242

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NONLINEAR GROWTH OF THE M = 1 TEARING MODE

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ABSTRACT

Numerical results are presented for the nonlinear evolution of the tearing mode with poloidal mode number one. Nonlinearly, the mode continues to grow exponentially at approximately the linear growth rate until it flattens the toroidal current inside the singular surface and increases the safety factor to unity at the plasma center. The hypothesis that the mode causes the internal disruption in tokamaks is supported by the fact that the time scale for the process agrees with experiment.

When the safety factor q is less than one at the plasma center, the fluctuations in the x-ray intensity from the ST tokamak shows a characteristic sawtooth behavior corresponding to internal (or minor) disruptions. The characteristics of the fluctuations as reported in reference 1 are:

a) At the plasma center the rise time of the sawtooth is approximately 1.5 msec., while the decay or disruption time is only about 0.03 msec.

b) The disruption is preceded by $m = 1$ oscillations with a growth time of 0.3 msec.

c) Outside the $q = 1$ singular surface, which is located at a radius of 2 centimeters, the sawtooth is reversed, i.e., the x-ray intensity rises abruptly and decays slowly.

Employing the parameters specified in reference 2 , one finds that the time for the current to increase by 10% at the origin due to the overall shrinkage of the current channel is of the same order as the rise time of the sawtooth. Presently, the shrinkage of the current channel is attributed to the cooling of the plasma exterior by impurity radiation.[3]

It has been suggested by von Goeler[1] and Kadomtsev[4] that the disruptive phase of the sawtooth is due to an $m = 1$ instability. Kadomtsev has argued heuristically that as the internal kink mode grows, resistivity allows the plasma to evolve from a state where the helical flux contours are circular to a lower energy state where the contours are again circular, thereby flattening the current density and increasing the safety factor q to unity at the origin. There are two major questions associated with this argument:

a) Is the lower energy state really accessible?

b) Is the nonlinear growth of the mode fast enough to explain the experimental data?

The numerical results presented here show that, indeed, a lower energy state is accessible and the instability does grow sufficiently rapidly.

In view of the results for the nonlinear growth of the tearing instability for poloidal mode numer $m = 2$, it is surprising that the answer to question b) is yes. Specifically, it has been shown both analytically[2] and numerically[5] that the nonlinear growth time of the $m = 2$ mode is only a few hundred times smaller than the skin time. In fact, for a reasonable model of the resistivity profile, $m = 2$ saturated states have been found corresponding to an island width of about 10% of the minor radius.[5] It is known, however, that the linear growth rate of the $m = 1$ mode is about ten times larger than the $m = 2$ mode.[6-8] The reason the $m = 1$ grows faster than the $m = 2$ is that for the case of perfect conductivity the $m = 1$ is marginally stable while the $m = 2$ is stable.[9-10] Furthermore, the growth rate for $m = 2$ decreases sharply when the width of the magnetic island produced by the mode exceeds the tearing layer width (defined in reference 6), the island width increasing only linearly rather than exponentially in time. By contrast, it is the principal result of this paper that the $m = 1$ island growth is very rapid even in the highly nonlinear phase. In fact, it continues to grow exponentially at more or less the linear growth rate until the current is completely flattened within the $q = 1$ singular surface.

In order to study the nonlinear resistive MHD behavior of toroidal systems, it is desirable to use a simplified system of equations. Employing cylindrical geometry, we assume that a perturbation has a given helical symmetry in that it depends only on r, the radial coordinate, and $\tau \equiv m\theta + kz$, where θ is the polodial coordinate, z is the longitudinal (toroidal) coordinate, and m and k are the poloidal and toroidal mode numbers, respectively. We adopt the standard tokamak ordering in which the ratio of the poloidal and toroidal magnetic fields is much less than one, while the safety factor is of order one, implying that $kr \ll 1$. Then, introducing the helical flux function ψ and the velocity stream function A and retaining only lowest order terms in the expansion parameter kr, we obtain[11]

