

BOOK REVIEWS

Chapter 4, and relaxation methods and applications in Chapter 5. There we find a detailed discussion of block relaxation methods, some result of convex analysis, and optimization techniques for quadratic functionals. In Chapter 6 the augment Lagrange method are introduced, a favorite of Glowinski and a subject on which he has written many papers. Lee's square approximations to nonlinear problems are discussed in Chapter 7 with applications to fluid dynamics. There some impressive transonic flow calculations are discussed which are handled by Lee square and finite element methods. Three appendices are provided, one on an introduction to linear variational problems, another on finite element methods with upwinding for second-order problems with convective terms, and a third on Navier-Stokes equations and their numerical treatment.

Again, this is a precise, carefully written book and a very welcome addition to the literature on nonlinear computational mechanics.

Nonlinear Elastic Deformations. By R. W. Ogden. John Wiley & Sons, New York, 1984. 532 pages. price: \$95.00

REVIEWED BY E. STERNBERG³

The revival of interest in the nonlinear theory of elasticity around the middle of this century was prompted in part by a concern with rubber elasticity and partly impelled by issues in the area of elastic instability. While these motivations are evident also in the present treatise, the latter attests as well to the influence of the contemporary axiomatic school of continuum mechanics that has provided an additional impetus for a reconsideration of finite elasticity theory in more recent years.

Professor Ogden's treatment of his subject comprises a mathematical exposition of the purely mechanical nonlinear theory, with primary focus on elastostatics. What sets this book apart from most related treatises, and lends it particular interest as a research monograph, is the emphasis on applications and the inclusion of topics not covered elsewhere, such as a chapter on experimental aspects of large-deformation elasticity. The occasionally unconventional selection of material discussed reflects the author's own research preoccupations. This bias is natural and indeed welcome in view of his significant contributions to the field of investigation at hand.

Chapter 1 is devoted to mathematical preliminaries from the algebra and calculus of tensors. Although the chief concentration here is on vectors and second-order tensors, some—if somewhat cursory—attention is given to higher-order Cartesian tensors and to general tensors. The author's decision to switch back and forth freely between an invariant and a coordinate-based development of these topics is apt to cause discomfort only among puristically inclined readers.

The kinematics of continuous media is taken up in Chapter 2. This chapter opens with a section on the role of "observers" and reference framings, in which the concepts of a body and of configuration maps, deformations, and motions of bodies are introduced. There follows a detailed analysis of deformations, strains, and strain-rates. The chapter ends with a section pertaining to objective tensor fields.

Chapter 3 is devoted to the balance laws of mass, momentum, and energy, the analysis of actual and nominal stress, as well as the Eulerian and Lagrangian versions of the

field equations of motion. A closing section on "conjugate stress analysis" contains material not accessible in the previous expository literature.

Relevant topics of constitutive theory are covered in Chapter 4. After a few introductory remarks on "simple materials," the author turns to a fairly comprehensive treatment of the constitutive laws associated with elastic and hyperelastic bodies. In this context material symmetry and isotropy, along with the constitutive implications of internal constraints, are discussed.

Chapter 5 deals largely with applications of the nonlinear theory to specific problems. It starts with general comments on displacement and traction boundary conditions, including some observations on the role of "dead loadings—a topic that would appear to remain in need of further exploration. Following a proof of Ericksen's theorem on deformations sustainable by *all* compressible, isotropic hyperelastic materials, a number of particular problems for such material are analyzed. The strain-energy density employed in this connection is a variant of that appropriate to the "harmonic material" introduced by Fritz John. This portion of Chapter 5 concludes with a section on the use of complex variables in plane-strain problems for harmonic materials. The remainder of the chapter is concerned with familiar universal solutions for *incompressible* hyperelastic solids, but for a final section on variational principles and conservation laws in the finite theory of elasticity.

The subject matter of Chapter 6, which occupies over 150 pages, is closely related to some of the author's own work. The chief objective of this chapter is the theory of incremental deformations superposed on a finite deformation. Here the structure of the ensuing linearized boundary-value problems is studied. Further, the foregoing theory is applied in deriving higher-order amendments of the classical linear theory and to the treatment of allied uniqueness, stability, and bifurcation issues. In this setting of the role of various constitutive inequalities, such as the strong-ellipticity postulate, is examined.

Chapter 7, the last chapter, and one that also benefits from the author's research experience, deals with available test results for highly deformable hyperelastic materials and with attempts to match such observed behavior within the framework of the nonlinear theory.

Throughout the book a sizable number of helpful exercises complement the text. Further, a useful—although somewhat haphazard—list of references to the pertinent literature is appended to each chapter.

Professor Ogden's exposition is evidently addressed to a mathematically rather erudite audience. Indeed, readers with an engineering background might at times find his concern with elaborate mathematical detail (e.g., the definition of a "regular surface" on p. 67) out of place in a treatise of this kind. On the other hand, the author's aspiration toward mathematical precision notwithstanding, there are conspicuous occasional lapses in care. The following examples will suffice to illustrate this point.

