Nonlinear Fuzzy H_∞ Guidance Law With Saturation of Actuators Against Maneuvering Targets

Bor-Sen Chen, *Fellow, IEEE*, Yung-Yue Chen, and Chun-Liang Lin

***Abstract*—A nonlinear H_∞ guidance law based on a fuzzy model is proposed for tactical missiles pursuing maneuvering targets in three-dimensional (3-D) space. In the proposed guidance scheme, the relative motion equations between the missile and target are first interpolated piecewise by Takagi–Sugeno linear fuzzy models. Then, a nonlinear fuzzy H_∞ guidance law is designed to eliminate the effects of approximation error and external disturbances to achieve the desired goal. The linear matrix inequality (LMI) technique is then employed to treat this H_∞ optimal guidance design in consideration of control constraints. Finally, the problem is further transformed into a standard eigenvalue problem so that it can be efficiently solved via a convex optimization algorithm, which is available from a numerical computation software.**

***Index Terms*—Fuzzy control, guidance, maneuvering target tracking, missile, nonlinear H_∞ control.**

NOMENCLATURE

r	Relative distance between the missile and the target.
ϕ	Pitch line-of-sight angle (PLOS).
θ	Yaw line-of-sight angle (YLOS).
$\ddot{r}$	Relative acceleration along to LOS.
$\dot{\phi}$	Angular velocity of ϕ.
$\dot{\theta}$	Angular velocity of θ.
$\ddot{\phi}$	Angular acceleration of ϕ.
$\ddot{\theta}$	Angular acceleration of θ.
$\vec{e}_r$	Unit vector along the LOS.
$\vec{e}_\phi$	Unit vector along the PLOS.
$\vec{e}_\theta$	Unit vector along the YLOS.
$\vec{a}_T = w_r\vec{e}_r + w_\theta\vec{e}_\theta + w_\phi\vec{e}_\phi$	Acceleration vector of target.
$\vec{a}_M = u_r\vec{e}_r + u_\theta\vec{e}_\theta + u_\phi\vec{e}_\phi$	Acceleration vector of missile.
$V_r = \dot{r}$	Relative velocity along to LOS.
$V_\phi = r\dot{\phi}$	Relative velocity normal to PLOS.
$V_\theta = r\dot{\theta}\cos\phi$	Relative velocity normal to YLOS.

Manuscript received June 1, 2001; revised March 4, 2002. Manuscript received in final form April 3, 2002. Recommended by Associate Editor N. Sundararajan. This work was supported by the National Science Council, Taiwan, R.O.C., under Grant NSC89-2213-E-007-039.

B.-S. Chen and Y.-Y. Chen are with the Department of Electrical Engineering, National Tsing-Hua University, Hsin Chu 30043, Taiwan, R.O.C. (e-mail: bschen@moti.ee.nthu.edu.tw).

C.-L. Lin is with the Institute of Automatic Control Engineering, Feng Chia University, Taichung 40724, Taiwan, R.O.C. (e-mail: chunlin@fcu.edu.tw).

Digital Object Identifier 10.1109/TCST.2002.804122

I. INTRODUCTION

THE PRINCIPLES of missile guidance are well known to control engineers. Since the basic principles were extensively covered in [1] and [2], many technologies have been developed to improve guidance performance and to accommodate environmental disturbances. These techniques are mainly based on classical control theory. Various guidance laws have been exploited with different design concepts over the years [2]. Currently, most popular terminal guidance laws defined by Locke [1] involve line-of-sight (LOS) guidance, LOS rate guidance, command-to-line-of-sight (CLOS) guidance [3] and other advanced guidances such as proportional navigation guidance (PNG) [1], augmented proportional navigation guidance (APNG) [4] and optimal guidance law based on the linear quadratic regulator theory [5], linear quadratic Gaussian theory [6], or linear exponential Gaussian theory [7].

Of the current techniques, guidance commands proportional to the LOS angle rate are generally used by most high-speed missiles today to correct missile course. This approach is referred to as PNG and is quite successful against nonmaneuvering targets. While PNG exhibits optimal performance with constant-velocity targets, it is not effective for uncertain target maneuvers and often leads to unacceptable miss distances [8]. Besides, the dynamic system representing relative motion between the pursuer (missile) and target is, in general, highly nonlinear and uncertain due to unmodeled dynamics and parametric perturbations resulting from the plant modeling. Therefore, as a well-considered guidance system design, robustness of engagement performance with respect to modeling uncertainties and external disturbances must also be considered.

Based on the reasons depicted above, it is highly desirable to apply advanced control techniques developing an effective guidance law to improve engagement performance for tactical missiles. As one of the powerful modern control techniques, the H_∞ control has been widely applied to treat the robust design problem of systems contaminated by modeling uncertainties and external disturbances. In aerospace applications, for example, the linear H_∞ control designs have been applied to control a space station [9] and a missile autopilot [10], [13]. A nonlinear H_∞ control design has also been used in the satellite attitude control problem [11], [12].

In this paper, the target accelerations are regarded as unpredictable disturbances. Based on this setting, the guidance problem is formulated as a nonlinear H_∞ disturbance attenuation problem. To solve the problem using the conventional H_∞ design approaches [11], a nonlinear Hamilton–Jacobi partial differential inequality (HJPDI) must be solved. Unfortunately, such an equation is usually difficult to be solved except for simple or special cases.

Recently, there has been rapidly growing interest in fuzzy control of nonlinear systems [17], [18]. Using this approach, a nonlinear plant can be approximated by a fuzzy model, and a model-based fuzzy controller can be developed to stabilize the overall system [21], [24]. In the present approach, the three-dimensional (3-D) missile–target dynamics is approximated by a perturbed fuzzy system motivated through a Takagi–Sugeno fuzzy model [16], which is obtained by interpolating several linearized systems at different operating points through fuzzy certainty functions. Using this approach, solutions of the nonlinear HJPDI from the conventional H_∞ guidance law design problem can be approximated by piecewise interpolating a set of linear Riccati-like equations via fuzzy certainty functions. The problem is also parameterized in terms of an eigenvalue problem (EVP) so that additional control constraints of actuators of missile control can be included in our design.

For the convenience of design, the proposed design framework is characterized as a linear matrix inequality problem (LMIP). For practical application, the saturation of actuators are also considered in the proposed nonlinear fuzzy H_∞ guidance law design. The LMIP is used to characterize a suboptimal H_∞ guidance law with control constraints so that the corresponding linear matrix inequalities (LMIs) are feasible. The EVPs or LMIPs are to be solved via a convex optimization technique supported by the LMI toolbox of Matlab [19] software. Finally, a simulation example is given to illustrate the design procedure and confirm the guidance performance.

II. Plant Modeling and Design Objective

The 3-D pursuit geometry is described in the spherical coordinates [29] where the relative position vector along the line of sight is expressed by $\vec{r}$. Fig. 1 is a pursuit–evasion geometry between the missile and the target. The differentiation of $\vec{r}$ gives the 3-D relative velocity as

$$\dot{\vec{r}} = \dot{r}\vec{e}_r + r\dot{\theta}\cos\phi\vec{e}_\theta + r\dot{\phi}\vec{e}_\phi. \tag{1}$$

Differentiating both sides of the above equation yields the relative accelerations as

$$\begin{aligned} \ddot{r} - r\dot{\phi}^2 - r\dot{\theta}^2\cos^2\phi &= w_r - u_r \\ r\ddot{\theta}\cos\phi + 2\dot{r}\dot{\theta}\cos\phi - 2r\dot{\phi}\dot{\theta}\sin\phi &= w_\theta - u_\theta \\ r\ddot{\phi} + 2\dot{r}\dot{\phi} + r\dot{\theta}^2\cos\phi\sin\phi &= w_\phi - u_\phi. \end{aligned} \tag{2}$$