$$\frac{d\psi}{dt} = -\eta J_z = \eta\left(\nabla^2\psi + \frac{2kB_z}{m}\right) \tag{1}$$

and

$$\rho \frac{d}{dt} \nabla^2 A = -\hat{z} \cdot (\underline{\nabla}\psi \times \underline{\nabla}\nabla^2 \psi) \ , \tag{2}$$

where d/dt denotes the convective time derivative and the spatial derivatives are perpendicular to z . Here, η is the resistivity, J_z is the toroidal current, B_z is the toroidal magnetic field, ρ is the mass density, and $\hat{z}$ is the unit vector in the toroidal direction. The radial and poloidal components of the magnetic field are give by $B_r = (1/r)\partial\psi/\partial\theta$ and $B_\theta = -\partial\psi/\partial r - krB_z/m$, respectively, and the fluid velocity $\underline{V}$ is given by $\underline{V} = \nabla A \times \hat{z}$. We assume that the cylindrical container is completely filled with plasma so that an equation for ψ in the vacuum is unnecessary. For the case where $\eta = 0$, these equations (together with the equation for ψ in the vacuum) were used in reference 11 to study the nonlinear development of surface kink modes.

For numerical integration of the case $\eta \neq 0$, an alternating direction implicit scheme is used to advance equation 1 while equation 2 is advanced explicitly. The resulting upper bound on the time step is large enough to allow the analysis of tearing modes.

The linear version of equations 1 - 2 can be solved <u>analytically</u> for poloidal mode number equal to one.[8] The linear growth rate γ is given by

$$\gamma = (r_w q' S)^{2/3} \ \tau_R^{-1} = (r_w q')^{2/3} \ S^{-1/3} \ \tau_{HP}^{-1} \ . \tag{3}$$

Here, $q \equiv rB_z/(RB_\theta)$ is the safety factor, $q' = dq/dr|_{r_s}$, r_w is the radius of the cylinder, r_s is the radius of the $q=1$ singular surface, $S \equiv \tau_R/\tau_{HP}$, $\tau_R = r_w^2/\bar{\eta}$, $\bar{\eta}$ is the characteristic magnitude of the resistivity, and $\tau_{HP} = m\rho^{1/2}/(kB_z)$. The time τ_R is the "skin" time, i.e., the characteristic time for the current channel to decay; τ_{HP} is the "poloidal" MHD time. Typically, the quantity S is of the order of 10^6, so that $\gamma \sim 10^4 \tau_R^{-1}$, which is about ten times larger than the growth rate for $m > 1$ modes.[9] The tearing layer width ε is equal to $r_w(r_w q' S)^{-1/3}$ and, typically, is about 1 millimeter.

For numerical results presented in this letter, the peaked model[9] for the unperturbed toroidal current density is employed:

$$J_{zo}(r) = \frac{J_{zo}(0)}{\left[1+(r/r_o)^2\right]^2} , \tag{4}$$

where $J_{zo}(0)$ is the magnitude of the current density at the origin. The current channel width r_o is taken to be $0.6r_w$; $J_{zo}(0)$ is adjusted so that q at the origin is 0.9, q at the wall is 3.4, and the radius of the singular surface is $0.2r_w$.

The resistivity η is modeled by assuming that it is independent of time and given by

$$\eta(r) = \eta(r_s) \frac{J_{zo}(r_s)}{J_{zo}(r)} . \tag{5}$$

The reason this model is employed is that for such a resistivity profile, the current remains constant if there is no tearing mode activity.

For the results presented in reference 1, the particle density is $6 \times 10^{13} cm^{-3}$, the electron temperature is 700eV, the poloidal β is 0.8, the wall radius is 13 cm, and the effective ion charge is 5. The corresponding S value is 6×10^5. The code, however, was run for $S = 5 \times 10^4$ in order to avoid numerical problems associated with a small tearing layer width. Nevertheless, since the tearing mode growth rate is proportional to $S^{-1/3}$, the disparity between the two S values should cause no qualitative difference in the results.

