It is shown that the Larmor and mechanical rotation affect the pulse dynamics via a parallel-to-transverse mode coupling diffusion term, which in fact diverges at $\omega_c \to \pm 2\Omega_0$. Pulses collapse at this limit, as nonlinearity fails to balance dispersion. The analysis is complemented by investigating critical plasma compositions, in fact near-symmetric ($T_- \approx T_+$) "pure" ($n_- \approx n_+$) pair plasmas, i.e. when the concentration of the 3rd background species is negligible, case in which the (quadratic) nonlinearity vanishes, so one needs to resort to higher order nonlinear theory. A modified ZK equation is derived and analyzed. Our results are of relevance in pair-ion (fullerene) experiments and also potentially in astrophysical environments, e.g. in pulsars.

Keywords: pair plasma, electron-positron plasma, soliton, Zakharov-Kuznetsov equation

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1. Introduction

Significant research effort has recently been devoted to *pair plasmas* (*p.p.*), a term denoting large ensembles of charged matter consisting of charge particle populations bearing equal masses and opposite charge signs [1, 2]. In contrast to ordinary (electron-ion, *e-i*) plasmas, where the large mass disparity between plasma constituents imposes distinct frequency scales, the pair species (of equal but opposite charge) respond on the same scale. Not quite expectedly, plasma wave characteristics cannot always be deduced from known results for *e-i* plasmas by simply taking a formal limit of equal masses. For instance, parallel propagating linear electromagnetic (EM) waves are not circularly but linearly polarized in pair plasmas, and Faraday rotation [3] is remarkably absent therein [4]. Furthermore, ion-acoustic waves have no counterpart in electron-positron (*e-p*) plasmas, where electrostatic wave dispersion may bear high frequency (Langmuir-type) characteristics [1, 2]. Remarkably, the recent production of pair

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fullerene-ion plasmas in laboratory [5] has enabled experimental studies of pair plasmas allowing one to get rid of intrinsic problems involved in *e-p* plasmas, namely pair recombination (annihilation) processes.

In general, *e-p* plasmas may also be characterized by the presence of positive ions, in addition to electrons and positrons. Electron-positron-ion (*e-p-i*) plasmas occur in various astrophysical contexts, such as the early universe [6], active galactic nuclei (AGN) [7] and in pulsar magnetospheres [8], and have also been created in the laboratory [9]. The standard description of *e-p-i* plasmas adopted here models them as fully ionized plasmas with two populations of different charge signs possessing equal masses and absolute charge values ($m_+ = m_- = m$, $q_+ = -q_- = Ze$), in addition to a population of positively charged ions, with $m_3 = M \gg m$ and $q_3 = +Ze$ (here e is the magnitude of the electron charge). On the other hand, one may anticipate the existence (intrinsically, or by intentional injection/doping) of a small fraction of charged massive particles (a heavier ion species, or dust particulates, defect) into fullerene pair-ion plasma [5] in order to realize three-component plasmas which may accommodate new physical phenomena. According to these considerations, we shall henceforth keep the charge sign $s = q_3/|q_3|$ arbitrary in our model.

Experimental investigations of low-amplitude (linear) electrostatic (ES) oscillations suggest the existence of three distinct modes [5]. Two of these modes, namely an acoustic mode and a Langmuir-like high-frequency mode, are predicted by theory [1, 2, 10, 11]. An intermediate-frequency mode also reported [5] is still a topic of controversial debate among theoreticians, and various alternative interpretations have been furnished, in terms of soliton-trains [12], ion acoustic waves accelerated by surplus electrons [13] or BernsteinGreeneKruskal (BGK)-like trapped ion modes [14]. Although, the experiments mentioned above rely on a symmetric pair-component plasma preparation ("pure" *p.p.*, i.e. equal number densities, and equal temperatures among the pair species), we shall prefer to leave the density and temperature ratio(s) of the positive-to-negative ion species arbitrary, i.e. not necessarily equal to unity (viz. pure *p.p.*).

Nonlinear excitations predicted to occur in pair plasmas and *e-p-i* plasmas include ES solitons [12, 15], EM solitons [16] and localized-envelope modulated wavepackets (envelope solitons) of either low-frequency ES [10,11], high-frequency ES [17] or EM [18] type, relying on an exhaustive use of the nonlinear plasma toolbox, including the Korteweg - de Vries (KdV) and Zakharov-Kuznetsov (ZK) Equation(s), the Sagdeev pseudopotential formalism and the nonlinear Schrödinger (NLS) Equation, respectively.

Introducing a different building block of our work, plasma *rotation* was first considered as a key element in plasma dynamics by Chandrasekhar [19] who suggested that Coriolis forces might play a role in cosmic phenomena, as indeed supported by subsequent studies regarding astrophysical plasma environments [20]. Several authors have therefore attempted to investigate wave propagation in rotating plasmas via linear [21] and nonlinear [22] models.

In this paper, we investigate the existence and properties of nonlinear electrostatic structures in rotating magnetized three-component pair-plasmas of either the doped *p.p.* (as described above) or the *e-p-i* kind. Nonlinear electrostatic structures are shown to exist, in the form of solitary waves, nonlinear periodic wave-forms and blow-up pulse excitations. Earlier results on envelope solitons in p.p. are also briefly reviewed. The role of the stationary background species (affecting the charge balance) is stressed.

2. The Model

We consider the propagation of electrostatic excitations in a magnetized, rotating, collisionless three-component plasma, consisting of positive ions (mass m, charge $+Ze$), negative ions (mass m, charge $-Ze$), and a third background species (mass m_3, charge $s_3 Z_3 e = \pm Z_3 e$). Here s_j (for $j = +, -, 3$) denotes the charge sign of species j, i.e. $s_+ = -s_- = +1$ and $s_3 = s = \pm 1$ (both cases being possible, in principle). In specific, this description applies to *e-p-i* plasmas for $Z = 1$ and $s = +1$, or in pair-ion (e.g. fullerene) plasmas ($Z = 1$), "doped" by the presence of a third charged particle species of higher mass (for $s = \pm 1$).

The external magnetic field is directed along the x−axis, i.e., $\boldsymbol{B} = B_0 \hat{x}$ (here B_0 is the magnitude of the ambient magnetic field and $\hat{x}$ is the unit vector along the x−axis). Intrinsic plasma rotation is taken into account via the addition of the Coriolis force felt by the fluid(s), involving the rotation frequency (angular velocity) $\Omega = \Omega_0 \hat{x}$. Both senses of rotation (with respect to the magnetic field orientation) may be understood, since corresponding, say, to Ω_0 (arbitrary real-valued) taking either positive or negative values.

