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ABSTRACT

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NONLINEAR FILTERING, THE LINK BETWEEN
KALMAN AND EXTENDED KALMAN FILTERS

By

Hosam E. Emara-Shabaik

March, 1982

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The problem of developing practical suboptimal filters for nonlinear systems is treated using a different approach. The developed filter (E1-F) is found to fill in the gap between the Kalman and the Extended Kalman filters. A numerical experiment to test the performance of the developed filter is conducted and the results are shown.

I. Introduction:

Estimation problems, and filtering among them, are basically concerned with extracting the best information from inaccurate observation of signals.

From the control theory point of view, the problem of estimating the state of dynamical systems plays an important role. Very often the optimal control law sought for a dynamical system is some sort of a feedback of its state. Take for example the control of a chemical process, a nuclear reactor, maneuvering of a space craft, guidance and navigation problems, and the problem of control and suppression of structural vibrations. Also, sometimes, it is of interest to know the state of a dynamic system. Take for example the tracking of moving objects like satellites in orbits, and enemy missiles. These are just a few examples of the application of this knowledge.

Fundamentally, the conditional probability density of the state conditioned on available observations holds the key for all kinds of state estimators. The case of a linear dynamical system, with measurements linear in the state variables, in the presence of additive Gaussian noise, and under the assumption of full knowledge of the system's parameters and noise statistics, has been optimally solved. In that particular case, the conditional probability density is Gaussian. A Gaussian density is characterized by only two quantities, namely its mean and covariance. Therefore, the optimal linear filter has a finite state, the conditional mean and the conditional covariance, and is widely known as the Kalman or the Kalman-Bucy filter. The

Kalman filter provides the minimum variance unbiased estimates. Also, the filter structures is linear, its gain and covariance can be processed independently of the estimate even before receiving the observations. These features make the Kalman filter desirable and easy to implement.

Unlike the linear case, the situation for nonlinear systems is completely different. The conditional probability density is no longer Gaussian even though the acting noise is itself Gaussian. In this case the evolution of the conditional probability density is governed by a stochastic integral-partial differential equation, Kushner's equation, or equivalently by an infinite set of stochastic differential equations for the moments of the density function. Therefore, the truly optimal nonlinear filter is of infinite dimensionality, and consequently is of a little practical interest. Therefore, the need for practical suboptimal filters is apparent.

Inspired by Kalman's results, a great deal of research effort has been directed towards extending the linear results and developing practical schemes for nonlinear filters. Developments have relied on two main schemes. One is concerned with approximations of the system nonlinearities. The other is concerned with approximations of the conditional probability density function. Several practical suboptimal schemes have been developed and a huge amount of numerical simulations have been reported. A brief account and discussion of these suboptimal filters is given in Emará-Shabaik (1979).

Still, the task of theoretical assessment of such suboptimal schemes - in the sense of providing a measure of how far a suboptimal filter is from being a truly optimal - has remained very hard to achieve. It inherits the very same practical difficulty of the optimal filter - infinite dimensionality - that one is trying to escape. Therefore, the support of any such schemes has to rely heavily on computer simulation and for that same reason not a single scheme can be claimed always superior. There are cases when a particular filter has performed better than others, while there are other cases where it has not. The final judgement is left to experience and the special case at hand. Consequently, the development of a new practical scheme will add to the list of contributions.

The main theme of this paper and its companion, under preparation, is to consider the nonlinear filtering problem from a different approach. The approach taken here is to consider the problem as the combination of approximating the system's description and solving the filtering problem for the approximate model. As a result some new schemes are developed. The problem formulation and the proposed solution are given next followed by some numerical results.

II. Problem Formulation:

Consider the general nonlinear dynamical system whose state $x(t)$ evolves in time according to the following differential equation,

$$\begin{aligned} dx(t) &= [A(t) x(t) + f(x(t), t)] dt + Q^{\frac{1}{2}}(t) dW(t) \\ x(t_0) &= x_0, \quad t \geq t_0 \end{aligned} \quad (1)$$

where

$x(t) \in R^n$ is an 'n' dimensional state vector.

$A(t)$ is an 'nxn' real matrix.

$f(x(t), t)$ is an 'n' dimensional vector valued real function.

$x_0 \in R^n$ is an 'n' dimensional Gaussian random vector (GRV) with
 $E\{x_0\} = \bar{x}_0^*$ (2)

and

$$\text{Cov}(x_0, x_0) \triangleq E\{(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)'\} = P_0^{\dagger} \quad (3)$$

$w(t) \in R^n$ is an 'n' dimensional Wiener process, and

$dW(t) = W(t+dt) - W(t)$. Therefore,

$$E\{dW(t)\} = 0 \text{ for all } t \geq t_0 \quad (4)$$

and

$$\text{Cov}(dW(t), dW(t)) \triangleq E\{dW(t) dW'(t)\} = (Idt) \quad (5)$$

Where I is the (nxn) unit matrix.

