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NONLINEAR INTERACTION OF A FAST MAGNETOGASDYNAMIC SHOCK WITH A TANGENTIAL DISCONTINUITY

by

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I

Abstract

A basic problem, which is of considerable interest in geoastrophysical applications of magnetogasdynamics, is the nonlinear interaction of a fast shock (S_f) with a tangential discontinuity (T). The problem is treated for an arbitrary S_f interacting with an arbitrary T under the assumption that in the frame of reference in which S_f and T are at rest the flow is super-fast on both sides of T and that a steady flow develops. As a result of the nonlinear analysis a flow pattern is obtained consisting of the incident discontinuities S_f1 and T2 and a transmitted fast shock S_f3, the modified tangential discontinuity T4 and a reflected fast shock S_f5 or fast rarefaction wave R_f5. The results can be discussed in terms of seven significant similarity parameters. The uniqueness of the solution is investigated as well as the region of dimensionless parameter space in which a solution of the type described above is possible. Varying the similarity parameters the solutions of type S_f1,T2,S_f3,T4,S_f5 are bounded in parameter space by the disappearance of S_f5, by the maximum total pressure ratio across S_f5 at which the flow direction on both sides of T4 can just be matched and by the limiting cases of S_f3 or S_f5 being switch-on shocks. The following analytical and numerical results concerning the dependence of the interaction on the type of tangential discontinuity T2 are obtained: A T2 with a velocity shear only in the direction perpendicular to the plane XY spanned by the normals of S_f1 and T2 leaves the incident shock unchanged and no reflected wave S_f5 or R_f5 develops. A velocity shear across T2 in the plane XY can be compensated by a density change such

that the incident shock is not deflected or changed at all and that no S_f5 or R_f5 develops although appreciable changes of density and velocity may occur across T2. In addition special cases like changes in magnetic field direction only, changes in density or velocity shear only etc. are discussed in some detail.

I. Introduction

In recent years plasma and magnetic field experiments on satellites and space probes have proved the existence of all proposed types of magnetogasdynamic discontinuities in the interplanetary plasma with the exception of the contact discontinuity with nonvanishing normal magnetic field. A comprehensive classification of the possible discontinuities taking into account the evolutionarity conditions is given in the book by Jeffney and Taniuti[1]. The pertinent space observations are reviewed in a recent article by Burlaga[2]. For many problems of the physics of discontinuities in the collisionless interplanetary plasma, magnetogasdynamics or even gasdynamics can give a reasonable answer, although it is realized that the properties of the discontinuities are modified by pressure anisotropies[3,4,5], the possible change in heat flux across the discontinuity[6] and a background wave spectrum[6,7].

The observations show more specifically that in addition to continuous fluctuations of the physical quantities the spatial and time structure of the solar wind is characterized by the frequent occurrence of tangential discontinuities and in somewhat smaller numbers of rotational discontinuities[8]. On the other hand, the interplanetary plasma is the propagation medium for fast magnetogasdynamics shocks which produce major disturbances in the space environment of the earth and therefore are of considerable interest.

A basic physical process during the propagation of interplanetary shocks is their interaction with the large number of tangential discontinuities. Other applications of this basic process are the interaction of tangential discontinuities with the earth's bow shock and the interaction of interplanetary shocks with the magnetopause of the earth after modification of the shock by its collision with the earth's bow shock.

It is the purpose of this paper to investigate the nonlinear interaction of a fast magnetogasdynamic shock wave with a tangential discontinuity in the nonrelativistic case. Both discontinuities are considered to be arbitrary in their orientation to each other and in all other physical parameters with the only restriction being that in the rest frame of both discontinuities a steady flow pattern is possible. Other possible solutions include a break-up of the shock, a propagation of part of a disturbance upstream along the initial discontinuities etc. The case of a steady flow is most important because the flow pattern depends on the local physical properties in the vicinity of the line of intersection of both discontinuities only.