The dynamics of ES waves is governed, within a two-fluid model, by a system of moment+Poisson equations for the negative and positive ion components, distinguished by the indices + and −, respectively. We consider the (five) equations

$$\frac{\partial n_\pm}{\partial t} + \nabla\cdot(n_\pm u_\pm) = 0, \tag{1}$$

$$m\left(\frac{\partial}{\partial t} + \boldsymbol{u}_\pm \cdot \nabla\right)\boldsymbol{u}_\pm = \mp Ze\nabla\phi - \frac{1}{n_\pm}\nabla p_\pm \pm \frac{Ze}{c}(\boldsymbol{u}_\pm \times B_0\hat{x}) + 2m(\boldsymbol{u}_\pm \times \Omega_0\hat{x}), \tag{2}$$

$$\nabla^2\phi = 4\pi e[Z(n_- - n_+) - s_3 Z_3 n_3]. \tag{3}$$

Here we have defined the number density $n_\pm$ and the fluid velocity variables $\boldsymbol{u}_\pm$ and the electrostatic potential ϕ, while c is the speed of light. The system is closed by the equation(s) of state $p_\alpha \sim n_\alpha^\gamma$, where $\gamma = (2 + f)/f$ (for f degrees of freedom. The Boltzmann constant k_B may be omitted where obvious. The choice $f = 2$ is made here, allowing for analytical tractability and physical insight[1]. An equilibrium state is assumed to exist, not excluding the existence of a finite plasma flow balancing pressure gradients in the above two-fluid system. For simplicity, we shall consider a quiescent equilibrium below, however.

Equations (1)–(3) are cast in a reduced (dimensionless) form, for convenience in manipulation. For the positive ion fluid, we have

$$\frac{\partial \tilde{n}_+}{\partial \tilde{t}} + \frac{\partial \tilde{n}_+ \tilde{u}_{+,x}}{\partial \tilde{x}} + \frac{\partial \tilde{n}_+ \tilde{u}_{+,y}}{\partial \tilde{y}} = 0\,, \tag{4}$$

$$\frac{\partial \tilde{u}_{+,x}}{\partial \tilde{t}} + \left(\tilde{u}_{+,x}\frac{\partial}{\partial \tilde{x}} + \tilde{u}_{+,y}\frac{\partial}{\partial \tilde{y}}\right)\tilde{u}_{+,x} + \frac{\partial \tilde{\phi}}{\partial \tilde{x}} + 2\sigma\frac{\partial \tilde{n}_+}{\partial \tilde{x}} = 0\,, \tag{5}$$

$$\frac{\partial \tilde{u}_{+,y}}{\partial \tilde{t}} + \left(\tilde{u}_{+,x}\frac{\partial}{\partial \tilde{x}} + \tilde{u}_{+,y}\frac{\partial}{\partial \tilde{y}}\right)\tilde{u}_{+,y} + \frac{\partial \tilde{\phi}}{\partial \tilde{y}} + 2\sigma\frac{\partial \tilde{n}_+}{\partial \tilde{y}} - \tilde{\Omega}_+\tilde{u}_{+,z} = 0\,, \tag{6}$$

and

$$\frac{\partial \tilde{u}_{+,z}}{\partial \tilde{t}} + \left(\tilde{u}_{+,x}\frac{\partial}{\partial \tilde{x}} + \tilde{u}_{+,y}\frac{\partial}{\partial \tilde{y}}\right)\tilde{u}_{+,z} + \tilde{\Omega}_+\tilde{u}_{+,y} = 0\,. \tag{7}$$

A similar set of (four) equations describe the negative ion fluid, upon formally shifting $+ \to -$ in the index everywhere, in addition to setting $\phi \to -\phi$ and $\sigma \to 1$. Finally, the Poisson equation becomes:

$$\tilde{\nabla}^2\tilde{\phi} = \tilde{n}_- - \tilde{n}_+ - s\frac{Z_3}{Z}\tilde{n}_3. \tag{8}$$

Recall that the density of the heavy plasma component "3" is taken to be stationary (of fixed density). Physically, this implies that the heavy background species will react extremely slowly to variations of the electric potential due to the fast ion dynamics, so that static equilibrium (for species 3) can be maintained at all times. The effective rotation frequencies $\tilde{\Omega}_- = 2\tilde{\Omega}_0 - \tilde{\omega}_c$ and $\tilde{\Omega}_+ = 2\tilde{\Omega}_0 + \tilde{\omega}_c$ take into account both Larmor gyration of the particles and the mechanical rotation of the plasma.

The variables appearing in Eqs. (4)–(7), denoted by a tilde, are all dimensionless, in fact scaled by appropriate quantities. Thus, the density n_j (for $j = +, -, 3$) is normalized by the unperturbed negative ion density n_0, $\boldsymbol{u}_\alpha$ is scaled by the negative ion thermal speed $C_{s-} = (k_B T_-/m)^{1/2}$, the potential ϕ by $\phi_0 = k_B T_-/(Ze)$, the rotation frequency Ω_0 and the ion cyclotron frequency $\omega_c = ZeB_0/(mc)$ by the negative ion plasma period $\omega_{p,-} = (4\pi Z^2 e^2 n_0/m)^{1/2}$. Finally, the space and time variables are scaled by the negative ion Debye radius $\lambda_{D-} = [k_B T_-/(4\pi Z^2 e^2 n_0)]^{1/2}$ and plasma period $\omega_{p,-}^{-1}$, respectively. We have defined the temperature ratio $\sigma = T_+/T_-$ (where T_+ and T_- are the positive and negative ion fluid temperatures, respectively). The neutrality condition implies

$$1 = \delta + \beta, \tag{9}$$

where $\delta = n_{+,0}/n_0$ (the index '0' denotes the unperturbed density states) and $\beta = s\frac{Z_3}{Z}\frac{\tilde{n}_3}{n_0}$. Recall that setting $\delta = 1$ (i.e. $\beta = 0$) recovers the pure pair-plasma limit (in the absence of background species), in which case the plasma frequencies of both ion species coincide. For $\delta \neq 1$ (i.e. $\beta \neq 0$) density disparity among the pair species occurs due to the background species. Note that $\delta > 1$ (and $\beta < 0$) for a negative background species, while $0 < \delta < 1$ (and $\beta > 0$) for a positive one (and the latter also holds for *e-p-i* plasmas). Retain that $\beta = 1 - \delta < 1$ by definition, although no lower boundary exists, for a negative species ($s = -1$).

The tilde will henceforth be omitted (thus all quantities below are understood as dimensionless).

3. Zakharov-Kuznetsov (ZK) Equation for the Electrostatic Potential

The independent variables will be stretched as:

$$X = \epsilon^{1/2}(x - \lambda t)\,,\; Y = \epsilon^{1/2}y\,,\; \tau = \epsilon^{3/2}t\,, \tag{10}$$

where ϵ is a small parameter and λ is the wave propagation speed. The state variables are expanded as

$$S = S^{(0)} + \epsilon S^{(1)} + \epsilon^2 S^{(2)} + \ldots \tag{11}$$

where $S^{(n)}$ is the n-th order contribution to any among the state variables $\{n_+, n_-, u_{+,x}, u_{-,x}, \phi\}$, (equal, at equilibrium, to $\{1, \delta, 0, 0, 0\}$). The transverse velocity (y and z components) is assumed to vary on a slower scale, hence:

$$u_{-,y} = \epsilon^{3/2}u_{-,y}^{(1)} + \epsilon^2 u_{-,y}^{(2)} + \epsilon^{5/2}u_{-,y}^{(3)} + \ldots\,, \tag{12}$$

(and similar for $- \to +$ and/or axis $y \to z$).