$Q^{\frac{1}{2}}(t)$ is a real matrix, and

$Q(t) \triangleq Q^{\frac{1}{2}}(t) Q^{\frac{1}{2}'}(t)$ is a positive semidefinite (nxn) matrix.

* $E\{\cdot\}$ denotes the expected value of (\cdot)

† $\text{Cov}(\cdot, \cdot)$ denotes the covariance of (\cdot) .

Also, consider the observations process $dy(t)$ to be given by

$$dy(t) = [C(t) x(t) + h(x(t),t)] dt + R^{\frac{1}{2}}(t) dv(t) \quad (6)$$

where

$dy(t) \in R^m$ is an 'm' dimensional observations vector.

$C(t)$ is an 'm x n' real matrix.

$h(x(t),t)$ is an 'm' dimensional vector valued real function.

$v(t) \in R^m$ is an 'm' dimensional Wiener process, and

$dv(t) = V(t + dt) - V(t)$. Therefore,

$$E \{ dv(t) \} = 0 \text{ for all } t \geq t_0 \quad (7)$$

and

$$\text{Cov}(dv(t), dv(t)) \triangleq E \{ dv(t) dv'(t) \} = (Idt) \quad (8)$$

$R^{\frac{1}{2}}(t)$ is a real matrix, and

$R(t) \triangleq R^{\frac{1}{2}}(t) R^{\frac{1}{2}'}(t)$ is a positive definite (n x n) matrix

We assume that x_0 , $w(t)$, and $v(t)$ are all independent of each other for all values of $t \geq t_0$. Also, the assumption that equation (1) satisfies the conditions for existence and uniqueness of solution given in Arnold (1974), and Jazwinski (1970) is being made. This means that our dynamical system (1) admits only one solution $x(t), t \geq t_0$ to be its state trajectory in the mean square sense. Furthermore, it is assumed that both $f(x(t),t)$ and $h(x(t),t)$ are continuous in $x(t)$.

As it is noticed from equations (1), and (6), the system structure is considered to be composed of two parts, a linear part plus a non-linear part. Furthermore, we assume that the system behavior is dominated by its linear part. That is to say,

$$\|f(x(t),t)\| < \|A(t)x(t)\| \quad (9)$$

and

$$\|h(x(t),t)\| < \|C(t)x(t)\| \quad (10)$$

where

$\|z\|$ is the norm of the vector z .

Equations (1) and (6) along with conditions (9) and (10) can be the original system's description, what is sometimes referred to as system's with conebounded nonlinearities. Also, it can be a representation obtained by linearization of a nonlinear system, where $f(x(t),t)$ and $h(x(t),t)$ represent second and higher order terms. In this case conditions (9) and (10) are valid as long as the system's state $x(t)$ remains within a small neighborhood of the nominal (linearizing) trajectory.

Accordingly, conditions (9), and (10) suggest that for a good guess of the system state $x^*(t)$ the following approximate equations for the dynamics and observations can be written as

$$dx_1(t) = [A(t)x_1(t) + f(x^*(t),t)] dt + Q^{1/2}(t) dw(t) \quad (11)$$

$$dy(t) = [C(t)x_1(t) + h(x^*(t),t)] dt + R^{1/2}(t) dv(t) \quad (12)$$

By virtue of continuity of the nonlinearities in $x(t)$, we should note the following. As $x^*(t)$ approaches $x_1(t)$, the approximate description given in (11), and (12) approaches the true description in (1), and (6). In fact, the following equation

$$\begin{aligned} dx_1(t) &= [A(t)x_1(t) + f(x_1(t),t)] dt + Q^{1/2}(t) dw(t), \\ x(t_0) &= x_0, \quad t \geq t_0 \end{aligned} \quad (13)$$

and equation (1) have the same solution both in the mean square sense and with probability one.

Thus follows, the filtering problem of the system (1), (6) can be considered as a unification of model approximation and state estimation of the approximate model. In other words, first we approximate the system description by finding a suitable $x^*(t)$. Then, solve the optimal filtering problem of the approximate model. The optimal filtering is basically to seek the minimum mean square error estimate of the state $x(t)$ based on the available observations, $Y_t = [y(s), t_0 \leq s \leq t]$.