The kinematics between the missile and the target in (2) can now be recast into the following nonlinear state-space equation:

$$\dot{x}(t) = F(x(t)) + Bu(t) + Dw(t) \tag{3}$$

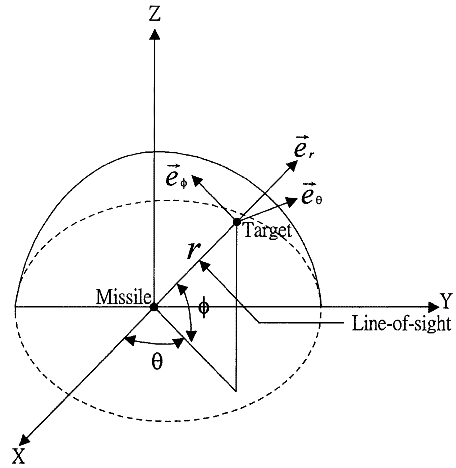


Fig. 1. 3-D pursuit–evasion geometry.

where the state vector $x(t)$, the vector field $F(x(t))$, the missile acceleration vector $u(t)$, and the target acceleration vector $w(t)$ are defined, respectively, as follows:

$$x(t) = \begin{bmatrix} r \\ \theta \\ \phi \\ V_r \\ V_\theta \\ V_\phi \end{bmatrix}, \qquad F(x(t)) = \begin{bmatrix} V_r \\ \dfrac{V_\theta}{r\cos\phi} \\ \dfrac{V_\phi}{r} \\ \dfrac{V_\theta^2 + V_\phi^2}{r} \\ -\dfrac{V_r V_\theta}{r} + \dfrac{V_\theta V_\phi \tan\phi}{r} \\ -\dfrac{V_r V_\phi}{r} - \dfrac{V_\theta^2 \tan\phi}{r} \end{bmatrix}$$

$$u(t) = \begin{bmatrix} u_r \\ u_\theta \\ u_\phi \end{bmatrix}, \qquad w(t) = \begin{bmatrix} w_r \\ w_\theta \\ w_\phi \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{4}$$

Let us denote the state variable $\eta(t) = [r \quad V_\theta \quad V_\phi]$ to be controlled as

$$\eta(t) = L'x(t) \tag{5}$$

where

$$L' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Remark 1: When $V_\theta, V_\phi \to 0$, it means missile and target in the head-on condition. Among three relative velocities of the nonlinear system in (3), only the relative velocity along to LOS (i.e., V_r) decreases the relative distance between missile and target. □

A good guidance law must guarantee a decreasing relative distance and at the same time, keep the pitch and yaw LOS angular rates as small as possible, i.e., in the head on condition. So our design objective is to specify the guidance command $u(t)$ so that the controlled variable $\eta(t)$ reduces to zero. Because the target accelerations are generally uncertain but bounded, it can be viewed as an external disturbance to the missile system. Since the H_∞ guidance law has been shown to be an effective control methodology to attenuate the effect of uncertain external disturbances on the desired control performance, the following H_∞ guidance performance index is considered here as the design objective [11], [23]

$$\int_0^{t_f} [\eta^T(t)Q\eta(t) + u^T(t)Ru(t)]\,dt \le x^T(0)Px(0) - x^T(t_f)Px(t_f) + \rho^2\int_0^{t_f} w^T(t)w(t)\,dt \quad \forall\, w(t)\in L_2[0,\,t_f]$$

where t_f denotes the flight time, the notation $w(t) \in L_2[0,\,t_f]$ denotes all possible $w(t)$ with $\int_0^{t_f} w^T(t)w(t)\,dt < \infty$; $P = P^T > 0$, $Q \ge 0$ and $R = R^T > 0$ are the weighting matrices, ρ^2 denotes the attenuation level which can be a prescribed value, i.e., from the energy viewpoint, the effect of uncertain target accelerations $w(t)$ on $\eta(t)$ and $u(t)$ must be less than ρ^2 for all possible $w(t) \in L_2[0,\,t_f]$.

Now, using (5), we obtain

$$\int_0^{t_f} [x^T(t)Q'x(t) + u^T(t)Ru(t)]\,dt \le x^T(0)Px(0) - x^T(t_f)Px(t_f) + \rho^2\int_0^{t_f} w^T(t)w(t)\,dt \quad \forall\, w(t)\in L_2[0,\,t_f] \tag{6}$$

where $Q' = Q'^T = L'^TQL' \ge 0$.

The closed-form solution of H_∞ guidance law satisfies the performance in (6) for the 3-D guidance system in (3) can be obtained by $u(t) = -(1/2)B^T(\partial U(x(t))/\partial x(t))$ via solving the following HJPDI [30]:

$$\frac{\partial U(x(t))}{\partial x(t)}F(x(t)) + \frac{1}{2}\frac{\partial U(x(t))}{\partial x(t)}\left(\frac{1}{\rho^2}DD^T - BB^T\right)\cdot\frac{\partial U(x(t))^T}{\partial x(t)} + \frac{1}{2}x^T(t)Q'x(t) \le 0 \tag{7}$$

where $U(x(t))$ is a Lyapunov function.

In general, it is almost impossible to obtain a closed-form solution $U(x(t))$ for 3-D guidance system from this nonlinear partial differential equation.

III. Nonlinear H_∞ Guidance Design Via Fuzzy Model Method

Construction for the H_∞ guidance law of the guidance system described in (3) needs to solve an HJPDI (7), whose solution is difficult to be solved even with numerical methods. To overcome this problem, a fuzzy model is employed here to approximate the nonlinear relative motion equation in (3). A fuzzy dynamic model proposed by Takagi and Sugeno [16] is applied to represent the locally linearized input–output relations around the operating points. This fuzzified linear model is described by a group of if–then rules and is used to deal with the H_∞ guidance design problem.

The ith rule of this fuzzy model for the nonlinear guidance system (3) is described by

Plant Rule i :

If $z_1(t)$ is G_{i1} and ... and $z_g(t)$ is G_{ig}

then $\dot{x}(t) = A_ix(t) + Bu(t) + Dw(t)$

for $i = 1, 2, \ldots, l$ (8)

where G_{ij} is the fuzzy set, $A_i \in R^{6\times 6}$, $B \in R^{6\times 3}$, l is the number of fuzzy rules, and $z_1(t), \ldots, z_g(t)$ are the premise variables. The overall fuzzy system can be inferred as follows [16], [18]:

$$\dot{x}(t) = \sum_{i=1}^{l} h_i(z(t))(A_ix(t)) + Bu(t) + Dw(t) \tag{9}$$

where

$$z(t) = [z_1(t)\ z_2(t) \cdots z_g(t)]^T$$

$$\mu_i(z(t)) = \prod_{j=1}^{g} G_{ij}(z_j(t))$$

$$h_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^{l}\mu_i(z(t))}$$

and $G_{ij}(z_j(t))$ is the membership grade of $z_j(t)$ in G_{ij}.

It is natural to assume

$$\mu_i(z(t)) \ge 0, \qquad \text{for } i = 1, \ldots, l \quad \text{for all } t.$$

Therefore, we get the certainty functions

$$h_i(z(t)) \ge 0, \qquad \text{for } i = 1, 2, \ldots, l \tag{10}$$

and

$$\sum_{i=1}^{l} h_i(z(t)) = 1. \tag{11}$$

The physical meaning of the fuzzy model (9) is that the locally linearized systems $\dot{x}(t) = A_ix(t) + Bu(t) + Dw(t)$ at different operation points (different fuzzy set G_{ij}) are interpolated piecewise via the certainty function $h_i(z(t))$ to approximate the original nonlinear system (3). Note that identifications of $G_{ij}(z_j(t))$ and A_i from $F(x(t))$ can be easily obtained using the clustering technique [14], [15].

Now, (3) can be rewritten as

$$\dot{x}(t) = \sum_{i=1}^{l} h_i(z(t))(A_ix(t)) + \Delta f + Bu(t) + Dw(t) \tag{12}$$

where $\Delta f = F(x(t)) - \sum_{i=1}^{l} h_i(z(t))A_ix(t)$ denotes the approximation error between the nonlinear missile–target model (3) and the fuzzified missile–target model (9).

With regard to the system (9), the following fuzzy guidance law is employed to deal with the guidance design problem:

Guidance Law Rule i :

$$\begin{aligned}&\text{If } z_1(t) \text{ is } G_{i1} \text{ and} \ldots \text{and } z_g(t) \text{ is } G_{ig}\\&\text{then } u(t) = K_i x(t)\\&\quad \text{for } i = 1, 2, \ldots, l.\end{aligned} \tag{13}$$

The overall fuzzy guidance law can be expressed as [16]

$$u(t) = \sum_{i=1}^{l} h_i(z(t)) K_i x(t) \tag{14}$$

where $h_i(z(t))$ is defined as in (10) and (11) and K_i, $i = 1, \ldots, l$ are the control parameters.