Substituting the above scaling ansatz into the evolution Eqs. (4)–(8), one may isolate different orders in ϵ and thus solve for the corresponding variable contributions. We choose to express all quantities in terms of the 1st-order potential perturbation $\psi = \phi^{(1)}$. The lowest-order equations in ϵ read

$$n_-^{(1)} = \frac{-1}{\lambda^2 - 2}\psi\,, \qquad u_{-,x}^{(1)} = \frac{-\lambda}{\lambda^2 - 2}\psi\,,$$

$$u_{-,z}^{(1)} = \frac{-\lambda^2}{\Omega_-(\lambda^2 - 2)}\frac{\partial \psi}{\partial Y}\,, \tag{13}$$

and

$$n_+^{(1)} = \frac{\delta}{\lambda^2 - 2\delta\sigma}\psi\,, \qquad u_{+,x}^{(1)} = \frac{\lambda}{\lambda^2 - 2\delta\sigma}\psi\,,$$

[1]The qualitative results thus obtained are not expected to differ substantially from the general choice $f = 3$ (for three-dimensional evolution) as known from earlier works.

$$u_{+,z}^{(1)} = \frac{\lambda^2}{\Omega_+(\lambda^2 - 2\delta\sigma)} \frac{\partial \psi}{\partial Y} \,. \tag{14}$$

The Poisson equation provides the relation

$$\frac{1}{\lambda^2 - 2} + \frac{\delta}{\lambda^2 - 2\delta\sigma} = 0, \tag{15}$$

which determines the pulse propagation speed λ as

$$\lambda^2 = \frac{2(1-\beta)(\sigma+1)}{2-\beta} \,. \tag{16}$$

A solution for λ exists, provided that $\lambda \neq \pm\sqrt{2}, \pm\sqrt{2\delta\sigma}$ (which excludes propagation in temperature-symmetric electron-positron plasmas, viz. $\delta = \sigma = 1$). Note the role of the background species (via β) and of the pair-ion temperature asymmetry (via σ). The expressions obtained in higher orders in ϵ are omitted here, for brevity.

Eliminating the second-order perturbed quantities and making use of the first-order results, we obtain a nonlinear partial-derivative equation (PDE) in the form of the *Zakharov-Kuznetsov (ZK) equation*,

$$\frac{\partial \psi}{\partial \tau} + A\psi \frac{\partial \psi}{\partial X} + \frac{\partial}{\partial X}\left(B \frac{\partial^2 \psi}{\partial X^2} + C \frac{\partial^2 \psi}{\partial Y^2} \right) = 0 \,. \tag{17}$$

The nonlinearity coefficient A and the diffusion coefficients B and D are given by

$$A = B\left[\frac{3\delta\lambda^2}{(\lambda^2 - 2\delta\sigma)^3} - \frac{3\lambda^2}{(\lambda^2-2)^3} \right], \tag{18}$$

$$B = \left[\frac{2\lambda}{(\lambda^2-2)^2} + \frac{2\delta\lambda}{(\lambda^2-2\delta\sigma)^2} \right]^{-1}, \tag{19}$$

$$C = B\left[1 + \frac{1}{\Omega_-^2} \frac{\lambda^4}{(\lambda^2-2)^2} + \frac{\delta\lambda^4}{\Omega_+^2(\lambda^2-2\delta\sigma)^2} \right]. \tag{20}$$

Note that λ may be eliminated, in favor of σ, by making use of the constraint (16). One thus obtains

$$A = -\frac{3\lambda^2 B}{(1-\beta)^2} \left[\frac{1+(1-\beta)^2}{(\lambda^2-2)^3} \right], \tag{21}$$

$$B = \frac{1-\beta}{2\lambda} \left[\frac{(\lambda^2-2)^2}{2-\beta} \right], \tag{22}$$

and

$$C = B\left\{ 1 + \frac{\lambda^4}{(\lambda^2-2)^2} \left[\frac{1}{\Omega_-^2} + \frac{1}{\Omega_+^2(1-\beta)} \right] \right\}. \tag{23}$$

Alternatively, it is straightforward to eliminate λ – since prescribed by (15) – so one obtains a set of expressions (omitted) in terms of σ and β.

Note that the dispersion coefficients B and C are always positive (since $\beta < 1$), while the sign of A is positive for λ below $\sqrt{2}$ (λ is assumed to be positive throughout this text), and negative above. As one may already anticipate (from earlier knowledge on the ZK or the KdV equation), this leads to positive (negative) pulses prescribed below (above) a critical Mach number (scaled pulse speed) threshold of $\sqrt{2}$. The sign of A can therefore be deduced upon simple inspection of Fig. 1 (b) (the regions above/below the left oblique curve therein correspond to negative/positive A). It turns out that A is generally positive for a positively charged species 3 (and, in *e-p-i* plasmas). For equal-pair-species-temperature plasmas (at $\sigma = 1$), the addition of negatively charged defects should lead to negative pulses being created, while positive background should have the inverse effect (positive pulses). Notice that B vanishes at $\lambda = \sqrt{2}$, while C remains finite, and A diverges as $\sim -1/(\lambda^2-2)$, thus intuitively suggesting that *no balance among dispersion and nonlinearity may occur for symmetric pure p.p.*. Inversely, if pulses are to be observed, the plasma should present an asymmetry among the pair-ions, either in concentration or in temperature.

The Zakharov-Kuznetsov equation (17) constitutes the key outcome of this model. The anticipated balance among dispersion and nonlinearity gives rise to localized solitary wave solutions, to be reviewed in the following.

4. Travelling Wave Analysis - Pulse Shaped Localized Solutions

We shall use the travelling wave transformation $\zeta = L_x X + L_y Y - MT$, where M is a real variable (representing a constant speed, scaled by the negative ion thermal speed), L_x and L_y are the directional cosines of the wave vector k along the X and Y axes, so that $L_x^2 + L_y^2 = 1$, $A_0 = AL_x$ and $B_0 = L_x R$, where $R = BL_x^2 + CL_y^2$ (> 0 here). Note that this ansatz represents propagation in a direction oblique with respect to the magnetic field, at a certain angle, say $\theta = \arctan(L_y/L_x)$. See that purely transverse propagation ($\perp \boldsymbol{B}$) is not covered by this model, since vanishing L_x would imply vanishing $A_0 = B_0 = 0$.

Equation (17) is now reduced to the ordinary partial differential equation:

$$-Mu' + A_0 uu' + B_0 u''' = 0, \tag{24}$$

where we have substituted ψ by $u = u(\zeta)$ for simplicity, and the prime here denotes differentiation with respect to ζ.