In the discussion of the numerical results we shall emphasize the case of only fast shocks in the resulting flow. Two limiting cases of this more general problem have been considered so far. Jaggi and Wolf[9] have investigated the problem of a weak tangential discontinuity interacting with a fast shock. The case with vanishing magnetic field is well known in the gasdynamic literature[10]. The special case, where both discontinuity planes are parallel, is included in the discussion of the general magnetogasdynamic Riemann problem by Gogosov[11].

In Section II we derive the resulting flow pattern from general arguments. In Section III the equations for the solution of the problem are formulated and seven dimensionless parameters are derived, which characterize the problem and lead to some interesting similarity laws. Section IV contains a graphical representation of the mathematical problem and a discussion of the uniqueness of the results. In addition, those regions in parameter space are identified in which solutions of the type considered are possible.

In Section V some numerical results for the interaction are presented.

Except for the requirement that each of the participating discontinuities has to be evolutionary, we shall not consider the stability of the solutions.

II. Derivation of Basic Flow Pattern

The problem under consideration is the interaction of a plane fast magnetogasdynamic shock and a plane tangential discontinuity, which according to the nomenclature of Jeffrey and Taniuti[12] is a contact discontinuity with a vanishing magnetic field component in the normal direction. Both discontinuities may have an arbitrary orientation to each other. If the planes are not parallel, it is always possible to find the line of intersection of both discontinuities, which we shall call the interaction line. We can then always find a frame of reference in which the fast shock and the tangential discontinuity are at rest. In this frame of reference we define a right-handed Cartesian coordinate system X,Y,Z with the Z-axis parallel to the interaction line and the X-axis in the direction of propagation. This is shown in Fig. 1, which will be used to develop the basic magnetogasdynamic flow pattern throughout this section. The incident fast shock is shown as a solid line called S_f1 and the initial tangential discontinuity T2 is shown as a dashed line. A straight line in the plane of Figure 1 evidently corresponds to a plane perpendicular to the XY plane, since the problem is two-dimensional. In defining our frame of reference we have not yet made use of the degree of freedom in the Z-direction. For definiteness we assume the velocity component $v_{z,1}$ in sector 1 between S_f1 and T2 to be zero without loss of generality.

The physical state in sector 1 characterized by the velocity vector $\underline{v}_1$, the magnetic field vector $\underline{B}_1$, the pressure p_1 and density ρ_1 is given as well as the physical quantities $\underline{v}_2$,

$\underline{B}_2$, p_2, ρ_2 on the other side of T2. It is clear that it is impossible to find a steady-state solution for all choices of initial parameters. The parameter range for which this is possible will also be investigated in this study.

It is illuminating to consider the cases in which one of the initial discontinuities S_f1 and T2 is weak, where the weakness is characterized by a parameter $\varepsilon << 1$. The case of T2 being weak[9] leads to a sector pattern, in which the shock S_f1 is changed by an amount of $O(\varepsilon)$ resulting in a transmitted fast shock S_f3. T2 is modified into the tangential discontinuity T4 with a strength of $O(\varepsilon)$. A reflected weak fast shock S_f5 or rarefaction wave R_f5 is generated with the strength of $O(\varepsilon)$. The direction of S_f5 or R_f5 is given by the fast Mach line going out from the interaction line and making an acute angle with the Y-axis. Here we have used the following convenient notation. The homogeneous sectors between the various disturbances are counted counterclockwise giving the subscripts for the physical quantities in these sectors. The disturbances are counted in the same sense and the characters R, S, T give the type of discontinuity. Physical quantities defining the disturbances themselves have subscripts according to this counting. The subscripts f and s define fast or slow shocks (S) or rarefaction waves (R) in the magnetogasdynamic sense.

The results for T2 being weak of $O(\varepsilon)$ suggest by the continuous dependence of the solutions on the initial conditions that for a range of tangential discontinuities T2, which are finite in amplitude, the flow pattern will be S_f1, T2, S_f3, T4, R_f5 or S_f5. For the appropriate range of velocities $\underline{v}_2$ there could also be real characteristics or Mach lines of the slow mode in sector 2 and therefore solutions involving slow shocks. However, we shall restrict ourselves to the case of fast transmitted waves only.