Substituting (14) into (12) yields the closed-loop system of the following form:

$$\dot{x}(t) = \sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z(t))h_j(z(t))\{(A_i + BK_j)x(t)\} + \Delta f + Dw(t). \tag{15}$$

Suppose there exists a design scalar α satisfying

$$\|\Delta f\| = \left\| F(x(t)) - \sum_{i=1}^{l} h_i(z(t))A_i x(t) \right\| \le \alpha \|x(t)\| \tag{16}$$

for all trajectories, where $\|\cdot\|$ denotes the L_2-vector norm.

Remark 2: If we assume $g = n$ and $z_1(t) = x_1(t)$, $z_2(t) = x_2(t), \ldots, z_n(t) = x_n(t)$, i.e., state variables are chosen as premise variables, then the plant rule can be represented as

$$\begin{aligned}&\text{Plant Rule } i\text{: If } x_1(t) \text{ is } G_{i1} \text{ and} \ldots \text{and } x_n(t) \text{ is } G_{in}\\&\quad\text{then } \dot{x}(t) = A_i x(t) + Bu(t) + Dw(t)\\&\quad\quad \text{for } i = 1, 2, \ldots, l.\end{aligned} \tag{17}$$

□

From (16), we get

$$\begin{aligned}\Delta f^T \Delta f &= \left\{ F(x(t)) - \sum_{i=1}^{l} h_i(z(t))A_i x(t) \right\}^T \\ &\quad \cdot \left\{ F(x(t)) - \sum_{i=1}^{l} h_i(z(t))A_i x(t) \right\} \\ &\le \alpha^2 x^T(t)x(t)\end{aligned} \tag{18}$$

i.e., the approximation error via the fuzzy interpolation model is bounded within a sector with slope $\pm\alpha$.

Stability is the most important issue in guidance system design. It is appealing for us to specify the control parameters K_i so that the stability of the closed-loop system (15) is ensured. In the following, we proceed to specify the fuzzy guidance law to stabilize the system (15) with the guarantee of H_∞ guidance performance index (6).

Let us first choose a Lyapunov function candidate as

$$V(t) = x^T(t)Px(t) \tag{19}$$

where the weighting matrix P is a positive-definite symmetric matrix, i.e., $P = P^T > 0$. The following lemma will be useful in the design procedure.

Lemma 1 [20]: For any matrices (or vectors) X and Y with appropriate dimensions, we have

$$X^T Y + Y^T X \le X^T J X + Y^T J^{-1} Y$$

where J is any positive-definite symmetric matrix. In this paper, we let J be an identity matrix. □

The time derivative of $V(t)$ is

$$\dot{V}(t) = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t). \tag{20}$$

By substituting (15) into (20), we get

$$\begin{aligned}\dot{V}(t) = \sum_{i=1}^{l}\sum_{j=1}^{l} & h_i(z(t))h_j(z(t)) \\ &\cdot \left\{ x^T(t)\left[P(A_i + BK_j) + (A_i + BK_j)^T P\right]x(t) \right.\\ &\quad \left. + \Delta f^T P x(t) + x^T(t)P\,\Delta f \right\} \\ & + w^T(t)D^T P x(t) + x^T(t)PDw(t).\end{aligned}$$

From (18) and Lemma 1, we obtain

$$\begin{aligned}\dot{V}(t) \le \sum_{i=1}^{l}\sum_{j=1}^{l} & h_i(z(t))h_j(z(t)) \\ &\cdot \left\{ x^T(t)\left[P(A_i + BK_j) + (A_i + BK_j)^T P\right]x(t) \right.\\ &\quad \left. + \Delta f^T \Delta f + x^T(t)PPx(t) \right\} \\ & + x^T(t)PDw(t) + w^T(t)D^T P x(t) \\ \le \sum_{i=1}^{l}\sum_{j=1}^{l} & h_i(z(t))h_j(z(t)) \\ &\cdot \left\{ x^T(t)\left(A_i^T P + PA_i + PBK_j \right.\right.\\ &\quad \left.\left. + K_j^T B^T P + \alpha^2 I + PP\right)x(t) \right\} \\ & + x^T(t)PDw(t) + w^T(t)D^T P x(t).\end{aligned} \tag{21}$$

Theorem 2: If the fuzzy guidance law (14) is employed in the nonlinear guidance system (3) and there exists a positive-definite matrix $P = P^T > 0$ such that the following matrix inequalities:

$$A_i^T P + PA_i + PBK_j + K_j^T B^T P + \alpha^2 I + P\left(I + \frac{1}{\rho^2}DD^T\right) \cdot P + Q' + K_j^T R K_j < 0 \tag{22}$$

are satisfied for $i, j = 1, 2, \ldots, l$ then the nonlinear closed-loop system (15) is quadratically stable in the absence of external disturbances, and the H_∞ guidance performance index (6) is guaranteed for a prescribed ρ^2 in the present of external disturbances. □

Proof: See Appendix A.

In general, it is not easy to analytically determine a common solution $P = P^T > 0$ from (22). Besides, the solution may not be unique. Fortunately, (22) can be reformulated as an

LMIP [22]. As a group of LMIs is constructed, the problem can then be solved in a computationally efficient manner using numerical techniques such as the interior point method. First, the Riccati-like inequalities (22) are transformed to the equivalent LMIs by introducing the new variables $W = P^{-1}$ and $Y_j = K_j W$

$$WA_i^T + A_i W + BY_j + Y_j^T B^T + \alpha^2 WW + \left(I + \frac{1}{\rho^2} DD^T\right) + WQ'W + Y_j^T RY_j < 0. \quad (23)$$

By the Schur complement [22], the quadratic inequalities are equivalent to the LMIs shown in (24) at the bottom of the page, for $i, j = 1, 2, \ldots, l$. Then, the H_∞ guidance problems with a prescribed disturbance attenuation level ρ is reduced to how to solve $W, Y_j, j = 1, 2 \ldots, l$ from LMIs in (24). If the LMIs in (24) have a common positive-definite solution W, then the nonlinear system described by (15) with $K_j = Y_j W^{-1}$ would be stable, and the H_∞ guidance performance index (6) is achieved.

To consider the saturation of actuators in practical applications, the constraints on control commands should also be imposed in the design of a guidance law. If $x(t)$ is restricted to stay in an invariant ellipsoid $\varepsilon_r = \{x(t) \in R^6 | x^T(t) W^{-1} x(t) \le 1$ or $\|W^{-(1/2)} x(t)\|^2 \le 1\}$ for all $t \ge 0$, then from (14), we get [22]

$$\begin{aligned}\max_{t\ge 0} \|u_k(t)\| &= \max_{t\ge 0} \left\| \left[\sum_{i=1}^{l} h_i(z(t)) K_i x(t) \right]_k \right\| \\ &\le \max_{t\ge 0} \sum_{i=1}^{l} \left[h_i(z(t)) \left\| \left[Y_i W^{-1} x(t) \right]_k \right\| \right] \\ &\le \left\{ \left[\max_i \max_{x\in\varepsilon_r} \left\| \left(Y_i W^{-1} x \right)_k \right\| \right] \sum_{i=1}^{l} h_i(z) \right\}. \end{aligned} \quad (25)$$

By (11), we obtain

$$\begin{aligned}\max_{t\ge 0} \|u_k(t)\| &\le \max_i \max_{x\in\varepsilon_r} \left\| \left(Y_i W^{-1} x \right)_k \right\| \\ &= \max_i \max_{x\in\varepsilon_r} \left\| \left(Y_i W^{-(1/2)} W^{-(1/2)} x \right)_k \right\| \\ &\le \max_i \left(Y_i W^{-1} Y_i^T \right)_k^{(1/2)} \end{aligned} \quad (26)$$

where we have used the fact $\|W^{-(1/2)} x(t)\| \le 1$.