Integrating Eq. (24) once, and assuming the boundary conditions u, u' and $u'' \to 0$ for $\zeta \to \pm\infty$, we obtain an energy-balance-like equation

$$\frac{1}{2}u'^2 + S(u) = 0. \tag{25}$$

The evolution of a solitary excitation is analogous to the problem of motion of a unit mass in a (Sagdeev-like) pseudopotential, given by

$$S(u) = \frac{1}{B_0}\left(\frac{-M}{2}u^2 + \frac{A_0}{6}u^3 \right). \tag{26}$$

A localized solution exists if $\mathrm{d}^2S/\mathrm{d}u^2\big|_{u=0} < 0$, i.e. if the origin in the $(u - S(u))$ plane is a local maximum (see that $\mathrm{d}S/\mathrm{d}u|_{u=0} = 0$). We have

$$\mathrm{d}^2S/\mathrm{d}u^2 = -M/B_0 < 0. \tag{27}$$

This relation is always satisfied here; the case $\beta = 1$ is meaningless physically (implying no positive ions, hence no pair plasma) and thus excluded. One concludes that stationary solitary waves can always propagate in pair plasmas (except for $\lambda = \sqrt{2}$, where finite dispersion fails to balance the significant nonlinearity).

Assuming the boundary conditions $\psi \to 0$ and $\mathrm{d}\psi/\mathrm{d}\zeta \to 0$ at $|\zeta| \to \infty$, we obtain a solitary wave solution of Eq. (25) as

$$\psi = \phi_0 \mathrm{sech}^2(\zeta/W), \tag{28}$$

where

$$\phi_0 = 3M/A_0, \quad W = \sqrt{4B_0/M} \tag{29}$$

provide the maximum potential amplitude ϕ_0 and the pulse width W. The localized pulses predicted here may be either positive or negative, depending on the sign of the nonlinearity coefficient A. The characteristics of these pulses will be discussed below.

5. An Alternative Solution Method

Going beyond the "traditional" solution of the ZK equation (17) by quadrature (see in the previous Section), we shall now adopt an alternative method, namely the improved Modified Extended Tanh-Function (iMETF) technique [23].

We anticipate a solution in the form

$$u(\zeta) = a_0 + a_1\omega + a_2\omega^2 + \frac{b_1}{\alpha_1 + \omega} + \frac{b_2}{(\alpha_2 + \omega)^2}, \tag{30}$$

with

$$\frac{\mathrm{d}\omega}{\mathrm{d}\zeta} = k + \omega^2, \tag{31}$$

where a_0, a_1, a_2, b_1, b_2 and k are arbitrary constants and ω, α_1 and α_2 are functions of ζ (to be determined).

The general solutions of the Riccati Eq. (31) are summarized in the following. For $k < 0$

$$\omega = -\sqrt{-k}\tanh(\sqrt{-k}\zeta), \tag{32}$$

or, alternatively, $\omega = -\sqrt{-k}\coth(\sqrt{-k}\zeta)$. For $k = 0$, $\omega = -1/\zeta$. Finally, for $k > 0$,

$$\omega = \sqrt{k}\tan(\sqrt{k}\zeta) \quad \text{or} \quad \omega = -\sqrt{k}\cot(\sqrt{k}\zeta). \tag{33}$$

Combining Eqs. (30) and (31) into (24), a polynomial equation in $\omega(\zeta)$ is obtained. Equating the coefficients of ω to zero will result in an overdetermined system of algebraic differential equations in terms of the parameters a_0, a_1, a_2, b_1, b_2, α_1, α_2, k, L_x, L_y and M. Combining with (32)–(33), we obtain a complete new set of solutions, to be presented in the following.

For $k < 0$, two different solutions are obtained,

$$u(\zeta) = \frac{M - 8kB_0}{A_0} + \frac{12kB_0\tanh^2(\sqrt{-k}\zeta)}{A_0}, \tag{34}$$

or

$$u(\zeta) = \frac{M - 8kB_0}{A_0} + \frac{12kB_0}{A_0}\left[\tanh^2(\sqrt{-k}\zeta) + \coth^2(\sqrt{-k}\zeta)\right]. \tag{35}$$

For $k = 0$, we have $u(\zeta) = \frac{M}{A_0}\left(1 - \frac{12B_0}{\zeta^2 M}\right)$. For $k > 0$,

$$u(\zeta) = \frac{M - 8kB_0}{A_0} - \frac{12kB_0\tan^2(\sqrt{k}\zeta)}{A_0}, \tag{36}$$

or

$$u(\zeta) = \frac{M - 8kB_0}{A_0} - \frac{12kB_0}{A_0}\left[\tan^2(\sqrt{k}\zeta) + \cot^2(\sqrt{k}\zeta)\right]. \tag{37}$$

Finally, for $k = -1$ and $M = 4B_0$, we obtain

$$u(\zeta) = \frac{12B_0}{A_0}(1 - c^2) \times \left[1 - \frac{2c}{c - \tanh\zeta} - \frac{1 - c^2}{(c - \tanh\zeta)^2}\right], \tag{38}$$

where c is a real function of ζ. Note that $A_0 \neq 0$ and $B_0 \neq 0$ is understood everywhere above.

5.1 Pulse-shaped localized solutions via the iMETF method

Anticipating localized solutions which vanish far from the origin, we may impose u, u' and $u'' \to 0$ for $\zeta \to \pm\infty$, leading to the constraint $k = -M/4B_0$. In this case, the solution (34) above reduces to the pulse-shaped solution (28) above, here smoothly recovered via the iMETF method. However, the method employed in this Section also provides other types of solutions, in addition to the latter one (potential pulse), which are discussed in [24].

In order to elucidate the characteristics of the solitary structure represented by Eq. (28), we have numerically analyzed the potential amplitude u_0 and investigated how the phase velocity λ and the ion-to-electron density ratio β change the profile of the maximum potential perturbation. Our main results are presented below.

The dependence of the wave propagation speed λ (given by Eq. (15)) on the density (β) and temperature (σ) ratios is analyzed in Fig. 1. The phase velocity λ is affected by the ion-to-electron density ratio β, and in fact decreases with increasing β. Therefore, in the case of a positive background species, the higher the fixed species' density (or, the more ions present in *e-p-i* plasmas), the slower solitary waves will be. The opposite effect (faster excitations) should occur, the higher negative background species concentration is. Supersonic excitations should occur, generally, for realistic (small) values of β (say, $\beta < 0.6 - 0.7$;