Generally, according to theorem (6.6) of Jazwinski (1970), pp. 184; and its specialization to linear systems; theorem (7.3) pp. 219 of the same reference, the optimal filter imitates the dynamics of the system and is linearly driven by the net observations. Therefore, guided by these results, we will seek the optimal filter for the system in (11) and (12) as a linear dynamic system driven linearly by the net observations. The optimality of the filter is in the sense of achieving minimum mean square error.

so, if we define the estimation error $e_1(t)$ as

$$e_1(t) \triangleq x_1(t) - \hat{x}_1(t) \quad (14)$$

and the covariance matrix $P(t)$ as

$$P(t) \triangleq E\{(e_1(t) - \bar{e}(t))(e_1(t) - \bar{e}(t))'\} \quad (15)$$

Where $\hat{x}_1(t)$ is an estimate of $x_1(t)$ based on Y_t , and

$$e_1(t) \triangleq E\{e_1(t)\} \quad (16)$$

then,

$$\begin{aligned} J(e_1(t)) &= \text{tr}(E\{e_1(t)e_1'(t)\}) \\ &= \text{tr}(P(t)) + \text{tr}(\bar{e}_1(t)\bar{e}_1'(t)) \end{aligned} \quad (17)$$

is to be minimized.

III. Proposed Solution, Derivation of the (E-1) Filter:

According to our approximation the dynamic measurements can be written as follows,

$$dx_1(t) = [A(t)x_1(t) + f(x^*(t), t)] dt + Q^{1/2}(t) dW(t) \quad (20)$$

$$dy_1(t) = dy(t) - h(x^*(t), t) dt = C(t) X_1(t) dt + R^{1/2}(t) dV(t) \quad (21)$$

Then a linear filter structure can be sought as

$$d\hat{x}_1(t) = [B(t) \hat{x}_1(t) + f(x^*(t), t)] dt + K(t) dy_1(t) \quad (22)$$

where

$B(t)$ is an 'nxn' matrix

$K(t)$ is an 'nxm' gain matrix

$B(t)$, and $K(t)$ are to be chosen to provide a minimum variance unbiased estimate. By definition, the estimation error is

$$e_1(t) = x_1(t) - \hat{x}_1(t) \quad (23)$$

Therefore,

$$de_1(t) = dx_1(t) - d\hat{x}_1(t) \quad (24)$$

And by substitution of (20), (21), and (22) in (24) above we get,

$$\begin{aligned} de_1(t) &= A(t)x_1(t)dt + Q^{\frac{1}{2}}(t) dw(t) - B(t) \hat{x}_1(t) dt \\ &\quad - K(t) C(t) x_1(t)dt - K(t) R^{\frac{1}{2}}(t) dv(t) \\ &= [A(t) - B(t) - K(t) C(t)] x_1(t)dt \\ &\quad + B(t) e_1(t)dt + Q^{\frac{1}{2}}(t) dw(t) \\ &\quad - K(t) R^{\frac{1}{2}}(t) dv(t) \end{aligned} \quad (25)$$

It is desirable to have the estimation error independent of the state. In this case large state variables can be estimated as accurate as small state variables. Therefore, we may choose

$$B(t) = A(t) - K(t) C(t) \quad (26)$$

Therefore, the dependence of the estimation error on the state is eliminated.

Also, by choosing

$$\hat{x}(t_0) = \bar{x}_0 \quad (27)$$

we have

$$\begin{aligned} de_1(t) &= [A(t) - K(t) C(t)] e_1(t)dt + Q^{\frac{1}{2}}(t) dw(t) \\ &\quad - K(t) R^{\frac{1}{2}}(t) dv(t), \quad e_1(t_0) = x_0 - \bar{x}_0 \end{aligned} \quad (28)$$

And

$$d\bar{e}_1(t) = [A(t) - K(t) C(t)] \bar{e}_1(t) dt, \bar{e}_1(t_0) = 0 \quad (29)$$

from which follows that

$$\bar{e}_1(t) = 0 \text{ for all } t \geq t_0 \quad (30)$$

Hence, the estimate is unbiased. Next, we seek

the filter gain matrix $K(t)$ that provides the minimum variance estimate.

By definition, the covariance matrix $P(t)$ is

$$P(t) \triangleq E \{ (e_1(t) - \bar{e}_1(t))(e_1(t) - \bar{e}_1(t))' \} \quad (31)$$

and due to (30)

$$P(t) = E \{ e_1(t) e_1'(t) \} \quad (32)$$

Straight forward manipulations show that $P(t)$ is given by the following differential equation,

$$dP(t) = \left([A(t) - K(t) C(t)] P(t) + P(t) [A(t) - K(t) C(t)]' + Q(t) + K(t) R(t) K'(t) \right) dt \quad (33)$$

As the matrix differential equation for the covariance matrix $P(t)$.

