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Nonlinear Field Dependence of Electric Conductivity above the Superconducting Transition

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The nonlinear field dependence of the so-called anomalous term in the electric conductivity due to the fluctuation of the superconducting order parameter is studied theoretically. It is found that in the limit of a large electric field E (i.e. $E \gg E_c = \sqrt{3/D}\ \varepsilon_0(T)^{3/2}/e$ where $\varepsilon_0(T) = 8/\pi(T-T_c)$ and D is the diffusion constant), the anomalous term becomes twice as large as the Aslamazov-Larkin term independenct of the dimensionality of the system and independent of the pair-breaking parameter δ.

It has been recognized recently[1)~5)] that the additional contribution to the electric conductivity due to the fluctuation of the order parameter consists of two distinct terms; the Aslamazov-Larkin term[6)] (AL) which can be interpreted as the electric conduction due to the virtual Cooper pairs and the so-called anomalous term[1),2)] coming from the renormalization of the current vertex due to the fluctuation. Furthermore it is well established that the AL term depends strongly on the external electric field[7)~11)] (i.e. the nonlinear field dependence). The electric current exhibit much slower field dependence than usual linear dependence (i.e. the Ohmic behavior).

In spite of importance of the anomalous term[1),2)] in those experiments[10),11)] on the nonlinearity, its nonlinear field dependence has not been clarified. In this short note we will report a theoretical calculation of the nonlinear field dependence of this anomalous term.

Before entering into the calculation we will resume the field dependence of the AL term. The AL conductivity in the presence of an electric field is given[7)~9)] as

$$\sigma_{\rm AL}^{(3)} = \frac{e^2 T}{4\pi^{3/2}\xi(T)\varepsilon_0} \int_0^\infty dt\, t^{-1/2} e^{-t-(E/E_c)^2 t^3}, \tag{1}$$

$$\sigma_{\rm AL}^{(2)} = \frac{e^2 T}{2\pi d\varepsilon_0} \int_0^\infty dt\, e^{-t-(E/E_c)^2 t^3}, \tag{2}$$

$$\sigma_{\rm AL}^{(1)} = \frac{e^2 T\xi(T)}{\sqrt{\pi} S\varepsilon_0} \int_0^\infty dt\, t^{1/2} e^{-t-(E/E_c)^2 t^3} \tag{3}$$

for the 3-dimensional, the 2-dimensional and the 1-dimensional system respectively. Here d is the thickness of the thin film, S is the area of the cross section of the wire and

$$\xi(T)=\sqrt{\frac{D}{\varepsilon_0}},$$

$$\varepsilon_0=\frac{8}{\pi}(T-T_c)$$

and

$$E_c=\sqrt{\frac{3}{D}}\,\frac{(\varepsilon_0)^{3/2}}{e}\,. \tag{4}$$

In order to evaluate the field dependence of the anomalous term it is convenient to introduce the integral representations of the fluctuation propagator[1] and the function associated with the fluctuation vertex[1] as follows:

$$D(\omega_n,\boldsymbol{q})=\int_0^\infty dt\,\exp(-t|\omega_n|)\,\widehat{D}(t,\boldsymbol{q}) \tag{5}$$

and

$$\Lambda(\omega_n,\boldsymbol{q})=\int_0^\infty dt\,\exp(-t|\omega_n|)\,\widehat{\Lambda}(t,\boldsymbol{q}), \tag{6}$$

where

$$\widehat{D}(t,\boldsymbol{q})=\frac{4\pi T}{[N(0)\psi^{(1)}(1/2+\rho)]}\exp\left\{-\int_0^t[D\boldsymbol{q}^2(t)+\varepsilon_0]dt\right\} \tag{7}$$

and

$$\widehat{\Lambda}(t,\boldsymbol{q})=\exp\left\{-\int_0^t[D\boldsymbol{q}^2(t)+\delta]dt\right\}. \tag{8}$$

Here the coefficient in front of exp in Eq. (7) is the one for dirty superconductors.[1] We also introduced the pair-breaking parameter δ in Eq. (8), following Thompson.[2]

In the absence of electric field $\boldsymbol{E}$, Eqs. (5) and (6) reduce to the ordinary expressions:[1]

$$D(\omega_n,\boldsymbol{q})=\frac{4\pi T}{[N(0)\psi^{(1)}(1/2+\rho)]}\,\frac{1}{(|\omega_n|+Dq^2+\varepsilon_0)} \tag{9}$$

and

$$\Lambda(\omega_n,\boldsymbol{q})=\frac{1}{|\omega_n|+Dq^2+\delta}, \tag{10}$$

respectively. Equations (7) and (8) are easily generalized in the presence of a constant electric field $\boldsymbol{E}$. We can then incorporate the effect of the electric field by replacing $\boldsymbol{q}$ by $\boldsymbol{q}\pm 2e\boldsymbol{E}t$, depending on whether $\boldsymbol{q}$ operates on $\Delta_{\boldsymbol{q}}$ or $\Delta_{\boldsymbol{q}}^{+}$. Making use of Eqs. (5) and (6) we can repeat exactly the same calculation as done in the case of the linear response[11),2)] and we find