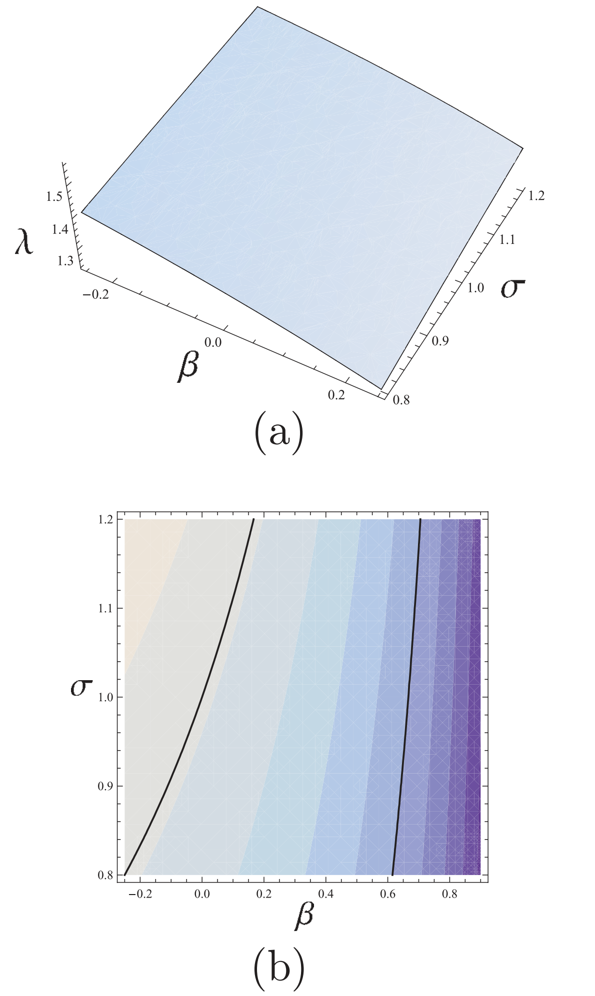

Fig. 1 The solitary wave velocity λ (given by Eq. (16)) is depicted vs. β and σ. Lighter regions in (b) correspond to higher values of λ. The right separatrix (thick line) shows the sonic limit $|\lambda| = 1$, attained for $\beta \simeq 0.6$ - 0.7 (unlikely, physically). Realistic (small) values of β correspond to *supersonic* excitations. The left line corresponds to $|\lambda| = \sqrt{2}$, separating regions of positive from negative A (and thus, potential pulse polarity; see in the text).

see Fig. 1 (b)); however, if the positive ion concentration exceeds, say, $\beta \simeq 0.6$ - 0.7, subsonic excitations may propagate in the plasma. The positive-to-negative ion (or, positron-to-electron in *e-p-i* plasmas) temperature ratio σ also affects the phase velocity λ, yet rather more for small (absolute) values of β; see Fig. 1 (b). For high values of β (> 0.6 approximately), σ does not quite affect λ. Admittedly, values of β are expected to be small (since species 3 is here introduced as a minority background population), so high values of β (and subsonic excitations) are rather unlikely to occur in doped pair plasmas.

From Fig. 2, it is seen that increasing the propagation speed λ leads to a decrease in the soliton amplitude u_0 for a given (any, fixed) value of the ion-to-electron density ratio β. At a critical velocity $\lambda = \sqrt{2}$, the sign of A – and hence of the pulse amplitude u_0 – shifts to negative, so the effect

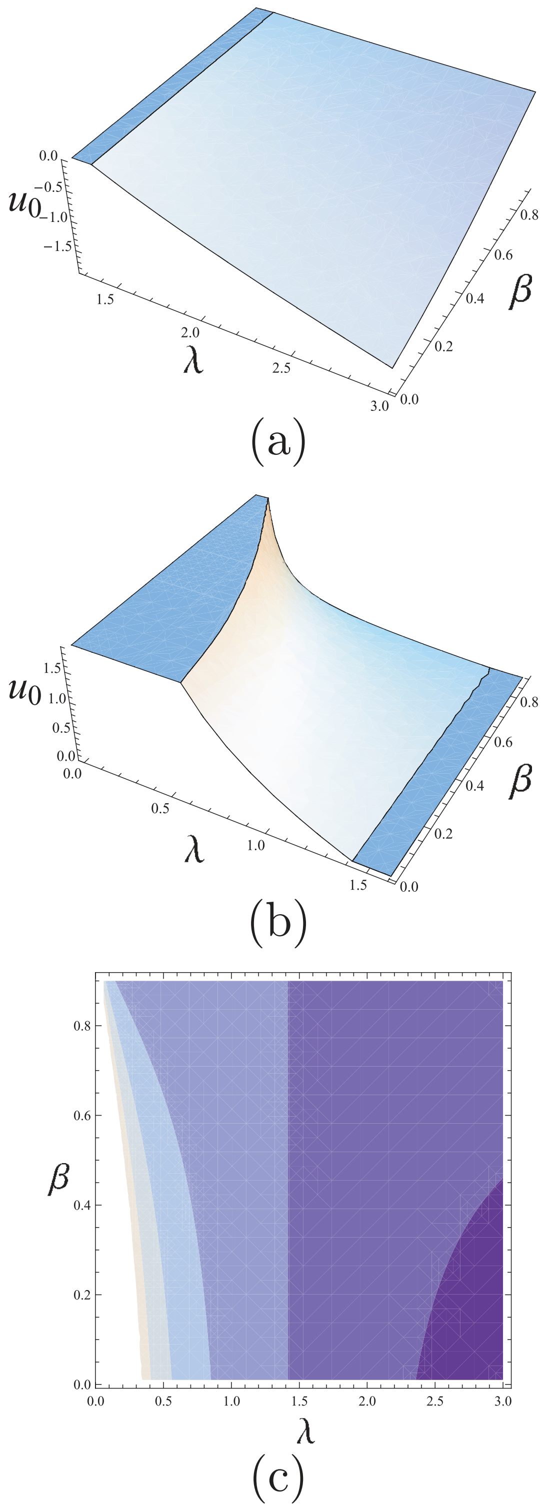

Fig. 2 The soliton amplitude u_0 (as given in Eq. (29)) is depicted against β and λ, for $L_x = 0.3$. The negative λ (1st frame) and positive λ (2nd frame) are separated by the curve $\lambda = \sqrt{2}$ (see in the text). Light-shaded regions in the 3rd frame correspond to higher values of the soliton amplitude u_0.

is inversed: higher speed λ then corresponds to a higher negative pulse amplitude (i.e., absolute value of u_0). The ratio β also has a two-fold effect on the potential amplitude u_0, i.e., for low propagation speed $\lambda < 1.44$ the amplitude