For the parameters specified in the preceding paragraph, the logarithm (base ten) of the fluid kinetic energy is plotted as a function of t/τ_R in Figure 1. Initially, the system is given an $m = 1$ perturbation such that the maximum width of the magnetic island is $4 \times 10^{-2} r_W$. The linear growth rate calculated from the initial slope of the curve in Figure 1 agrees with the value obtained from equation 3 to within 16%; the difference in the two rates is probably due to the fact that equation 3 was derived assuming constant resistivity.

In summary, the numerical results show that nonlinearly the kinetic energy grows exponentially at approximately the linear growth rate until a maximum of 1 eV per particle is reached. (Kadomtsev's heuristic theory predicts that about 3 eV per particle should be released by the magnetic field.) By the time the kinetic energy is maximum, the toroidal current has flattened inside the singular surface and the safety factor at the plasma center has increased to approximately unity. The magnetic island structure produced by the instability is very complicated and fills the interior of the plasma inside the singular surface. However, the helical field is very small so that practically speaking the structure is not appreciably different from a uniform $q = 1$ situation. The time for this process to occur is of the same order as the time for the internal disruption. After reaching a maximum, the kinetic energy decays by a factor of 5 at an average rate of about 1/16 of the linear growth rate.

In Figure 2, the contours of constant helical flux are plotted in the poloidal plane at the times indicated by the circles in Figure 1. The radius of the outermost circle is $0.4r_W$. At $t = 1.92 \times 10^{-3} \tau_R$, the contours show that the instability is essentially the linear $m = 1$ disturbance. However, when the kinetic energy reaches a maximum near $t = 3.8 \times 10^{-3} \tau_R$, the contour structure is complicated and fills the region inside the singular surface. As the kinetic energy decreases, the contours evolve slowly but remain complex. A strict interpretation of the results leads to the conclusion that the plasma does not evolve to a state where the flux contours are circular, as suggested by Kadomtsev.[2] The flux function, however, is fairly uniform inside the singular surface at $t = 1.36 \times 10^{-2} \tau_R^{-1}$ so that any distinction between circular contours and these contours is not important except perhaps for the analysis of the heat transport parallel to the magnetic field.

Figure 3 shows the fluid flow patterns corresponding to the flux contours in Figure 2. The shape of the flow patterns remains essentially the same as the instability evolves except that relative to the velocity in the singular layer, the velocity at the plasma center increases.

In Figure 4, the current density is plotted as a function of r along the line going through the x-point and the center of the magnetic island. The times are the same as those in Figures 2 and 3. At $t = 1.92\times10^{-3}\ \tau_R$, the current begins to flatten in the center of the island while a skin current develops at the x-point. When the kinetic energy reaches the maximum, the current is flat through most of the plasma interior and the skin current is quite large. Then, as the kinetic energy decreases, the skin current disappears and the current through the plasma interior remains fairly flat. The magnitude of the current at the plasma center corresponds to a q of approximately one.

Presumably, the flattening in the current corresponds to convection leading to a decrease in temperature inside and an increase outside the singular surface. (Notice that the total current in Figure 4 does not remain constant because in the code the electric field at the wall, rather than the total current, is held fixed.) The time required for the flattening to occur is essentially the time for the kinetic energy to reach the maximum value, i.e., the time for the magnetic island width to increase to approximately $2\,r_s$. Consequently, for an initial island width of about 1 millimeter and $S = 6\times10^5$, the time for the flattening to occur in ST is $7\gamma^{-1} \approx 0.04$ msec, in agreement with the experimental disruption time.

It should be emphasized that the preceding result can be affected by finite gyro radius and toroidal corrections. One should also take into account the modifications in Ohm's law due to the fact that the ratio of the mode growth rate to the electron collision frequency is not small.

Furthermore, in order to obtain a more detailed explanation of the internal disruption, transport processes must be included in the analysis. Specifically, an explanation of the slowly growing $m = 1$ oscillations preceding the disruption must include the self-consistent time evolution of the temperature and, thus, the resistivity. In addition, in order to produce a series of sawtooth oscillations, the long time scale effect of the transport on the evolution of the current density must be incorporated into the code.