$$\sigma_{MT}=4e^2TD\sum_q\int_0^\infty dt\exp\left\{-\int_0^t[D(\boldsymbol{q}-2e\boldsymbol{E}t)^2+\delta]dt\right\}$$

$$\times\int_0^\infty ds\exp\left\{-\int_0^s[D(\boldsymbol{q}+2e\boldsymbol{E}s)^2+\varepsilon_0]ds\right\}$$

$$=4e^2TD\sum_q\int_0^\infty dt\int_0^\infty ds\exp\left\{-\delta t-\varepsilon_0 s-(t+s)\right.$$

$$\left.\times D\left[(\boldsymbol{q}-e\boldsymbol{E}(t-s))^2+\frac{(eE)^2}{3}(t+s)^2\right]\right\}. \tag{11}$$

In the limit $E\to 0$, it is easy to see that Eq. (11) reduces to the familiar expression[1),2)]

$$\sigma_{MT}(E\to 0)=4e^2TD\sum_q\frac{1}{Dq^2+\delta}\cdot\frac{1}{Dq^2+\varepsilon_0}. \tag{12}$$

It is important to note in the above calculation that $\boldsymbol{q}$ in Λ operates on Δ while that in D operates on Δ^*. In order to carry out further integrals, we will first consider the 3-dimensional case. The summation over $\boldsymbol{q}$ is replaced by the 3-dimensional integral and we have

$$\sigma_{MT}^{(3)}=\frac{4e^2TD}{(2\pi)^3}\int d^3q\int_0^\infty dt\int_0^\infty ds$$

$$\times\exp\left\{-\delta t-\varepsilon_0 s-(t+s)D\left[(\boldsymbol{q}-e\boldsymbol{E}(t-s))^2+\frac{(eE)^2}{3}(t+s)^2\right]\right\}$$

$$=\frac{4e^2TD}{(2\pi)^3}\frac{\pi^{3/2}}{D^{3/2}}\int_0^\infty dt\int_0^\infty ds(t+s)^{-3/2}\exp\left\{-\delta t-\varepsilon_0 s-\frac{D(eE)^2}{3}(t+s)^3\right\}$$

$$=\frac{e^2T}{2\pi^{3/2}D^{1/2}}\int_0^\infty du\int_0^1 d\lambda\exp\left\{-[\delta\lambda+\varepsilon_0(1-\lambda)]u-\frac{D(eE)^2}{3}u^3\right\}$$

$$=\frac{e^2T}{2\pi^{3/2}\xi(T)}\frac{1}{\varepsilon_0-\delta}\int_0^\infty dt\, t^{-3/2}\{e^{-\delta/\varepsilon_0 t}-e^{-t}\}e^{-(E/E_c)^2t^3}. \tag{13}$$

We can carry out similar transformations for the 2-dimensional (i.e. the thin film with the thickness d) and the 1 dimensional system (i.e. the wire with the cross section S) and we obtain

$$\sigma_{MT}^{(2)}=\frac{e^2T}{\pi d}\frac{1}{\varepsilon_0-\delta}\int_0^\infty dt\, t^{-1}(e^{-\delta/\varepsilon_0 t}-e^{-t})e^{-(E/E_c)^2t^3} \tag{14}$$

and

$$\sigma_{MT}^{(1)}=\frac{2e^2T}{\sqrt{\pi}S}\frac{\xi(T)}{\varepsilon_0-\delta}\int_0^\infty dt\, t^{-1/2}(e^{-\delta/\varepsilon_0 t}-e^{-t})e^{-(E/E_c)^2t^3}, \tag{15}$$

respectively.

It is of interest to point out that Eqs. (13), (14) and (15) converge even in the limit δ tends to 0. In particular in the limit $\delta=0$ and $E\ll E_c$ we have

$$\sigma_{\mathrm{MT}}^{(3)}|_{\delta=0}=\frac{e^2T}{\pi\xi(T)\varepsilon_0}, \tag{16}$$

$$\sigma_{\mathrm{MT}}^{(2)}|_{\delta=0}=\frac{2e^2T}{3\pi d}\ln\left(\frac{\gamma E_c}{E}\right), \qquad \gamma=1.76\cdots \tag{17}$$

and

$$\sigma_{\mathrm{MT}}^{(1)}|_{\delta=0}=\frac{2e^2T}{\sqrt{\pi}S}\frac{\xi(T)}{\varepsilon_0-\delta}\left[\frac{1}{3}\Gamma\left(\frac{1}{6}\right)\left(\frac{E_c}{E}\right)^{1/3}-\sqrt{\pi}\right], \tag{18}$$

respectively. On the other hand in a large electric field (i.e. $E\gg E_c$), $\sigma_{\mathrm{MT}}^{(n)}$ are exactly twice of $\sigma_{\mathrm{AL}}^{(n)}$ in the same limit independent of the dimension of the system.

$$\sigma_{\mathrm{MT}}^{(n)}=2\sigma_{\mathrm{AL}}^{(n)} \quad \text{for } E\gg E_c \quad (n=1,\ 2 \text{ and } 3) \tag{19}$$

which is independent of δ. Therefore we can conclude that the anomalous term becomes extremely important in experiments with large electric fields.