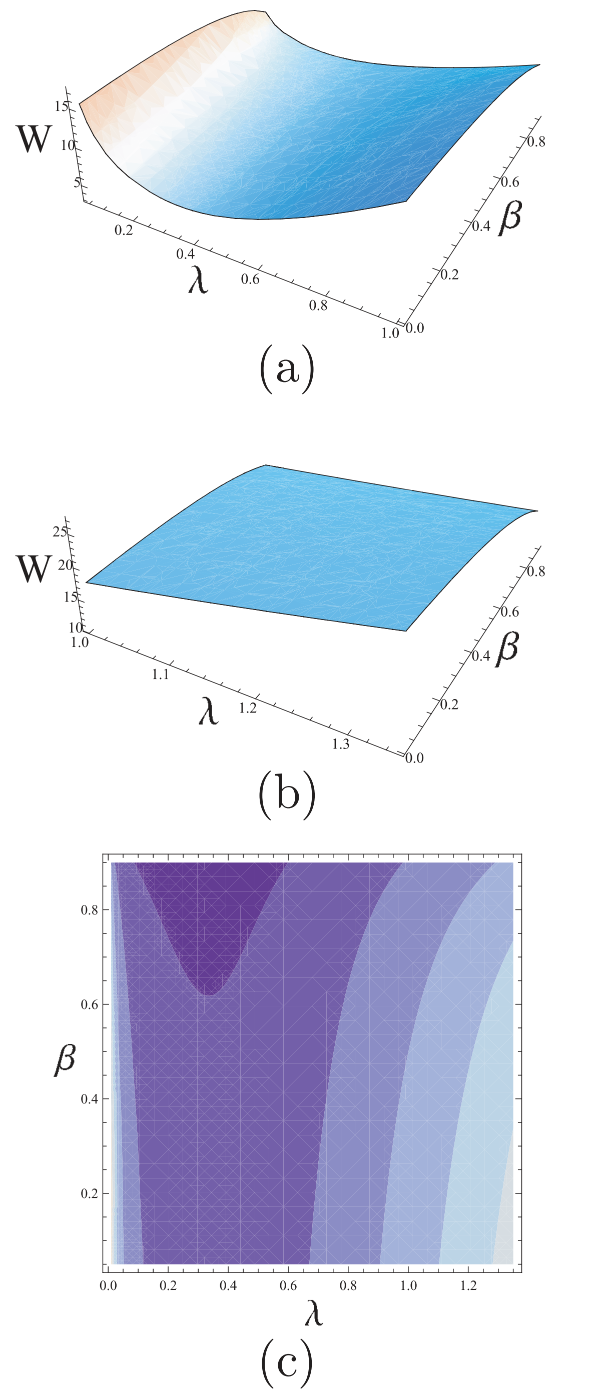

Fig. 3 The soliton width W (defined in Eq. (29)) is depicted vs. β and λ, for $L_x = 0.3$, $\Omega_0 = 0.1$, and $\omega_c = 0.3$. Light-colored regions in the lower frame correspond to higher values of W.

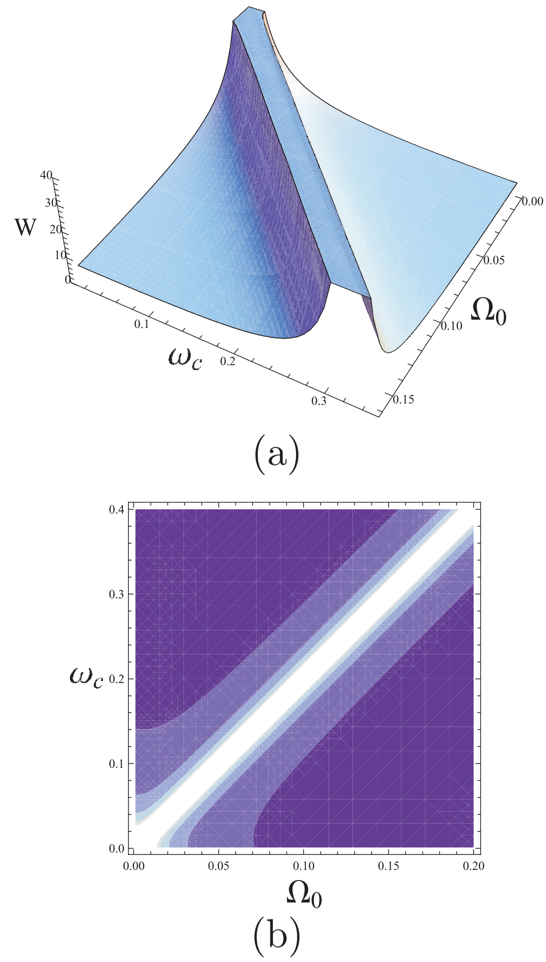

Fig. 4 The soliton width W (defined by Eq. (29)) against Ω_0 and ω_c is depicted, for $\beta = 0.4$, $\lambda = 0.6$, and $L_x = 0.7$. Light-colored regions in the lower frame correspond to higher values of W.

increases as the positive-to-negative ion density ratio β increases. However, for high propagation speed $\lambda > 1.44$, increasing β leads to an increase of the soliton amplitude u_0.

The dependence of the spatial extension (width) W on the phase velocity λ, the ion-to-electron density ratio β, the rotation frequency Ω_0 and the electron (positron) gyrofrequency ω_c are displayed in Figs. 3 and 4. In Fig. 3, we see that increasing the propagation speed λ leads to a decrease of the width W, for low λ (< 0.6 approximately; though rather unlikely to occur: see discussion above). For higher λ (and everywhere in the supersonic region), by increasing the speed λ the width increases. Increasing the positive-to-negative ion density ratio β leads to a decrease of the soliton width, in both cases. For certain values of λ (low phase speed) and β (high ion-to-electron density ratio), one can notice a reduction of the width; see Fig. 3 (c). In Fig. 4, we analyze the effect of the rotation frequency Ω_0 and of the gyrofrequency ω_c on the soliton width W. We see that the width W is affected by both the rotation frequency Ω_0 and the cyclotron frequency ω_c. In particular, an interesting effect is witnessed when $\omega_c - 2\Omega_0 \to 0$, where the width W diverges. This occurs when the dispersion is negligible, and thus fail to balance nonlinearity; our solutions are not expected to occur in that region, i.e. near the white region in Fig. 4 (lower frame).

It is also appropriate to analyze the nonlinearity coefficient A in the $\beta - \sigma$ plane. Eliminating λ, it is straight-

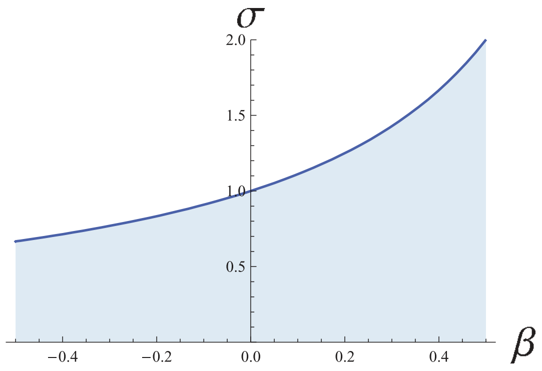

Fig. 5 The critical temperature ratio threshold $\sigma_{\text{cr},1}$ is shown against β. The shaded (white) region below (above) the curve corresponds to values of σ where A is positive (negative), i.e. where positive (negative) potential pulses are predicted to occur.

forward to show that A (= $A(\sigma,\beta)$ then) changes sign, for given say β, at a critical temperature ratio

$$\sigma_{\text{cr},1} = 1/1 - b\,.$$

The value of $\sigma_{\text{cr},1}$ increases with β everywhere; see in Fig. 5. Recall that positive/negative values of β correspond to a positive/negative background species charge (so, only the positive semi-axis is relevant in *e-p-i* plasmas, for instance). The interpretation of Fig. 5 is straightforward: for values of σ above $\sigma_{\text{cr},1}$, A is negative, and thus negative pulses will occur, while for σ below $\sigma_{\text{cr},1}$, $A > 0$. For small β (minority background species), $\sigma_{\text{cr},1}$ is in the vicinity of 1, as expected. T herefore, in "pure" p.p., say ($\beta = 1$), a temperature mis-balance $T_+ > T_-$ will lead to negative pulses, and vice versa. Increasing the concentration of positive dust leads to positive pulses occuring in an extended region, while increasing the concentration of negative dust shrinks that region.