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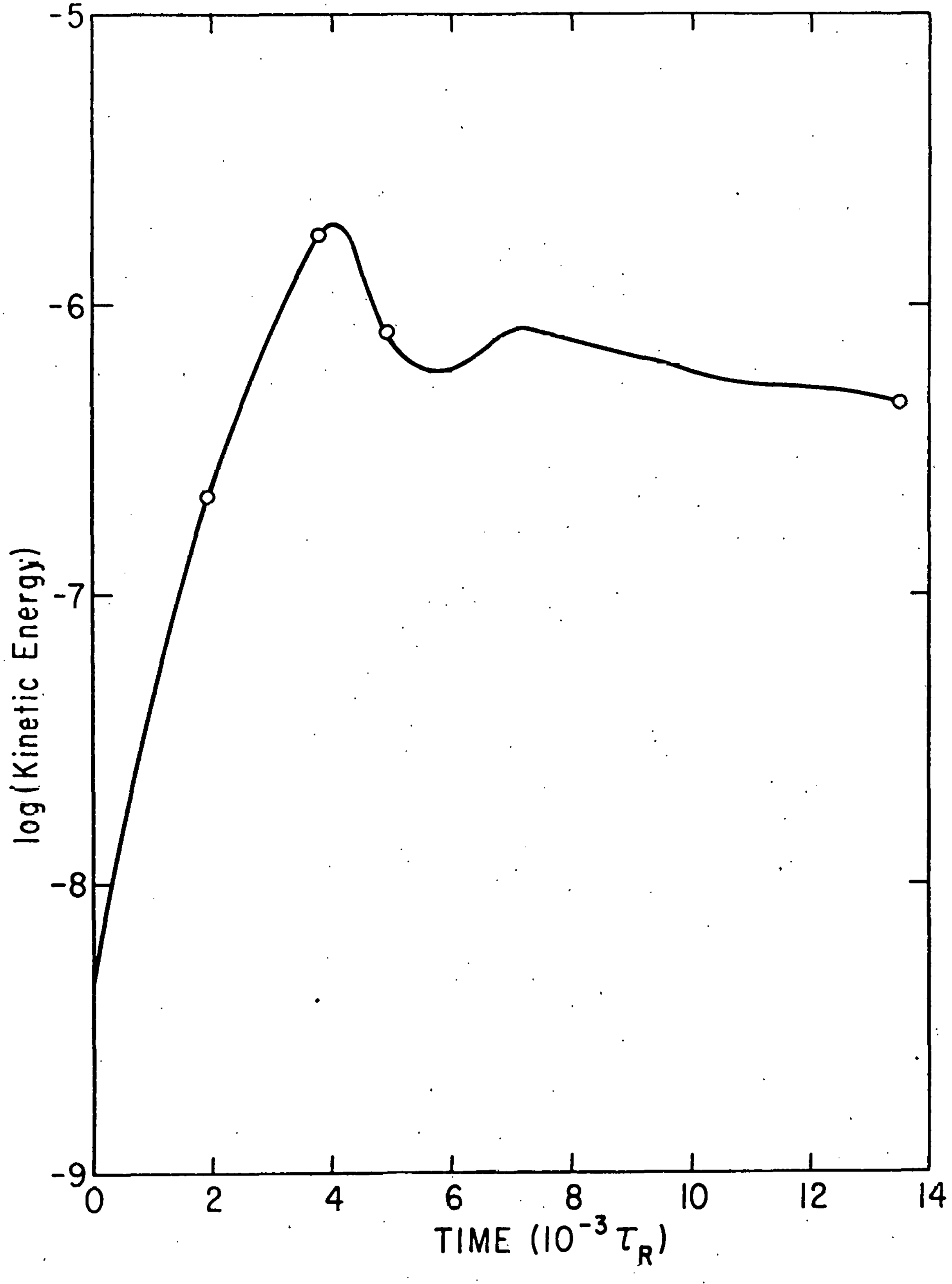

FIGURE 1 762033

Logarithm (base 10) of the fluid kinetic energy as a function of time.