The initial condition for (33) is given by

$$P(t_0) = E \{ (x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T \} = P_0 \quad (34)$$

Now, the optimization problem involving the choice of the gain matrix $K(t)$ can be stated as follows.

$$\begin{aligned} & \min \quad \text{tr}(P(t)) \\ & K(s) \\ & t_0 \leq s \leq t \end{aligned}$$

Subject to conditions as given by equations (33) and (34) which is the same as

$$\min_{\substack{K(s) \\ t_0 \leq s \leq t}} \text{tr} \left(P_0 + \int_{t_0}^t dP(t) \right) \equiv \text{tr}(P_0) + \min_{\substack{K(s) \\ t_0 \leq s \leq t}} \text{tr} \left(\int_{t_0}^t dP(t) \right)$$

Therefore, we seek $K(s)$, $t_0 \leq s \leq t$ that minimizes

$$\text{tr} \left(\int_{t_0}^t dP(t) \right) = \int_{t_0}^t \text{tr}(dP(t)) \quad (35)$$

substituting for $dP(t)$ in equation (35) we get

$$\begin{aligned} \int_{t_0}^t \text{tr}(dP(t)) &= \int_{t_0}^t \left(\text{tr}([A(t) - K(t) C(t)]P(t)) \right. \\ &\quad \left. + \text{tr}(P(t)[A(t) - K(t) C(t)]') \right. \\ &\quad \left. + \text{tr}(K(t) R(t) K'(t)) + \text{tr}(Q(t)) \right) dt \end{aligned} \quad (36)$$

The integrand in (36) is a convex quadratic in $K(t)$. According to the theory of calculus of variations, the minimizing $K(s)$, $t_0 \leq s \leq t$ is given as the solution of the Euler's equation which reduces to a simple algebraic equation in the present case, namely

$$\frac{\partial}{\partial K(t)} \text{tr}(dP(t)) = 0 \quad (37)$$

Using the concept of gradient matrices and the formulae developed in Athans, et.al.(1965), we get

$$\frac{\partial}{\partial K(t)} \text{tr}(K(t) C(t) P(t)) = P(t) C'(t) \quad (38)$$

$$\frac{\partial}{\partial K(t)} \text{tr}(P(t) C'(t) K'(t)) = P(t) C'(t) \quad (39)$$

and

$$\frac{\partial}{\partial K(t)} \text{tr}(K(t) R(t) K'(t)) = 2K(t) R(t) \quad (40)$$

Therefore the optimal gain is given by

$$-2P(t)C'(t) + 2K(t)R(t) = 0 \quad (41)$$

i.e.,

$$K(t) = P(t)C'(t)R^{-1}(t) \quad (42)$$

Substituting this result into equation (33), the covariance matrix satisfies the following matrix Riccati differential equation,

$$dP(t) = \left[A(t)P(t) + P(t)A'(t) - P(t)C'(t)R^{-1}(t)C(t)P(t) + Q(t) \right] dt, P(t_0) = P_0 \quad (43)$$

To summarize, the solution for the filtering problem of the approximate model is given by

$$\left. \begin{aligned} d\hat{x}_1(t) &= \left[A(t)\hat{x}_1(t) + f(x^*(t), t) \right] dt + K(t) \left[dy(t) \right. \\ &\quad \left. - C(t)\hat{x}_1(t)dt - h(x^*(t), t)dt \right] \\ \hat{x}_1(t_0) &= \bar{x}_0 \\ K(t) &= P(t)C'(t)R^{-1}(t) \\ dP(t) &= \left[A(t)P(t) + P(t)A'(t) - P(t)C'(t)R^{-1}(t)C(t)P(t) \right. \\ &\quad \left. + Q(t) \right] dt \\ P(t_0) &= P_0 \end{aligned} \right\} (44)$$

According to the argument following equations (11) and (12), $x^*(t)$ is required to provide the optimal solution of the following minimization problem.