6. Critical Plasma Compositions

It is obvious, from expressions (18) and (19) (in combination with (15, 16), for a rigorous study) that the nonlinearity coefficient may acquire very small values if δ and σ take values near unity, or for $|\lambda|$ in the vicinity of $\sqrt{2}$ (which is the same; cf. (16)). In that region, nonlinearity may fail to balance dispersion, so higher order nonlinearity may have to be included in the description. Physically, this seems to suggest that for near-symmetric ($T_- \approx T_+$) "pure" ($n_- \approx n_+$) pair plasmas (or ideal *e-p* plasmas), i.e. unless a finite (non-negligible) concentration of the "3rd" background species is present, the above results may not be valid. Therefore, it may be appropriate to investigate the situation when higher-order nonlinearity is also taken into account.

Plasmas at critical compositions, where the quadratic nonlinearity coefficient acquires negligible values, may require a different perturbative scaling. A detailed discussion can be found e.g. in [25]. In this Section, we shall adopt this scenario, by adjusting our parameter scaling appropriately.

We shall adopt the stretched variables

$$X = \epsilon(x - \lambda t)\,, \qquad Y = \epsilon y\,, \qquad \tau = \epsilon^3 t\,, \tag{39}$$

and will use the expansion (11), together with

$$u_\alpha = \epsilon^2 u_\alpha^{(1)} + \epsilon^3 u_\alpha^{(2)} + \epsilon^4 u_\alpha^{(3)} + \ldots\ . \tag{40}$$

Substituting from (39), (11) and (40) into the fluid evolution equations, the lowest-order in ϵ yields the relations (13)–(14). The higher orders in ϵ yields a set of expressions (here omitted) relating the 2nd and 3rd order contributions to the first-order potential contribution ψ.

Eliminating the 3rd-order variables, we obtain an *extended* Zakharov-Kuznetsov (EZK) equation in the form

$$\frac{\partial\psi}{\partial\tau} + A\psi\frac{\partial\psi}{\partial X} + D\psi^2\frac{\partial\psi}{\partial X} + \frac{\partial}{\partial X}\left(B\frac{\partial^2\psi}{\partial X^2} + C\frac{\partial^2\psi}{\partial Y^2}\right) = 0, \tag{41}$$

where the coefficients A, B and C are given by (18)–(20) above; the cubic nonlinearity coefficient D reads

$$D = \frac{3}{2}B\left[\frac{\delta\lambda^2(5\lambda^2 + 8\delta\sigma)}{(\lambda^2 - 2\delta\sigma)^5} + \frac{\lambda^2(5\lambda^2 + 8)}{(\lambda^2 - 2)^5}\right]. \tag{42}$$

7. Solution of the EZK Equation

The EZK equation (41) may be solved by using the traveling-wave transformation introduced in Sec. 4 (refer to the definitions therein), and integrating. Assuming vanishing boundary conditions, we have

$$\frac{1}{2}\left(\frac{d\psi}{d\zeta}\right)^2 + S(\psi) = 0. \tag{43}$$

The evolution of a localized excitation is therefore analogous to the problem of motion of a unit mass in the pseudopotential

$$S(u) = \frac{1}{B_0}\left(\frac{-M}{2}u^2 + \frac{A_0}{6}u^3 + \frac{DL_x}{12}u^4\right). \tag{44}$$

Here, A_0, B_0 and L_x are as defined in Sec. 4 above. Compare to (26) to see the contribution of the cubic nonlinearity via the last term in the *rhs*.

We shall now discuss the solutions of Eq. (44).

7.1 Existence conditions for a solitary-wave solution

A localized solution of Eq. (44) exists if $S''(\psi = 0) < 0$. We thus obtain an explicit condition for solitary wave existence. Equation (44) can take the form

$$\begin{aligned}\left(\frac{d\psi}{d\zeta}\right)^2 &= \tfrac{-D}{6R}\psi^2\left(\psi^2 + \tfrac{2A}{D}\psi - \tfrac{6M}{DL_x}\right)\\ &= \tfrac{-D}{6R}\psi^2(\psi - \varphi_1)(\psi - \varphi_2),\end{aligned} \tag{45}$$

where $\varphi_{1/2} = -\frac{1}{D}(A \pm \sqrt{\Delta})$, $\Delta = A^2 + \frac{6MD}{L_x}$. We are looking for stationary soliton solutions, obeying $\psi \to 0$ and $\psi'(\zeta) \to 0$ at $|\zeta| \to \infty$. These are found by integrating Eq. (45) appropriately. In the case[2] $A > 0$ and $D > 0$, the details of the solution method can be found in [26] and are thus omitted here. One (then) obtains the (two) expressions

$$\phi^{(1)}_{+/-} = \frac{6M}{D}\left[\varphi_{1/2}\sinh^2\left(\frac{1}{2}\sqrt{\frac{M}{R}}\zeta\right) - \varphi_{2/1}\cosh^2\left(\frac{1}{2}\sqrt{\frac{M}{R}}\zeta\right)\right]^{-1}, \qquad (46)$$

which represent, respectively (for the upper/lower subscripts), positive and negative pulse-shaped excitations for the electric potential. Note that, interestingly, although for positive A, say, only positive pulse solitons would be prescribed by the ZK Eq. (see previous Section), here both options in (46) are possible, provided that $A^2 + \frac{6MD}{L_x} \geq 0$ is ensured, for reality. The same is true for negative[3] A.

7.2 Double-layer (kink) solutions

For a kink-shaped potential double-layer (DL) solution to exist, the potential $S(\psi)$ should take negative values among $\psi = 0$ and some double root ϕ_m. We therefore impose $S(u) = S'(u) = 0$ at $u = 0$ *and* at $u = \phi_m$ (to be defined), in addition to $S''(0) < 0$. Applying constant boundary conditions, we obtain the condition for the existence of double layers,

$$\phi_m = -A/D \quad \text{and} \quad M = -A^2 L_x/(6D), \qquad (47)$$

i.e., $\Delta = 0$ (see above). The DL propagation speed (correction) M is prescribed by (47). The pseudopotential in (44) now takes the form

$$V(\psi) = \frac{D}{12R}\psi^2(\psi - \phi_m)^2. \qquad (48)$$

See that $D < 0$ must be imposed for reality, viz. $V(\psi) < 0$. Therefore, (47) suggests that the double root ϕ_m [of $S(\psi)$, corresponding to the potential polarity: see (48)] bears the same sign as the nonlinearity coefficient A (i.e., $\psi > 0$ in (48) if $A > 0$ – as in Fig. 8 – and vice versa). In regions where $D < 0$ and $A > 0$, positive DLs (kinks) may occur, while for $D < 0$ and $A < 0$ negative DLs (antikinks) exist.

The DL solution reads (recall that $D < 0 < R$)

$$\psi = \frac{\phi_m}{2}\left[1 \pm \tanh(\zeta/W_{\rm DL})\right], \qquad (49)$$

where the width of the DL is $W_{\rm DL} = 2\sqrt{-6RD}/|A|$.