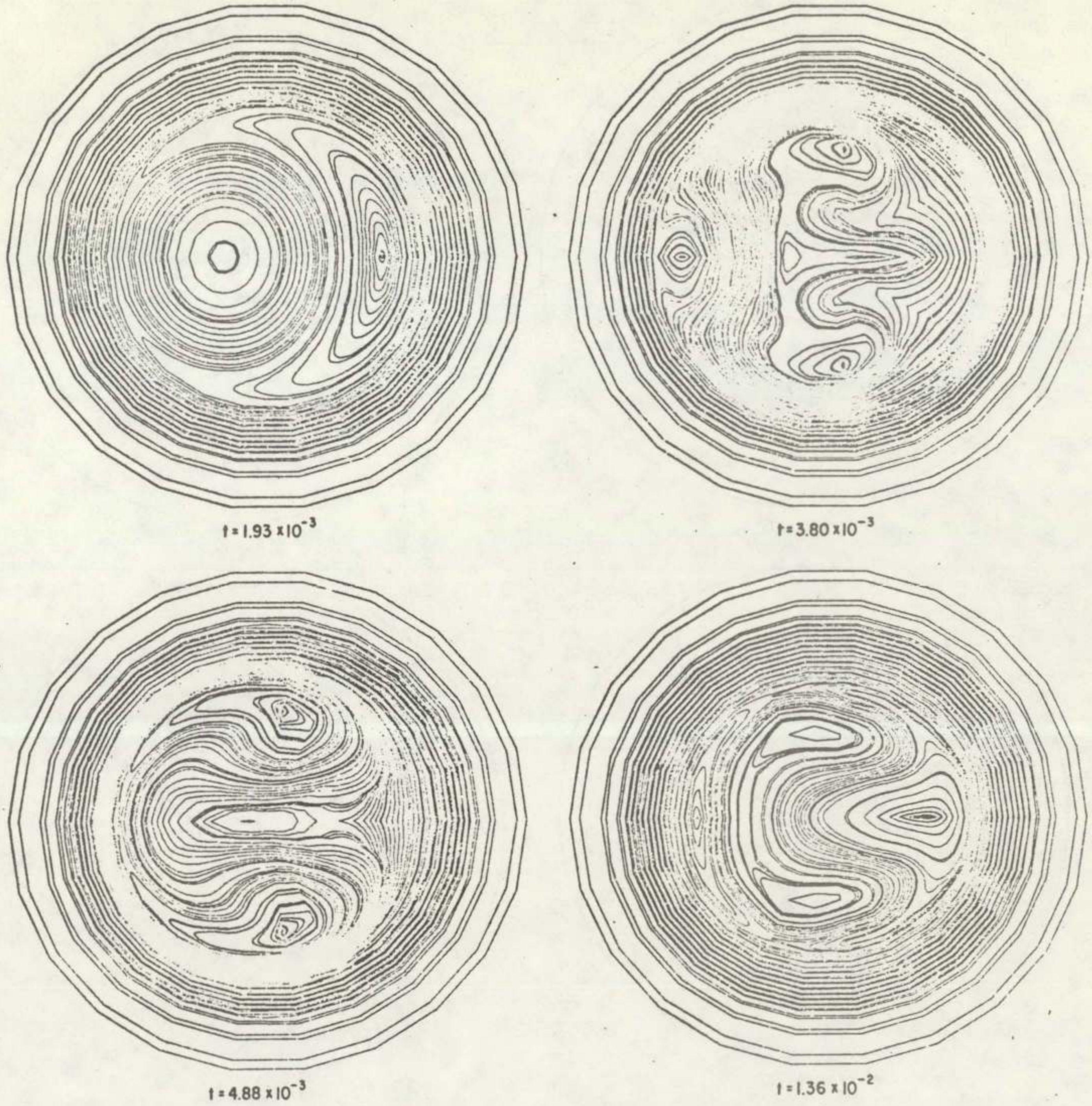

FIGURE 2 762030

Helical flux contours in the poloidal plane at selected times.