Therefore, the constraints on control commands $\|u_k(t)\| \le \nu$ for all k are enforced at all time $t \ge 0$ if the following LMIs [22] hold for $i = 1, \ldots, l$:

$$\begin{bmatrix} 1 & x^T(0) \\ x(0) & W \end{bmatrix} \ge 0 \quad \begin{bmatrix} X & Y_i^T \\ Y_i & W \end{bmatrix} \ge 0 \quad (27)$$

and

$$X_{kk} \le \nu^2 \quad (28)$$

where X_{kk} denotes the kth diagonal element of X. This is, if the control constraints $\|u_k(t)\| \le \nu$ are enforced for the H_∞ guidance law, then parameters W, Y_j and $K_i = Y_i W^{-1}$ must be solved under the LMI constraints (24), (27), and (28), simultaneously.

In general, it is appealing to eliminate the influence of external disturbances on the guidance performance as possible, i.e., make the attenuation level ρ^2 as small as possible to achieve the H_∞ optimal guidance performance. In this situation, the H_∞ robustness optimization design for the guidance system (3) is formulated as a constrained optimization problem

$$\min_{\{Y_1, \ldots, Y_l\}} \rho^2 \quad \text{subject to } W = W^T > 0, \text{(24), (27), and (28).} \quad (29)$$

The above constrained optimization problem is called as an EVP. This EVP can also be solved very efficiently by convex optimization algorithms such as the interior algorithm [22]. Software packages, such as the LMI optimization toolbox of Matlab [19], have been developed for this purpose and can be utilized to solve the problem. After the EVP in (29) has been solved by the optimal W, Y_i, for $i = 1, 2, \ldots, l$, then the corresponding control parameters are calculated as $K_i = Y_i W^{-1}$, for $i = 1, 2, \ldots, l$. Based on the analysis above, the optimal H_∞ fuzzy guidance design can be summarized as follows.

Step 1) Select the fuzzy plant rules and membership functions via the Takagi–Sugeno fuzzy model (8) for the system (3) and find the upper bound α from (16).

Step 2) Select the weighting matrices Q and R according to the design purpose.

Step 3) Transform the Riccati-like equations (23) into the LMI (24).

Step 4) Solve the EVP (29) to get optimal W, Y_i, for $i = 1, 2, \ldots, l$ and the corresponding minimum ρ^2.

Step 5) Obtain the control parameters $K_i = Y_i W^{-1}$, for $i = 1, 2, \ldots, l$.

Step 6) Realize the fuzzy H_∞ robustness optimization guidance law (14).

Remark 3: The EVP [22] is to minimize the maximum eigenvalue of a matrix that depends affinely on a variable, subject to an LMI constraint (or determine that the constraint is infeasible), i.e.,

$$\text{minimize } \lambda \quad \text{subject to } \lambda I - A(x) > 0, \qquad B(x) > 0$$

$$\begin{bmatrix} WA_i^T + A_i W + BY_j + Y_j^T B^T + I + \frac{1}{\rho^2} DD^T & W & Y_j^T \\ W & -(\alpha^2 I + Q')^{-1} & 0 \\ Y_j & 0 & -R^{-1} \end{bmatrix} < 0 \quad (24)$$

where $A(x)$ and $B(x)$ are symmetric matrices that depend affinely on the optimization variable x. This is a convex optimization problem. □

Remark 4: The above design procedure is based on perfect state measurement. In the noisy measurement, an observer-based guidance law is introduced as follows.

Suppose only r, V_r, V_θ, V_ϕ can be measured directly but corrupted by measurement noises,
i.e.,

$$\dot{x}(t) = F(x(t)) + Bu(t) + Dw(t)$$
$$y(t) = Cx(t) + n(t)$$

where $n(t)$ denotes the external noises, for example, the tracker noises or target glint noises and so on in the navigation process, whose statistical characteristics are unknown or with uncertainty. By the same technique presented in this paper, we can obtain the overall fuzzy system, fuzzy observer, and observer-based guidance law, respectively, as follows.

Fuzzy system:

$$\dot{x}(t) = \sum_{i=1}^{l} h_i(z(t))A_i x(t) + Bu(t) + Dw(t)$$
$$y(t) = Cx(t) + n(t).$$

Fuzzy observer:

$$\dot{\hat{x}}(t) = \sum_{i=1}^{l} h_i(z(t)) \left\{ A_i\hat{x}(t) + Bu(t) + L_i(y(t) - \hat{y}(t)) \right\}.$$

Observer-based guidance law:

$$u(t) = \sum_{i=1}^{l} h_i(z(t))K_i\hat{x}(t).$$

After some manipulations, we can obtain the robust estimation gains L_i first and control gains K_i next. By this arrangement, the robust observer-based guidance law can be found. □

IV. Simulation Example

Engagement performance and robustness of the proposed fuzzy nonlinear H_∞ guidance law $(FH_\infty G.)$ and the APNG [4] against different types of targets are compared. Three maneuvering strategies of targets in 3-D [2] are employed to examine the robustness and tracking performance of the guidance laws, i.e., the external disturbances $w(t)$ in 2) is generated by the following maneuvering targets to test the robustness in this example.

1) Step target:

$$w_r = \lambda_T \vec{e}_r$$
$$w_\theta = \lambda_T \frac{-\dot{\phi}}{\sqrt{\dot{\phi}^2 + \dot{\theta}\cos^2\phi}}\,\vec{e}_\theta$$
$$w_\phi = \lambda_T \frac{\dot{\theta}\cos\phi}{\sqrt{\dot{\phi}^2 + \dot{\theta}\cos^2\phi}}\,\vec{e}_\phi.$$

2) Ramp target:

$$w_r = \lambda_T t\vec{e}_r$$
$$w_\theta = \lambda_T t \frac{-\dot{\phi}}{\sqrt{\dot{\phi}^2 + \dot{\theta}\cos^2\phi}}\,\vec{e}_\theta$$
$$w_\phi = \lambda_T t \frac{\dot{\theta}\cos\phi}{\sqrt{\dot{\phi}^2 + \dot{\theta}\cos^2\phi}}\,\vec{e}_\phi.$$

3) Sinusoidal target:

$$w_r = \lambda_T \sin(\Omega t)\vec{e}_r$$
$$w_\theta = \lambda_T \sin(\Omega t) \frac{-\dot{\phi}}{\sqrt{\dot{\phi}^2 + \dot{\theta}\cos^2\phi}}\,\vec{e}_\theta$$
$$w_\phi = \lambda_T \sin(\Omega t) \frac{\dot{\theta}\cos\phi}{\sqrt{\dot{\phi}^2 + \dot{\theta}\cos^2\phi}}\,\vec{e}_\phi$$

where λ_T is the target's navigation gain and $\Omega = 20$ (rad/s). In our simulation, we set the navigation gain as a random value within 0∼$4G$.

To demonstrate performance robustness of the proposed method, the following scenarios are considered.

Case 1: Target escapes from missile $(w_r > 0)$

$$r = 1 \text{ km} \quad \theta = \pi/3 \quad \phi = \pi/3$$
$$V_r = -500 \text{ m/s} \quad V_\theta = 200 \text{ m/s} \quad V_\phi = 300 \text{ m/s}.$$

Case 2: Target escapes from missile $(w_r > 0)$

$$r = 4 \text{ km} \quad \phi = \pi/3 \quad \phi = \pi/3$$
$$V_r = -500 \text{ m/s} \quad V_\theta = 200 \text{ m/s} \quad V_\phi = 300 \text{ m/s}.$$

Case 3: Target is toward to missile $(w_r < 0)$

$$r = 10 \text{ km} \quad \theta = \pi/3 \quad \phi = \pi/3$$
$$V_r = -1000 \text{ m/s} \quad V_\theta = 200 \text{ m/s} \quad V_\phi = 300 \text{ m/s}.$$

In all simulations, the constraints $\|u_k(t)\| \le 6G$ have been imposed on the control commands.

Based on the predescribed design procedure, we design an H_∞ guidance law via the following steps.