[2]Note that only the case $A > 0$ and $D > 0$ was studied in [26]. We therefore consider the solution (46) for $D > 0$ here. The general case will be investigated in a more detailed report, to appear. (The assumption $A > 0$ in fact poses no restriction: see the next footnote.)

[3]In the case $A < 0$, one can use the transformation $\psi \to -\psi$ in the ZK Eq. (41) to obtain $A \to -A > 0$ in the new equation thus obtained, and thus be led to the same results (same expressions, eventually slightly different quantitative interpretation).

Let us analyze D in the $\beta - \sigma$ plane. Eliminating λ from (16) (as explained above), it is straightforward to show that D (= $D(\sigma,\beta)$ then) is, in fact, $\sim -(\sigma - \sigma_{\rm cr,2})/(\sigma - \sigma_{\rm cr,1})$, and thus changes sign, for given say β, at $\sigma = \sigma_{\rm cr,1}(\beta)$ (defined previously) *and* also at a second temperature ratio threshold $\sigma_{\rm cr,2}(\beta) = (4 - 22\beta + 22\beta^2 - 9\beta^3)/(4 + 10\beta - 10\beta^2 + 5\beta^3)$, which is depicted in Fig. 6. The interpretation is straightforward, upon inspection of Figs. 5 and 6. Four regions are distinguished in Fig. 7. In the upper white region, both D and A are negative, and thus negative double layers will occur. In the lower white region, $D < 0 < A$, and thus positive double layers will occur (in addition to positive pulses). See that this happens up to a certain value $\beta \simeq 0.23$. D is positive, and thus DL existence is *excluded*, for σ values in the right shaded area in Fig. 7 and in the left shaded area (localized pulses do exist, nevertheless, in these regions; see above). For small β (minority background species), $\sigma_{\rm cr,1}$ is in the vicinity of 1. Therefore, in "pure" p.p., say ($\beta = 1$), a temperature perturbation (such that $T_+ \neq T_-$) will en-

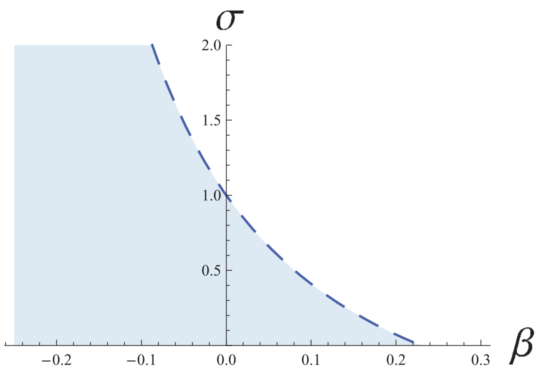

Fig. 6 The temperature ratio threshold $\sigma_{\rm cr,2}$ is depicted vs. β. The shaded region corresponds to values of σ where the quantity $\sigma - \sigma_{\rm cr,2}$ is negative. Since, in fact, $D \sim -(\sigma - \sigma_{\rm cr,2})/(\sigma - \sigma_{\rm cr,1})$, combining with Fig. 6, one obtains the regions where D is negative, i.e. where double layers may exist.

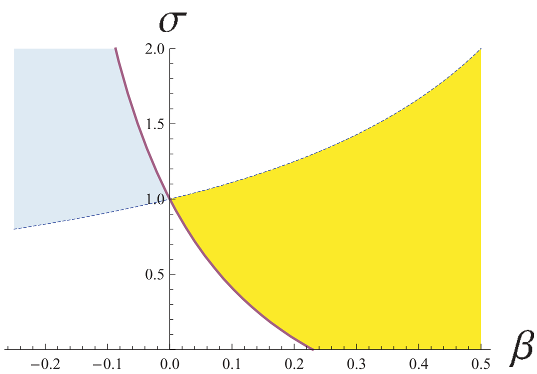

Fig. 7 The combination of Figs. 5 and 6 is shown. No DLs exist in the shaded areas, while negative (positive) DLs exist in the upper (lower) white regions.

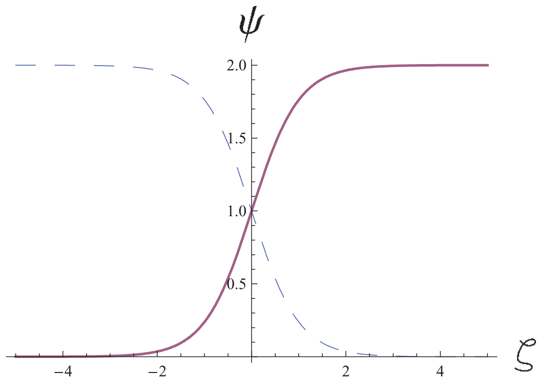

Fig. 8 The double layer excitation (49) (for the upper plus sign, here) is depicted (arbitrary parameter variables), in its kink/antikink form (continuous/dashed line, respectively).

able the occurrence of double layers (by shifting, say, the original state $(0, 1)$ vertically in Fig. 7), while a positive background species variation ($\beta \neq 0$) might then destabilize the excitations occurring in the plasma (moving across the separatrix).

8. Discussion and Summary

We have studied the nonlinear propagation of electrostatic excitations in rotating magnetized doped pair-ion (or, electron-positron-ion) plasmas. A two-fluid plasma model was employed, incorporating both Lorentz and Coriolis forces, to take into account the interplay between the gyroscopic (Larmor) frequency ω_c and the intrinsic rotation frequency Ω_0.

A Zakharov-Kuznetsov (ZK) type equation was derived for the evolution of the electric potential perturbation. Assuming an arbitrary direction of propagation, with respect to the magnetic field, we have derived the exact form of pulse-shaped solutions, and discussed their characteristics. It was shown that the Larmor and mechanical rotation affect the pulse dynamics via a parallel-to-transverse mode coupling diffusion term in the ZK equation, which in fact diverges at $\omega_c \to \pm 2\Omega_0$. Pulses collapse at this limit, as nonlinearity fails to balance dispersion.

The analysis was complemented by investigating critical plasma compositions, in which the (quadratic) nonlinearity vanishes, so one needs to resort to higher order nonlinear theory. An extended ZK equation was derived and its solutions were discussed.

A parametric analysis was carried out, as regards the effect of the pair-plasma composition (background number density), species temperature(s) and the relative strength of rotation to Larmor frequencies. It was shown that the plasma composition (meaning the pair-ion density and temperature balance, essentially) affects the existence and the dynamics of solitary waves both qualitatively and quantitatively. It is therefore suggested that the occurrence of impurities or pair-component asymmetry (either intentional or intrinsic) should be considered in p.p. experiments, for rigor.

Our results are of relevance in pair-ion (fullerene) experiments, wherein a small population of massive charged defects may modify the dynamics dramatically. They may also be relevant in astrophysical environments, in particular in pulsar magnetospheres, where a co-existence of electrons, positrons and ions in a rotating plasma may occur.

Acknowledgments

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