These considerations could be used to establish the basic magnetogasdynamic flow pattern in the frame of reference chosen by their extension to the general case of S_f1 and T2 being of finite strength. A more satisfactory and instructive way is to derive this basic flow pattern from basic principles and the known properties of magnetogasdynamic discontinuities and rarefaction waves. This is especially important for the question of the uniqueness of the solutions.

We derive the basic type of solution by the following five arguments.

1. Since the initial conditions given by the angle α_2 (see Fig. 1) and the physical quantities $\underline{v}_1$, $\underline{B}_1$, ρ_1, p_1 and $\underline{v}_2$, $\underline{B}_2$, ρ_2, p_2 do not contain any characteristic length and since the magnetogasdynamic equations contain no characteristic length, we immediately get the result that the solution to our problem depends on the angle φ only and not on r, where r, φ and Z are cylindrical coordinates around the Z-axis, i.e., we have $\frac{\partial}{\partial r} \equiv 0$ and $\frac{\partial}{\partial Z} \equiv 0$. The solution must then be composed of a sector pattern of discontinuities and rarefaction waves centered at the origin, and homogeneous regions in between.

2. The vector $\underline{v}_1$ and $\underline{B}_1$ must be parallel to the plane of T2 by the definition of a tangential discontinuity, i.e., the projections of $\underline{v}_1$ and $\underline{B}_1$ on the X, Y-plane called $\underline{v}_{p,1}$ and $\underline{B}_{p,1}$, respectively, must be parallel or antiparallel to each other. Because of $\underline{E} = -\underline{v} \times \underline{B}$ we must therefore have $E_Z = 0$ in sectors 1 and 2 and because of Faraday's law $E_Z \equiv 0$ everywhere.

Therefore $\underline{v}_p$ and $\underline{B}_p$ must be parallel or antiparallel everywhere. This immediately excludes a contact discontinuity with nonvanishing normal component of the magnetic field $B_n \neq 0$ from further consideration, because it would imply $E_Z = -(\underline{v} \times \underline{B})_Z \neq 0$.

3. There can be at most one tangential discontinuity in addition to T2 as the result of the interaction. Suppose, there are more than one newly created tangential discontinuities among them T_k and T_n with $n > k > 2$, say. Since a tangential discontinuity is characterized by vanishing normal components of $\underline{v}$ and $\underline{B}$ and the continuity of the total pressure $p_t = p + \frac{B^2}{8\pi}$ across the discontinuity only, the state between T_k and T_n is not completely determined by the initial conditions. There is for example no way to determine the density or densities between T_k and T_n. As we have to require at least piecewise continuous dependence of the solutions on the initial conditions such a configuration is impossible unless it is required by additional initial conditions. The statement at the beginning of this paragraph is proven thereby.

4. An important quantity in the classification of discontinuities is the Alfvén Mach number A_n based on the normal components of the velocity and magnetic field with respect to a shock or rarefaction wave under consideration. Using the normal vector $\underline{n}$, which points in the direction of propagation of the shock or rarefaction wave, we obtain

$$A_n = \frac{|\underline{v} \cdot \underline{n}|}{|\underline{B} \cdot \underline{n}|} \sqrt{4\pi\rho} = \frac{|\underline{v}_p \cdot \underline{n}|}{|\underline{B}_p \cdot \underline{n}|} \sqrt{4\pi\rho} = \frac{|\underline{v}_p|}{|\underline{B}_p|} \sqrt{4\pi\rho} \tag{1}$$

since the direction cosines of $\underline{v}_p$ and $\underline{B}_p$ with respect to an arbitrary normal $\underline{n}$ are equal in magnitude as shown under item 2.