The assertion on p. 26 that "the eigenvectors [of a symmetric tensor] are mutually orthogonal and the eigenvalues are real" does not convey the intended meaning. Actually, since the underlying space is assumed on p. 1 to be a *real* Euclidean vector space, complex eigenvalues are automatically excluded, as is reaffirmed on p. 24 by the requirement that eigenvalues be "scalars." Accordingly, the argument in support of the spectral theorem sketched on pp. 26, 27, however commonly employed, is open to objection. Next, the field hypotheses in the statement of the generalized divergence theorem on p. 70 are inadequate and do not even assure the existence of the volume integral in (1.5.66). Further, on p. 77 a "configuration" of a body B is defined as a one-to-one mapping of B into the Euclidean reference space.

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Since no metric or topology is assigned directly to the abstract set of "particles" B , the meaning of the differentiability assumptions imposed on configuration maps is left unclear. Finally, the inequalities cited in the footnote on p. 191 are merely necessary, but not sufficient, in order that I_1, I_2, I_3 be the principal scalar invariants of a symmetric positive-definite tensor, and thus the domain of definition of the function Φ remains unspecified.

Such minor criticisms cannot, and are not intended to, detract from the value of this worthwhile contribution to the literature on an important subject of growing current interest.

Computational Techniques for Differential Equations. Edited by John Noye. North-Holland, The Netherlands, 1984. 680 Pages. Price \$74.50.

REVIEWED BY G. MAJDA⁴

The textbook *Computational Techniques for Differential Equations*, edited by J. Noye, is a very valuable addition to the list of textbooks on computational methods in engineering and applied science. It introduces researchers to five different areas of numerical methods:

- (1) the numerical solution of ordinary differential equations,
- (2) finite difference techniques for partial differential equations,
- (3) the Galerkin method,
- (4) the finite element method,
- (5) the boundary element method.

The text also contains two chapters on direct and iterative methods for the solution of systems of linear algebraic equations.

Each chapter is written by an expert in the particular field. All techniques are illustrated with model problems rather than with complex problems which often occur in applications. As an example, Poisson's equation is used to illustrate the basic properties of the finite element and the boundary element method. However, practical applications are not ignored. Some applications are presented in the text and generous references to the literature appear throughout the book. The theory behind the numerical methods is either given a heuristic motivation or it is briefly sketched. Again, generous references to the literature are given where theoretical results can be studied in detail.

The primary contribution of this book is that it covers a broad set of important topics in an expository fashion. All chapters start in a way that assumes that the reader has no previous experience with the material. Noye's text can be used either as a textbook for a graduate class on the numerical solution of differential equations or as a reference for scientists using numerical methods in their research. It has the special feature of including, in one textbook and in a very readable manner, material that usually appears in five or six different books. It should serve as a useful guide for students

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and researchers who want to get a good overview of most methods that are typically used in the numerical solution of differential equations.

My primary criticism of this book is that despite its breadth, important topics or references to these topics are omitted. The numerical solution of two-point boundary value problems is never mentioned. The numerical solution of systems of hyperbolic conservation laws is given only a one-paragraph discussion and the only reference to the literature on this important topic is a 1967 reference to the method of shock fitting. Very little discussion appears on appropriate methods for imposing the extra boundary conditions that are often required for finite difference approximations of initial-boundary value problems for parabolic and hyperbolic partial differential equations.

Atmospheric Dispersion of Heavy Gases and Small Particles. (A Symposium at Delft, August 29–September 2, 1983, under the sponsorship of the International Union of Theoretical and Applied Mechanics.) Edited by G. Ooms and H. Tennekes. Springer-Verlag, New York, 1984. 440 Pages. Price \$38.50.

REVIEWED BY R. A. DOBBINS⁵

This conference consisted of four review articles and over two dozen research papers from contributors based in Europe and the United States. The topics include: gravity spreading of dense gases; dense gas dispersion; turbulence models and dispersion of dense gas clouds; laboratory and large-scale experiments of heavy gas dispersion; liquified gas spills on the sea; and many related topics. The substantial number of papers and the numerous references in the review and research papers provides an excellent summary of the status of heavy gas dispersion in 1983. All viewpoints, from rigorous theoretical understanding to practical calculation techniques, seem to be represented. There is frequent reference to works in progress which will be reported at future meetings. Thus, this conference not only summarizes the past results but provides at least a limited view on the nature of the works to be reported in the future. A noteworthy editorial defect does emerge from casual examination of the conference proceedings. In one article there is reference to the now unavailable original paper as the source of the equations from which the quoted graphical results originate.

An early motivation for the study of heavy gas dispersion was and remains the inadvertent release of liquified natural gas. News of a catastrophic release of methyl isocyanate at Bhopal, India and similar types of episodes serve as a reminder that heavy gas dispersion is potentially a factor in a variety of accidents in industrial plants. This volume provides a status report of the knowledge of this topic as of September, 1983.

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