In the following we show the asymptotic expressions of the total fluctuation contribution to the electric conductivity.

$$\sigma_{\mathrm{fl}}^{(3)}\,(\equiv\sigma_{\mathrm{AL}}^{(3)}+\sigma_{\mathrm{MT}}^{(3)})$$

$$=\frac{e^2T}{4\pi\xi(T)\varepsilon_0}\left\{1+\frac{4}{1+(\delta/\varepsilon_0)^{1/2}}-\left(\frac{15}{8}+\frac{3}{2}\frac{(\varepsilon_0/\delta)^{5/2}-1}{1-\delta/\varepsilon_0}\right)\left(\frac{E}{E_c}\right)^2\right\} \quad \text{for } E\ll E_c \tag{20}$$

$$=\frac{e^2T}{4\pi^{3/2}\xi(T)\varepsilon_0}\left\{\Gamma\left(\frac{1}{6}\right)\left(\frac{E_c}{E}\right)^{1/3}-\frac{\sqrt{\pi}}{3}(2+\delta/\varepsilon_0)\left(\frac{E_c}{E}\right)\right\} \quad \text{for } E\gg E_c, \tag{21}$$

$$\sigma_{\mathrm{fl}}^{(2)}=\frac{e^2T}{2\pi d\varepsilon_0}\left\{1+\frac{2}{1-\delta/\varepsilon_0}\ln\left(\frac{\varepsilon_0}{\delta}\right)-4\left[\frac{5}{2}+\frac{\delta}{\varepsilon_0}+\left(\frac{\delta}{\varepsilon_0}\right)^2\right]\left(\frac{E}{E_c}\right)^2\right\} \quad \text{for } E\ll E_c \tag{22}$$

$$=\frac{e^2T}{2\pi d\varepsilon_0}\left\{\Gamma\left(\frac{1}{3}\right)\left(\frac{E_c}{E}\right)^{2/3}-\frac{1}{3}\left(2+\frac{\delta}{\varepsilon_0}\right)\Gamma\left(\frac{2}{3}\right)\left(\frac{E_c}{E}\right)^{4/3}\right\} \quad \text{for } E\gg E_c \tag{23}$$

and

$$\sigma_{\mathrm{fl}}^{(1)}=\frac{e^2T\xi(T)}{2S\varepsilon_0}\left\{1+\frac{4\sqrt{\varepsilon_0/\delta}}{1+\sqrt{\delta/\varepsilon_0}}-\left(\frac{105}{8}+\frac{15}{2}\frac{(\varepsilon_0/\delta)^{7/2}-1}{1-\delta/\varepsilon_0}\right)\left(\frac{E}{E_c}\right)^2\right\} \quad \text{for } E\ll E_c \tag{24}$$

$$= \frac{e^2 T \xi(T)}{S\varepsilon_0} \left\{ \frac{E_c}{E} - \frac{\Gamma(5/6)}{3\sqrt{\pi}} \left(2 + \frac{\delta}{\varepsilon_0}\right) \left(\frac{E_c}{E}\right)^{5/3} \right\}$$

$$\text{for} \quad E \gg E_c . \qquad (25)$$

In particular in the case of the wire, the current becomes constant independent of E in the limit $E \gg E_c$.

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Note added in proof:

The present calculation on the electric conductivity coming from the anomalous term can be easily extended into the region below T_c, if the electric field $\boldsymbol{E}$ is reasonably large. In this case our expression (11) for σ_{MT} is still valid, if we replace ε_0 by ε_0' defined by

$$\varepsilon_0' = \varepsilon_0 + \frac{14\zeta(3)}{\pi^4 N(0)} \sum_q \int_0^\infty dt \; \exp\left(-\int_0^\infty [D(\boldsymbol{q} - 2e\boldsymbol{E}t)^2 + \varepsilon_0'] dt\right).$$

In particular for a thin film with the thickness d we have

$$\varepsilon_0' = \varepsilon_0 + \frac{7\zeta(3)}{2\pi^5} \frac{1}{dN(0)D} \int_{\varepsilon_0'/\varepsilon_c}^\infty \frac{dt}{t} e^{-t-(E/E_c')^2 t^3}$$

where $E_c' = \sqrt{3/D}\ (\varepsilon_0')^{3/2}/e$ and ε_c is the cut off energy of the order of $(8/\pi) T_c$. The above definition of ε_0' is equivalent to that used in Ref. 11).