Step 1) Select the fuzzy plant rules and membership functions for the guidance system model. To reduce the design effort and complexity, rules of the fuzzy system are used as few as possible. After some tests by ANFIS algorithm [14], [15], 18 rules based on the premise variables $z(t) = (z_1(t), z_2(t), z_3(t)) = [r, V_\theta, V_\phi] = [x_1(t), x_5(t), x_6(t)]$ are used here to approximate the system. Membership functions for the state variables r, V_θ, and V_ϕ are shown in Fig. 2.

The ith rule is

Rule i: If r is r_i and V_θ is $V_{\theta i}$ and V_ϕ is $V_{\phi i}$;
then $\dot{x}(t) = A_i x(t) + Bu(t) + Dw(t)$, $i = 1, \ldots, 18$

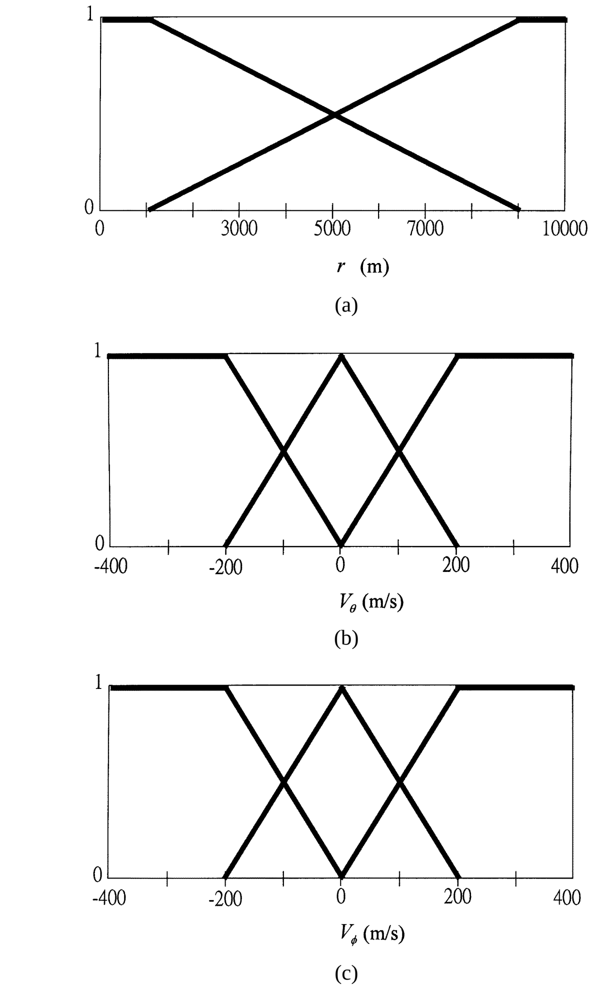


Fig. 2. Membership functions for (a) r, (b) V_θ, and (c) V_ϕ.

where operation points r, V_θ, V_ϕ are given by

$$
\begin{aligned}
r_1 &= r_2 = r_3 = r_4 = r_5 = r_6 = r_7 = r_8 = r_9 = 1000\\
r_{10} &= r_{11} = r_{12} = r_{13} = r_{14} = r_{15} = r_{16} = r_{17} = r_{18} = 9000\\
V_{\theta 1} &= V_{\theta 2} = V_{\theta 3} = V_{\theta 10} = V_{\theta 11} = V_{\theta 12} = -200\\
V_{\theta 4} &= V_{\theta 5} = V_{\theta 6} = V_{\theta 13} = V_{\theta 14} = V_{\theta 15} = 0\\
V_{\theta 7} &= V_{\theta 8} = V_{\theta 9} = V_{\theta 16} = V_{\theta 17} = V_{\theta 18} = 200\\
V_{\phi 1} &= V_{\phi 4} = V_{\phi 7} = V_{\phi 10} = V_{\phi 13} = V_{\phi 16} = -200\\
V_{\phi 2} &= V_{\phi 5} = V_{\phi 8} = V_{\phi 11} = V_{\phi 14} = V_{\phi 17} = 0\\
V_{\phi 3} &= V_{\phi 6} = V_{\phi 9} = V_{\phi 12} = V_{\phi 15} = V_{\phi 18} = 200.
\end{aligned}
$$

Remark 5: Operating conditions were chosen according to the portioned flight envelope. The available flight envelop of the guided missile was characterized according to the system specification (such as speed, max acceleration capability, and achievable altitude, etc.) determined during the concept exploration phase based on the missile aerodynamic configuration, propulsion, weight, and structure, etc. The available flight envelope was then uniformly partitioned into several flight conditions. In the current study, the available flight envelope was chosen as r for relative distance, V_θ and V_ϕ for tangential velocities in yaw and pitch axes, respectively. They are shown by membership functions in Fig. 2. Over the flight envelope, the flight conditions were uniformly partitioned into 18 parts as those in the fuzzy rules in Step 1. □

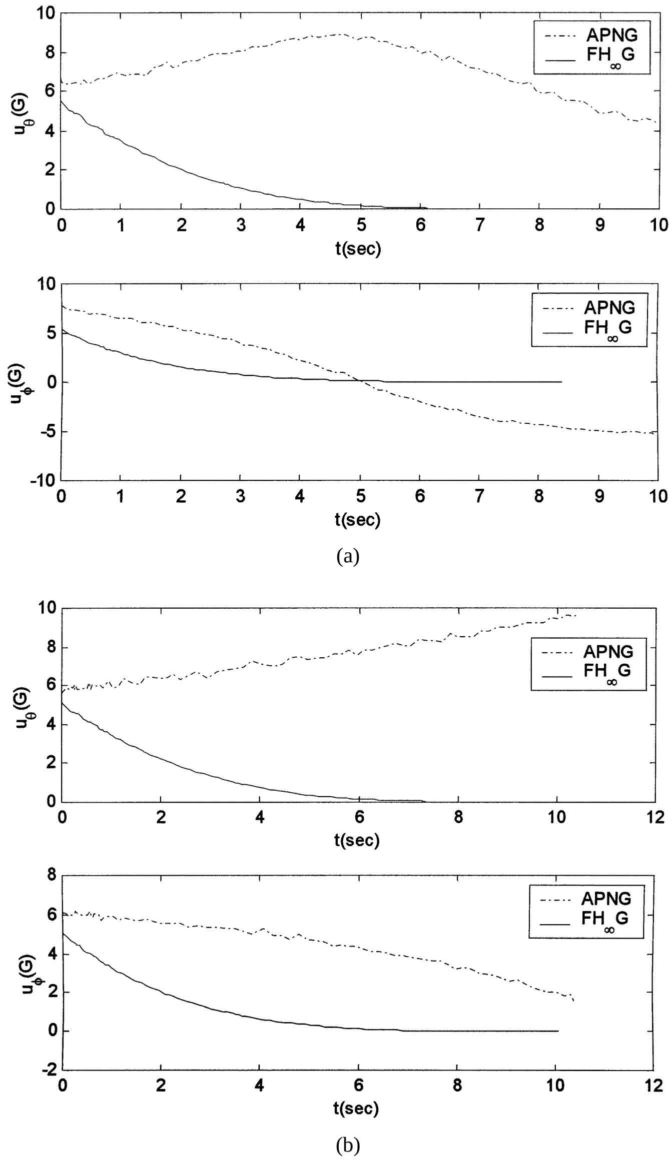


Fig. 3. Control commands for $FH_\infty G$ and APNG versus sinusoidal target with different initial conditions. (a) Case 2. (b) Case 3.

Step 2) Select the weighting matrices Q and R

$$
Q = \begin{bmatrix} 0.005 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix} \quad R = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}.
$$

Step 3) Solve the EVP in (29) to get $W, Y_1, \ldots, Y_{18}$.

Step 4) Obtain the fuzzy control parameters $K_j = Y_j W^{-1}$, $j = 1, \ldots, 18$ and the corresponding minimum $\rho^2 = 0.7225$.