$$\min_{x^*(t)} J(x^*(t)) = E_{Y_t} \left\{ (x_1(t) - x^*(t))' (x_1(t) - x^*(t)) \right\} \quad (45)$$

then for every $t \geq t_0$ setting $\partial J(x^*(t)) / \partial x^*(t) = 0$ we get

$$x^*(t) = E_{Y_t} \left\{ x_1(t) \right\} = \hat{x}_1(t) \quad (46)$$

Therefore, combining the results of equations (44), and (46) we get the first of the developed filters, to be denoted as the (E1-F) filter namely,

$$d\hat{x}_1(t) = \left[A(t)\hat{x}_1(t) + f(\hat{x}_1(t), t) \right] dt + K(t) \left[dy(t) - C(t)\hat{x}_1(t) dt - h(\hat{x}_1(t), t) dt \right], \hat{x}_1(t_0) = \bar{x}_0 \quad (47)$$

$$K(t) = P(t)C'(t)R^{-1}(t) \quad (48)$$

$$dP(t) = \left[A(t)P(t) + P(t)A'(t) - P(t)C'(t)R^{-1}(t)C(t)P(t) + Q(t) \right] dt$$

$$P(t_0) = P_0 \quad (49)$$

It is straightforward to recognize that in case of a linear system, i.e. $f(x(t), t)$ and $h(x(t), t)$ are identically zero or only functions of time, equations (47), (48) and (49) reduce to the well known Kalman filter.

Although the (E1-F) is developed using a different approach, it bears a close relationship with the extended Kalman filter (EKF) given in Jazwinski (1970). The equations for the state estimate of both the (E1-F) and the (EKF) have the same structure. While the equations for the gain and covariance of the (E1-F) are different from those for the (EKF), they are identical to those of the Kalman filter (KF). Therefore, unlike the (EKF), the gain and covariance for the (EL-F) can be processed off line and prior to receiving the observations. This is due to the fact that, the matrices $A(t)$ and $C(t)$ in the (E1-F) are different from the corresponding matrices $\tilde{A}(\hat{x}(t),t)$ and $\tilde{C}(\hat{x}(t),t)$ in the (EKF), they are no longer estimate dependent. The two sets of matrices are related as follows,

$$\tilde{A}(\hat{x}(t),t) = A(t) + \frac{\partial f(x(t),t)}{\partial x(t)} \Big|_{x(t) = \hat{x}(t)} \quad (50)$$

and

$$\tilde{C}(\hat{x}(t),t) = C(t) + \frac{\partial h(x(t),t)}{\partial x(t)} \Big|_{x(t) = \hat{x}(t)} \quad (51)$$

Therefore, the (E1-F) has the gain and covariance computational facility enjoyed by the linear filter, and moreover, it is of higher sophistication since it accounts for otherwise neglected nonlinearities.

Therefore, the E1-F will be of an advantage over the EKF when on line computations of the gain and covariance are not affordable due to capacity limitations of on line computers. This is usually the case of airborne and spaceborne computers.

Furthermore, while the (EKF) has to be strictly interpreted in the Ito sense, Emera-Shabaik (1980), it is not the case with the (E1-F). This is so because the gain $K(t)$ as given by equation (48) is not estimate dependent. On the other hand, if equations (1) and (6) are obtained through linearization of some nonlinear system, where the system is being continuously relinearized around the most recent available estimate then the (E1-F) and (EKF) are identically the same. So, in a sense the (E1-F) provides the missing link between the Kalman and extended Kalman filters, and this credits the new approach of looking at nonlinear filtering.

IV. Numerical Experiment:

The Van der Pol oscillator:

The Van der Pol oscillator is characterized by the following differential equation, Cunningham (1958).

$$\ddot{x}(t) - \epsilon \dot{x}(t)(1 - x^2(t)) + x(t) = 0 \quad (52)$$

which describes a dynamical system with state dependent damping coefficient equals $-\epsilon(1-x^2(t))$ where ϵ is a positive parameter. The damping in the system goes from negative to zero to positive values as the value of $x^2(t)$ changes from less than to greater than unity. The oscillator's response is characterized by a limit cycle in the $x(t)$, $\dot{x}(t)$ plane (the phase plane). The limit cycle approaches a circular shape as ϵ becomes very small, it has a maximum value for $x(t)$ equals 2.0 irrespective of the value of ϵ . This type of oscillations occur in electronic tubes which exhibit also what is known as thermal noise. Denoting $x(t)$ as $x_1(t)$, and $\dot{x}(t)$ as $x_2(t)$, equation (52) can be rewritten in a state space formulation. Also, considering the existence of some noise forcing on the system, we get the following representation for the Van der Pol oscillator.

$$\begin{bmatrix} dx_1(t) \\ dx_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & \epsilon \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} dt + \begin{bmatrix} 0 \\ -\epsilon x_1^2(t) x_2(t) \end{bmatrix} dt + Q^{\frac{1}{2}} \begin{bmatrix} dW_1(t) \\ dW_2(t) \end{bmatrix} \quad (53)$$

Also suppose that the following measurement is taken

$$dy(t) = [x_1(t) + x_1^3(t)] dt + R^{1/2} dv(t) \quad (54)$$

In (53) and (54) above $[W_1(t) W_2(t)]^T$ is considered to be a two dimensional Wiener process. Also, $V(t)$ is a one dimensional Wiener process. R is a positive nonzero real value, and Q is a (2x2) matrix. The following values for noise statistics are considered.