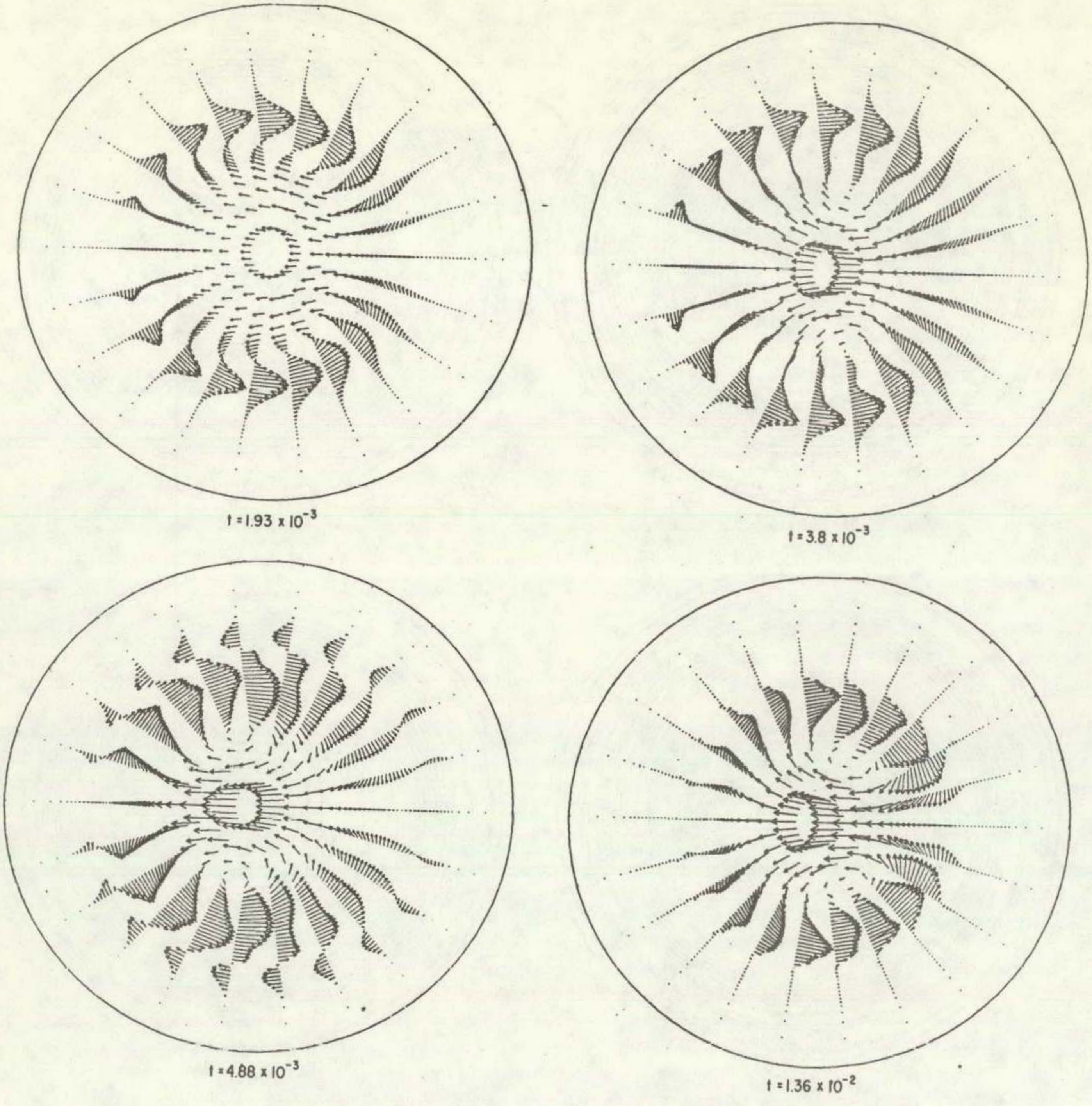

FIGURE 3 762035

Fluid flow patterns in the poloidal plane at selected times.