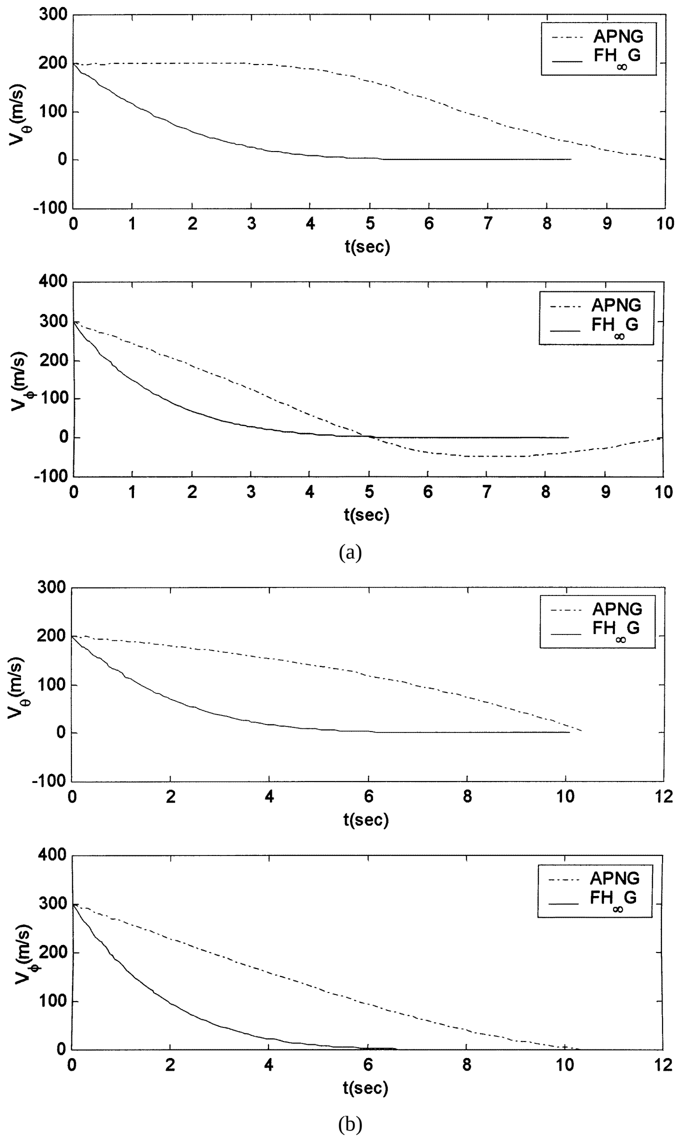


Fig. 4. Tangential relative velocities of $FH_\infty G$and APNG versus sinusoidal target with different initial conditions. (a) Case 2. (b) Case 3.

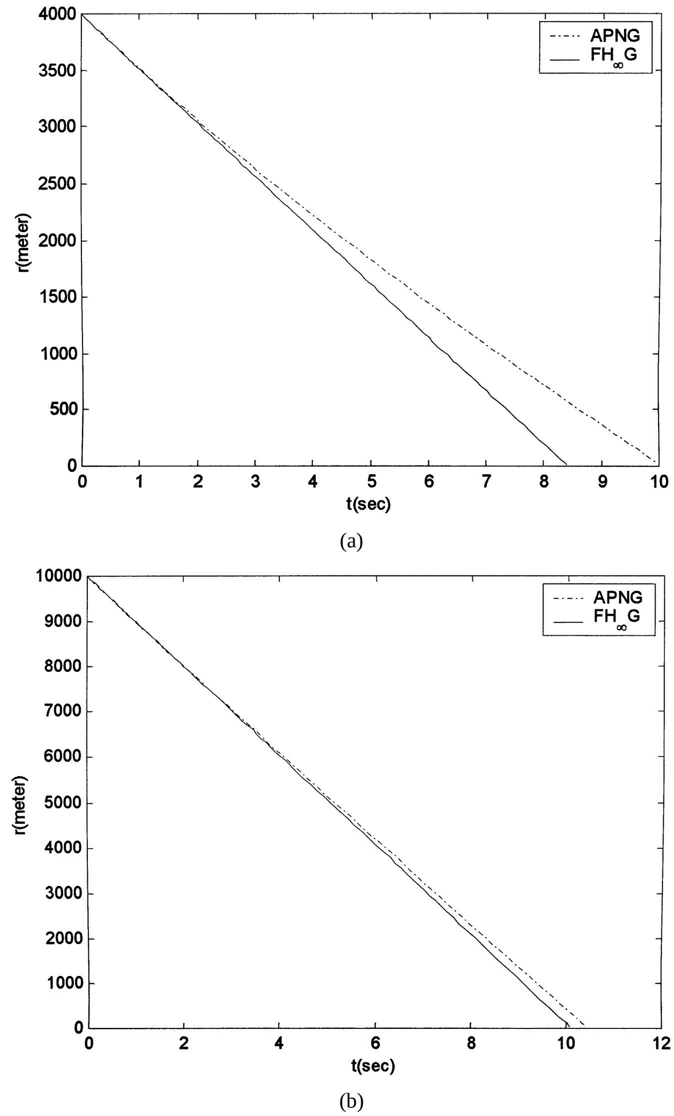


Fig. 5. Trajectories of relative distances between missile and target for $FH_\infty G$ and APNG versus sinusoidal target with different initial conditions. (a) Case 2. (b) Case 3.

Step 5) Realize the fuzzy H_∞ robustness optimization guidance law

$$u(t) = \sum_{i=1}^{18} h_i(r, V_\theta, V_\phi)(K_i x(t)).$$

Step 6) For brevity, A_i and K_j, for $i = 1, \ldots, 18$ are not presented here and will be available from the authors. □

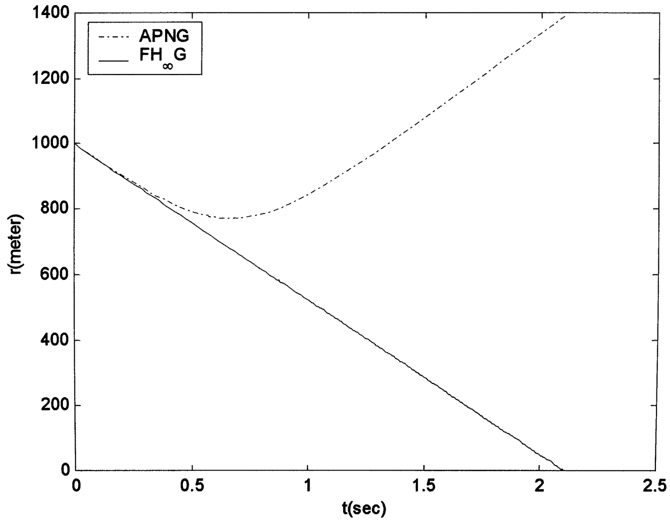
Some discussion follows.

A. Comparisons of Control Efforts

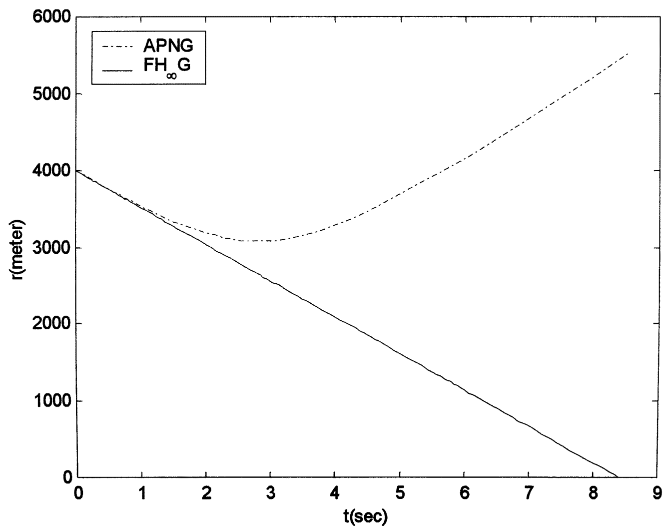
Comparisons between $FH_\infty G$ and APNG versus the sinusoidal target are discussed. Control commands for both guidance laws are shown, respectively, in Fig. 3(a) and (b). Fig. 3(a) illustrates the simulation result with the initial condition of case 2, in which the target escapes from the missile. Fig. 3(b) illustrates the simulation result with the initial condition of Case 3, in which the target is toward to the missile. For both cases we see that the guidance commands in the APNG design are all larger than that of $FH_\infty G$. Therefore, concerning about energy consumption, $FH_\infty G$ yields better results. This is owing to the fact that the factor of control energy consumption has been included in the designed performance index; on the other hand, larger acceleration commands issued from the APNG lead to a higher control energy consumption.