Case #	Q_{11}	Q_{12}	Q_{22}	R	figures
Van der Pol 1	0.5	0.0	0.5	4.0	1 to 2
Van der Pol 2	5.0	2.0	5.0	10.0	3 to 4

Also ϵ is taken to be 0.2

In the figures, the following symbols are used.

$XI \equiv$ the i^{th} state, $I = 1, 2$

$XIK \equiv$ the estimate of the i^{th} state provided by the (K-F)

$XIE \equiv$ the estimate of the i^{th} state provided by the (E1-F)

$XIEK \equiv$ the estimate of the i^{th} state provided by the (EKF)

In both cases; as indicated by figures 1, 2, 3, and 4, both the (E1-F) and (EKF) provide very accurate tracking of the system's states while the (KF) provides crude estimates.

V. Conclusions:

A new approach for nonlinear filtering is developed. Basically, it consists of a model approximation technique combined with optimal filtering of the approximate model. The resulting nonlinear filter (EI-F) has a structure that fits into the gap between the Kalman and the Extended Kalman filters. On one hand it enjoys the same computational facility for the gain and covariance enjoyed by the Kalman filter (KF). While on the other hand it provides estimates on the same level of accuracy as provided by the extended Kalman filter (EKF).

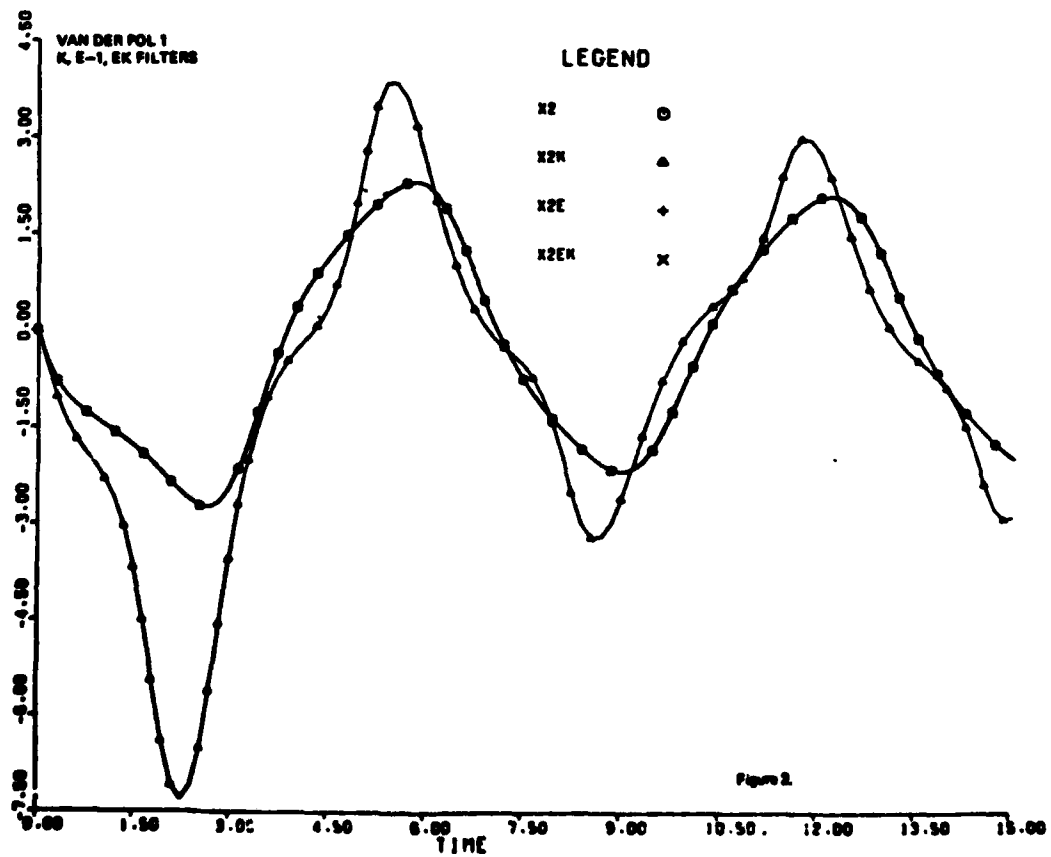
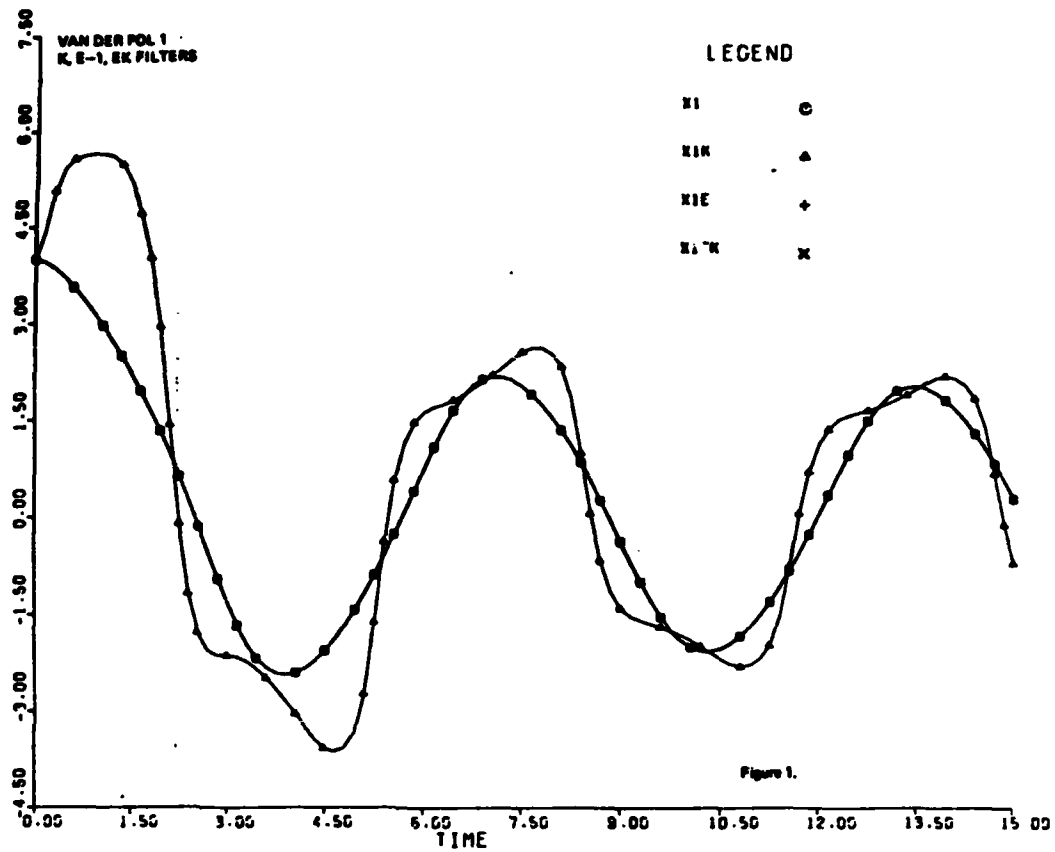