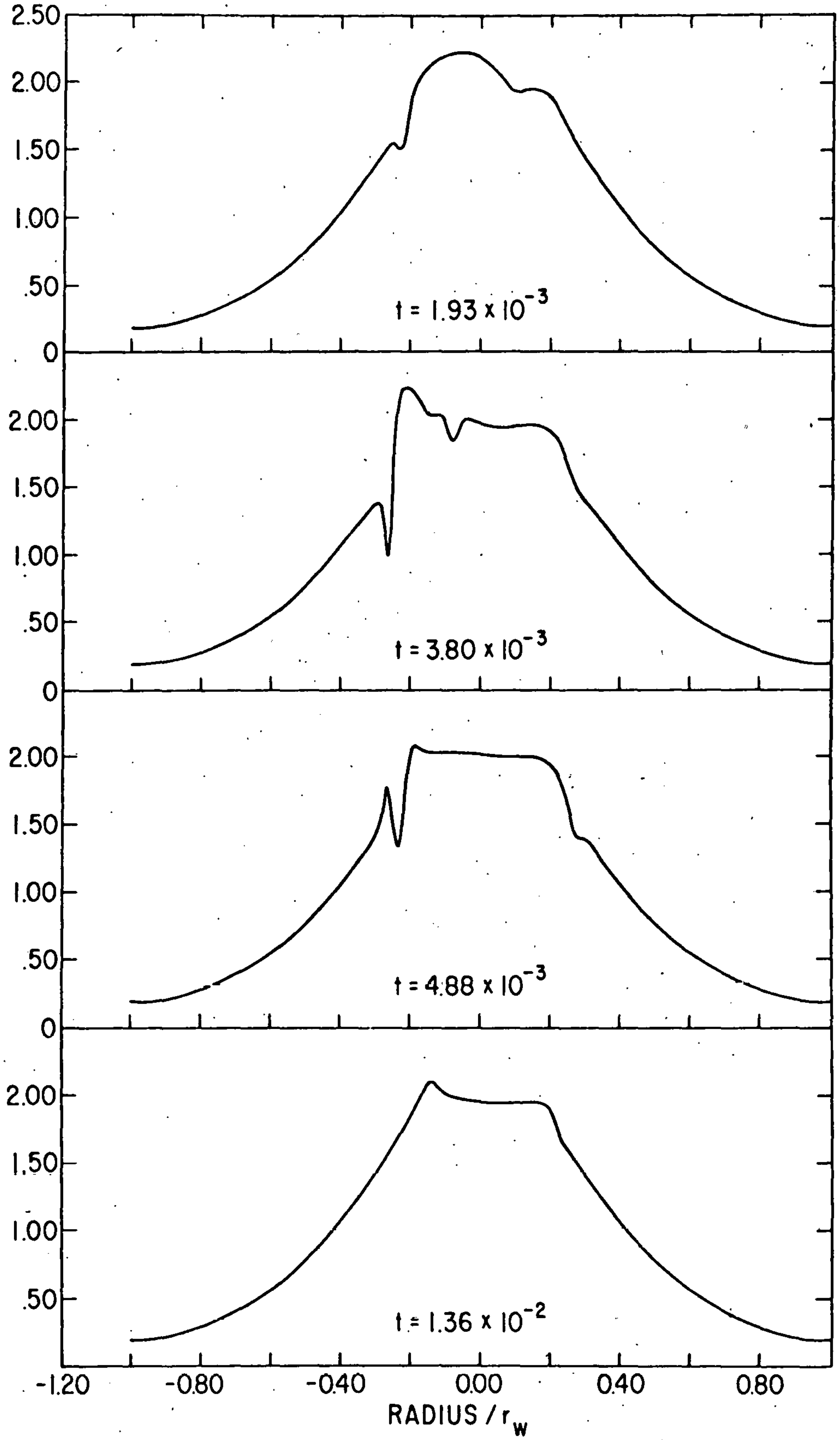

FIGURE 4 762034

Toroidal current density as a function of r at selected times.