B. Comparisons of Tracking Errors

Our design objective is to develop an effective guidance law to keep the pitch LOS angular rate, yaw LOS angular rate, and relative distance as small as possible under uncertain target accelerations. From Fig. 4(a) and (b), it is obvious that the angular decaying rates V_θ and V_ϕ in the pitch and yaw axes of the proposed design method all converge to zero rapidly than the conventional one. This finding reveals that the proposed method possesses excellent target tracking ability, and it is possible to get smaller miss distances than that of the APNG.



(a)



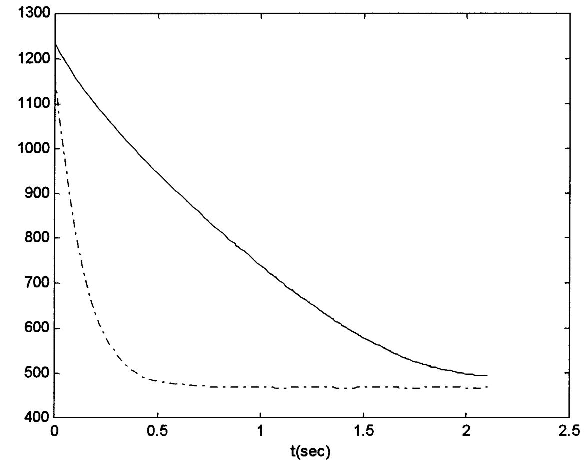
(b)

Fig. 6. Trajectories of relative distances between missile and target for $FH_\infty G$ and APNG versus step target (Case 1) and ramp target (Case 2) with different initial conditions. (a) Case 1. (b) Case 2.

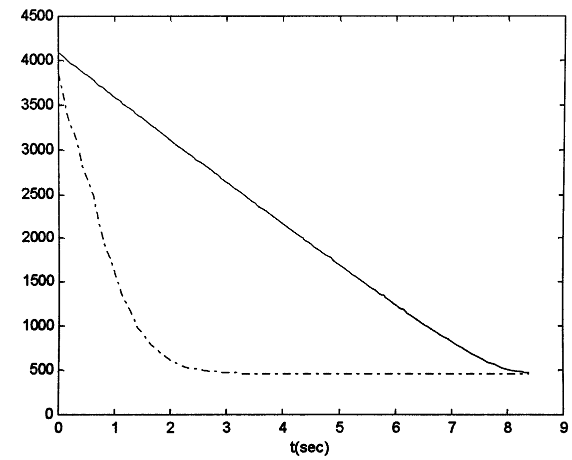
Although Fig. 4 shows that the APNG can make the pitch and yaw LOS angular rates converge to zero finally, however, the guidance law generates large control commands (see Fig. 3). Fig. 5 illustrates, respectively, convergence of the relative distance for the initial conditions in cases 2 and 3.

C. Robustness

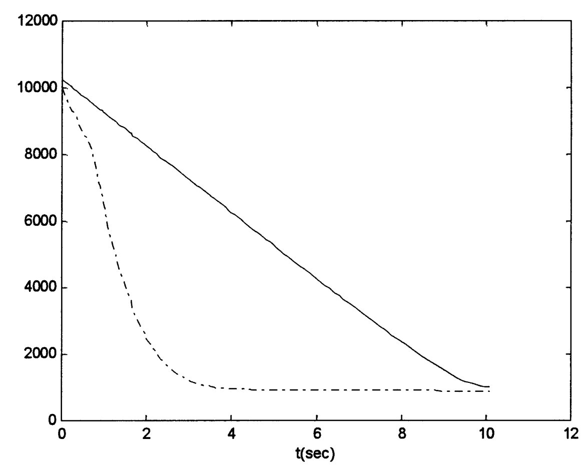
Robustness of the guidance design is examined by three types of target acceleration commands. According to the definition of performance robustness index, a robust guidance law should keep the engagement performance with less sensitivity to the external disturbances, i.e., the target acceleration commands. It has been shown in [4] that APNG can successfully engage targets with sinusoidally evasive acceleration when the navigation gain N lies in the range of 3.3 $\sim$ 3.8. However, the successful engagement is based on the assumption that information about the target acceleration profiles is precisely known. Simulation results in Fig. 6 have indicated it is hard for the APNG to track step and ramp targets, but the proposed robust guidance law still can accomplish the missions. Hence, the proposed guidance law is more robust to uncertain target accelerations than the conventional one. It is easy to find the upper bound $\alpha = 1.01$ for fuzzy approximation errors by computer simulations (ANFIS approach method), and the relative simulation results for different initial conditions are shown in Fig. 7.



(a)



(b)



(c)

Fig. 7. The plots of $\|\Delta f\|$ (dash line) $\leq \alpha\|x(t)\|$ (solid line) for the missile tracking the target with different initial conditions. (a) Case 1. (b) Case 2. (c) Case 3.

V. Conclusion

A nonlinear fuzzy H_∞ guidance law with control constraints against maneuvering targets without solving the complicated HJPDI as that of the conventional H_∞ control design is successfully developed in this paper. The problem of nonlinear fuzzy H_∞ guidance law design with saturation of actuators is first transformed to an EVP so that it could be efficiently solved with available computer software. The proposed guidance law possesses higher maneuverability and results in smaller LOS angular rates than the traditional APNG. It also consumes less control energy and offers better performance against uncertain target accelerations. Simulation results show that the proposed guidance law offers the potential to be applied in the high-performance missile system designs.

Appendix

A. Proof of Theorem 2

Proof: From (21), we get

$$\begin{aligned}\dot{V}(t) \le& \sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z(t))h_j(z(t))\left\{x^T(t)(A_i^TP + PA_i\right.\\ &\left.+ PBK_j + K_j^TB^TP + \alpha^2 I + PP)x(t)\right\}\\ &+ \Big[x^T(t)PDw(t) + w^T(t)D^TPx(t)\\ &- \rho^2 w^T(t)w(t) - \frac{1}{\rho^2}x^T(t)PDD^TPx(t)\Big]\\ &+ \frac{1}{\rho^2}x^T(t)PDD^TPx(t) + \rho^2 w^T(t)w(t)\\ =& \sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z(t))h_j(z(t))\left\{x^T(t)(A_i^TP + PA_i\right.\\ &\left.+ PBK_j + K_j^TB^TP + \alpha^2 I + PP)x(t)\right\}\\ &- \left(\frac{1}{\rho}D^TPx(t) - \rho w(t)\right)^T\\ &\cdot\left(\frac{1}{\rho}D^TPx(t) - \rho w(t)\right)\\ &+ \frac{1}{\rho^2}x^T(t)PDD^TPx(t) + \rho^2 w^T(t)w(t)\\ \le& \sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z(t))h_j(z(t))\left\{x^T(t)(A_i^TP + PA_i\right.\\ &\left.+ PBK_j + K_j^TB^TP + \alpha^2 I + PP)x(t)\right\}\\ &+ \frac{1}{\rho^2}x^T(t)PDD^TPx(t) + \rho^2 w^T(t)w(t). \quad (30)\end{aligned}$$

From (22), we further have

$$\begin{aligned}\dot{V}(t) \le& -x^T(t)Q'x(t) - \sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z(t))h_j(z(t))x^T(t)\\ &\cdot\left\{K_j^TRK_j\right\}x(t) + \rho^2 w^T(t)w(t)\\ \le& -x^T(t)Q'x(t) - \sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z(t))h_j(z(t))x^T(t)\\ &\cdot\left\{K_i^TRK_j\right\}x(t) + \rho^2 w^T(t)w(t). \quad (31)\end{aligned}$$

From the properties of $h_i(z(t))$ in (10) and (11), (31) implies

$$\dot{V}(t) \le -x^T(t)Q'x(t) - u^T(t)Ru(t) + \rho^2 w^T(t)w(t). \quad (32)$$

Assuming $w(t) = 0$, we get

$$\dot{V}(t) \le -x^T(t)Q'x(t) - u^T(t)Ru(t) < 0.$$

This demonstrates that the closed-loop system is quadratically stable while there is in the absence of $w(t)$.