Figure 1. First state and estimates by Kalman, E1, Extended Kalman filters.

Figure 2. Second state and estimates by Kalman, E1, Extended Kalman filters.

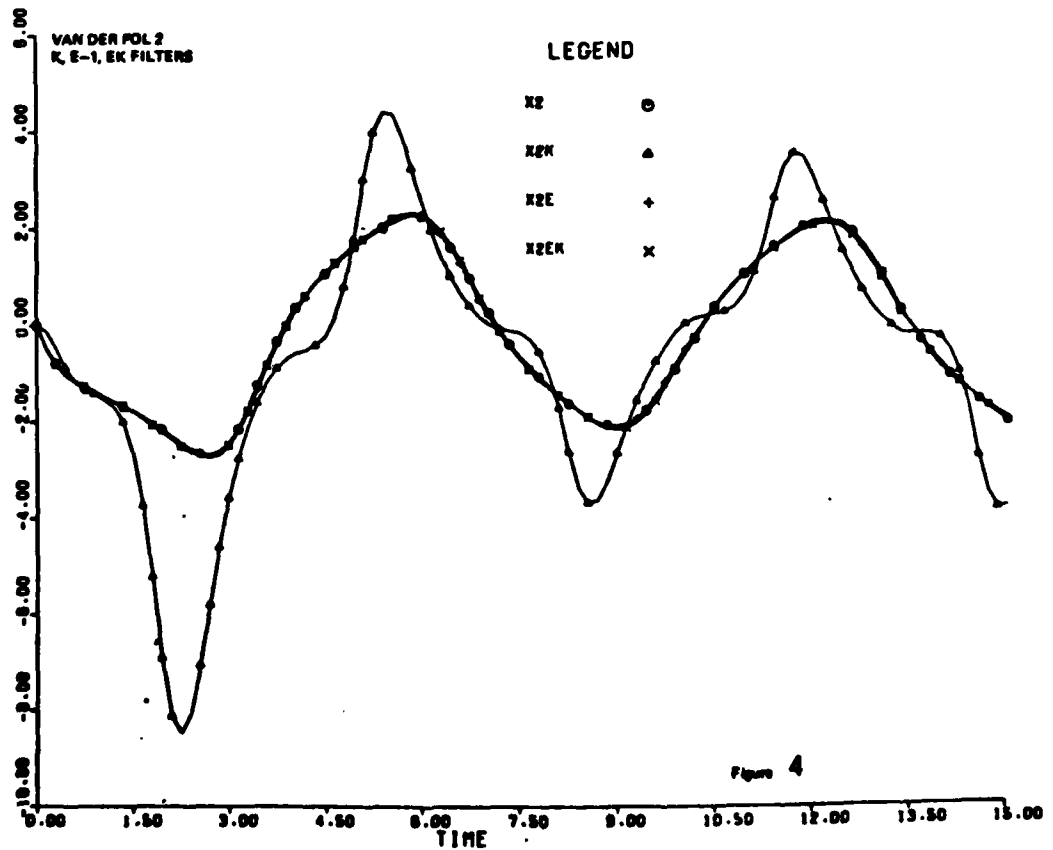
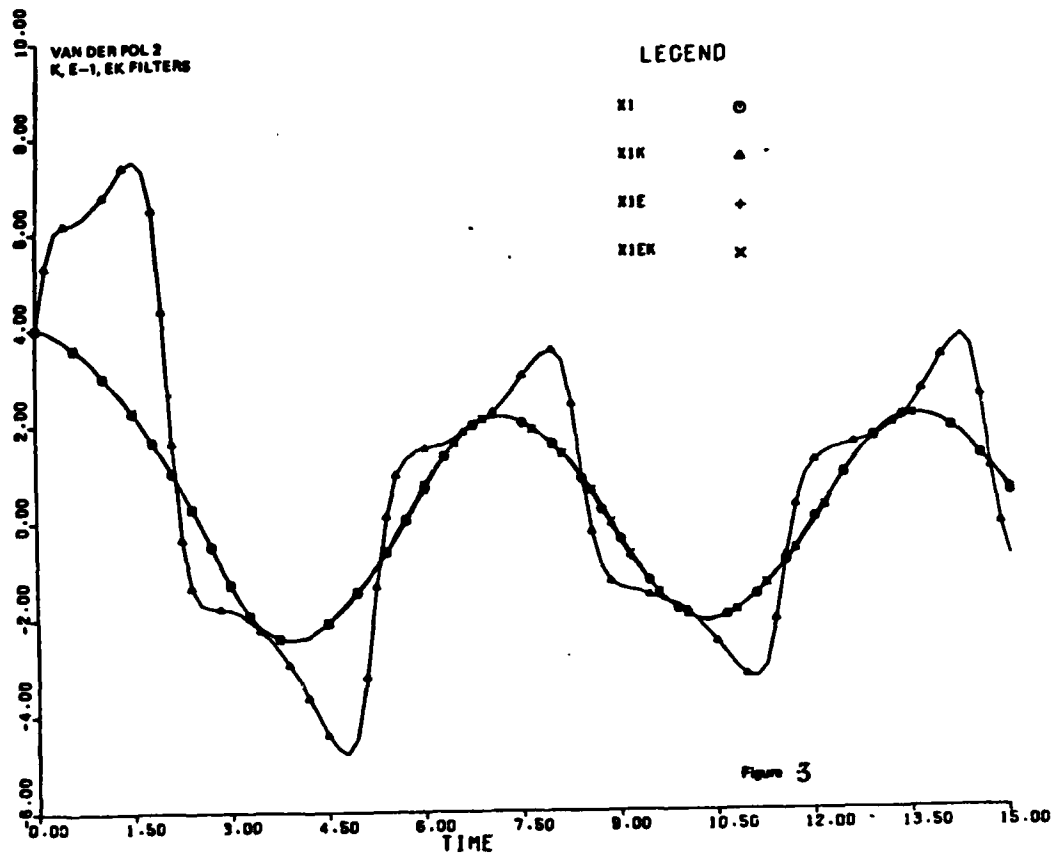


Figure 3. First state and estimates by Kalman, E1, Extended Kalman filters.

Figure 4. Second state and estimates by Kalman, E1, Extended Kalman filters.

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