Integrating (32) from $t = 0$ to $t = t_f$ yields

$$\begin{aligned}V(t_f) - V(0) \le& -\int_0^{t_f}\left[x^T(t)Q'x(t) + u^T(t)Ru(t)\right]dt\\ &+\rho^2\int_0^{t_f} w^T(t)w(t)\,dt. \quad (33)\end{aligned}$$

That is

$$\begin{aligned}\int_0^{t_f}\left[x^T(t)Q'x(t) + u^T(t)Ru(t)\right]dt \le& x^T(0)Px(0)\\ &-x^T(t_f)Px(t_f) + \rho^2\int_0^{t_f} w^T(t)w(t)\,dt. \quad (34)\end{aligned}$$

This demonstrates that the H_∞ guidance performance is achieved with a prescribed ρ^2. □

References

[1] A. S. Locke, *Guidance*. Princeton, NJ: Van Nostrand, 1955.

[2] C. F. Lin, *Modern Navigation, Guidance, and Control Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1991, pp. 252–413.

[3] I. Ha and S. Chong, "Design of a CLOS guidance law via feedback linearization," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 28, pp. 51–63, Jan. 1992.

[4] P. Zarchan, *Tactical and Strategic Missile Guidance*. Washington, DC: AIAA, 1990, pp. 87–110.

[5] A. E. Bryson, Jr. and Y. C. Ho, *Applied Optimal Control*. Waltham, MA: Blaisdell, 1969, pp. 148–176.

[6] J. E. Potter, "A guidance-navigation separation theorem," AIAA, Washington, DC, AIAA Paper 64-653, 1964.

[7] J. L. Speyer, W. M. Greenwell, and D. G. Hull, "Adaptive noise estimation and guidance for homing missile," *J. Guidance Contr. Dyn. Conf.*, pp. 48–56, 1982.

[8] F. W. Nesline and P. Zarchan, "A new look at classical vs. modern homing missile guidance," *J. Guidance Contr. Dyn.*, vol. 4, no. 1, pp. 78–85, 1981.

[9] S. R. Vadali and H. S. Oh, "Space station attitude control and momentum management: A nonlinear look," *J. Guidance Contr. Dyn.*, vol. 15, no. 3, pp. 577–586, 1992.

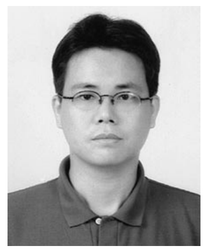
[10] G. Ferreres and M. M'Saad, "Parametric robustness evaluation of an H_∞ missile autopilot," *J. Guidance Contr. Dyn.*, vol. 19, no. 3, pp. 621–627, 1996.

[11] W. Kang, "Nonlinear H_∞ control and its applications to rigid spacecraft," *IEEE Trans. Automat. Contr.*, vol. 40, pp. 1281–1285, July 1995.
[12] M. Dalsmo and O. Egeland, "State space feedback H_∞ control of a rigid spacecraft," in *Proc. 34th Conf. Decision Contr.*, New Orleans, LA, 1995.
[13] J. B. Burl and D. L. Krueger, "Parametric uncertainty reduction in robust missile autopilot design," *J. Guidance Contr. Dyn.*, vol. 19, no. 3, pp. 733–736, 1996.
[14] J.-S. R. Jang, "ANFIS: Adaptive-network-based fuzzy inference systems," *IEEE Trans. Syst. Man, Cybern.*, vol. 23, pp. 665–685, May 1993.
[15] J.-S. R. Jang, C. T. Sun, and E. Mizutani, *Neuro-Fuzzy and Soft Computing*. Upper Saddle River, NJ: Prentice-Hall, 1997.
[16] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-15, pp. 116–132, Jan. 1985.
[17] G. C. Hwang and S. C. Lin, "A stability approach to fuzzy control design for nonlinear systems," *Fuzzy Sets Syst.*, vol. 48, no. 3, pp. 279–287, 1992.
[18] H. O. Wang, K. Tanaka, and M. F. Griffin, "An approach to fuzzy control of nonlinear systems: Stability and design issues," *Trans. Fuzzy Syst.*, vol. 4, pp. 14–23, Feb. 1996.
[19] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, *LMI Control Toolbox*. Natick, MA: The Math Works, 1995, ch. 8.
[20] K. Zhou and P. P. Khargonekar, "Robust stabilization of linear systems with norm bounded time varying uncertainty," *Syst. Contr. Lett.*, vol. 10, no. 1, pp. 17–20, 1988.
[21] H. Ying, "Analytical analysis and feedback linearization tracking control of general Takagi–Sugeno fuzzy dynamic systems," *IEEE Trans. Syst., Man, Cybern. C*, vol. 29, pp. 290–298, Jan. 1999.
[22] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia, PA: SIAM, 1994, ch. 2.
[23] A. Stoorvogel, *The H_∞ Control Problem: A State Space Approach*. Englewood Cliffs, NJ: Prentice-Hall, 1992.
[24] B. S. Chen, C. S. Tseng, and H. J. Uang, "Robustness design of nonlinear dynamic systems via fuzzy linear control," *IEEE Trans. Fuzzy Syst.*, vol. 7, pp. 571–585, Aug. 1999.
[25] M. Guelman, "Proportional navigation with maneuvering target," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-8, pp. 364–371, 1972.
[26] P. J. Yuan and J. S. Chern, "Ideal proportional navigation," *J. Guidance Contr., Dyn.*, vol. 15, no. 5, pp. 1161–1165, 1992.
[27] D. G. Hull, J. L. Speyer, and C. Y. Teng, "Maximum-information guidance for homing missiles," *J. Guidance Contr., Dyn.*, vol. 8, no. 4, pp. 494–497, 1985.
[28] U. S. Shukla and P. R. Mahapatra, "Generalized linear solution of proportional navigation," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 24, pp. 231–238, 1988.
[29] F. P. Adler, "Missile guidance by three-dimensional proportional navigation," *J. Appl. Phys.*, vol. 27, no. 3, pp. 500–507, 1956.
[30] A. J. van der Schaft, "L_2-gain analysis of nonlinear systems and nonlinear state feedback H_∞ control," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 770–784, June 1992.

Bor-Sen Chen (M'82–F'01) received the B.S. degree from Tatung Institute of Technology, Tatung, R.O.C., in 1970, the M.S. degree from National Central University, Taiwan, R.O.C., in 1973, and the Ph.D. degree from the University of Southern California, Los Angeles, in 1982.

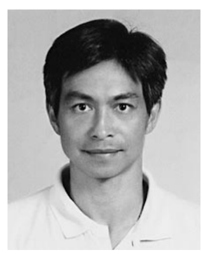
From 1973 to 1987, he was a Lecturer, Associate Professor, and Professor at Tatung Institute of Technology. He is now a Professor at National Tsing-Hua University, Hsinchu, Taiwan, R.O.C.. His current research interests include control and signal processing.

Dr. Chen has received the Distinguished Research Award from National Science Council of Taiwan four times. He is a Research Fellow of the National Science Council and the Chair of the Outstanding Scholarship Foundation. He is now an Associate Editor of the IEEE TRANSACTIONS ON FUZZY SYSTEMS and Editor of the *Asian Journal of Control*.



Yung-Yue Chen was born in Pingtong, Taiwan, R.O.C. He received the B.S. and M.S. degrees from Feng Chia University, Taiwan, R.O.C., in 1996 and 1998, respectively, and is pursuing the Ph.D. degree in electrical engineering at National Tsing-Hua University, Hsinchu, Taiwan, R.O.C.

His research interests include guidance, intelligent control, and robust control.



Chun-Liang Lin was born in Tainan, Taiwan, R.O.C., in 1958. He received the Ph.D. degree in aeronautical and astronautical engineering from the National Cheng Kung University, Tainan, Taiwan, R.O.C., in 1991.

He is currently a Professor in the Department of Automatic Control Engineering and a Section Chief in the Office of Research and Development, Feng Chia University, Taichung, Taiwan, R.O.C. His research interests include guidance and control, vibration control, intelligent control, and robust control.