That is, the Alfvén Mach number A_n is independent of the normal direction, if the state in front of the wave is specified. This property has interesting consequences. Before we investigate these consequences some inequalities for shocks and rarefaction waves (including strength zero) have to be recalled:

$$S_f\text{: upstream:} \quad A_n \geq 1, \ \frac{v_n}{c_f} \geq 1 \tag{2}$$

$$\text{downstream:} \ A_n \geq 1, \ \frac{v_n}{c_f} \leq 1$$

$$R_f\text{: upstream and downstream:} \ A_n \geq 1, \ \frac{v_n}{c_f} = 1 \tag{3}$$

$$S_s\text{: upstream:} \quad A_n \geq 1, \ \frac{v_n}{c_s} \geq 1 \tag{4}$$

$$\text{downstream:} \ A_n \geq 1, \ \frac{v_n}{c_s} \leq 1$$

$$R_s\text{: upstream and downstream:} \ A_n \leq 1, \ \frac{v_n}{c_s} = 1 \tag{5}$$

Here c_f and c_s are the fast and slow magnetoacoustic speeds, respectively. For finite shocks $A_n = 1$ is valid on the downstream side for switch-on shocks only and on the upstream side for switch-off shocks only.

Using equation 1 and inequalities 2, 3, 4, 5 it immediately turns out that a streamline starting in sector 1 can pass through fast waves (i.e., fast shocks or rarefaction waves) only, because the initial shock wave S_f1 has been chosen to be of the fast type, i.e., $A_n > 1$ everywhere on the streamline starting in sector 1. Here we always assume that no switch-on or switch-off waves occur as part of the general solution. We shall return to this question later. A stream line starting in sector 2 will also pass through fast waves only, if

$$A_{n,2} \geq \max\ (1\ \sqrt{s_2}) \tag{6}$$

where s_2 is given by

$$s_2 = 5/3 \frac{p_2}{B_2^2/4\pi} \tag{7}$$

as the square of the ratio of sound-speed over Alfvén speed[1,13]. It has already been mentioned that we shall consider the case of fast transmitted waves only which requires inequality 6 to be valid.

5. We now have to introduce the important notion of "outgoing" waves and "incoming" waves, where by waves we mean shock waves or rarefaction waves. "Outgoing" fast waves are defined such that a weak fast disturbance can travel along the wave away from the interaction line and thereby modify the wave. It is based on the requirement that a change in the initial conditions must influence the various disturbances resulting in the S_f1, T2 interaction. This implies, that behind the shock or downstream of the shock the state must be sub-fast, super-Alfvénic and super-slow. This condition restricts the allowable angle with the streamlines of the incident plasma as schematically shown in Figure 2a. Since the fan which characterizes a fast rarefaction wave is made out of fast characteristics, we have the situation shown in Figure 2b. We shall call "incoming" waves those waves which are not "outgoing".

Since in the interaction only outgoing waves can be created, i.e., disturbances which are connected to the interaction line by causality, the number of possible shocks or rarefaction waves going out from the interaction line can be further reduced. For example, behind an outgoing fast shock or rarefaction wave $A_n > 1$ and $\frac{v_n}{c_f} < 1$ for all possible normal directions of a wave following the outgoing fast wave. Therefore, no further shock or rarefaction wave is possible on a streamline passing through an outgoing fast shock.

A streamline starting in sector 2 can therefore pass through one outgoing fast wave S_f3 or R_f3 only and has to reach infinity in sector 3. If S_f1 is an outgoing shock no further fast wave could be crossed by a streamline until infinity is reached behind S_f1. This is impossible, since there is only one unknown namely the strength of S_f3 or R_f3, whereas two conditions have to be fulfilled when sector 3 and the state behind S_f1 are separated by a tangential discontinuity or seven conditions if all physical quantities have to be continuous from sector 3 to the state behind S_f1. Here we have used $\underline{v}_p \times \underline{B}_p = 0$ everywhere.

Consequently S_f1 must be an incoming shock in agreement with the fact that S_f1 is given by the initial conditions. The general solution can then be given as S_f1, T2 leading to S_f3, T4, R_f5 or S_f5. We also have five homogeneous sectors 1 to 5, where 5 is the sector behind S_f1. With the arbitrary strength parameters ε_3 and ε_5 for S_f3 and S_f5 or R_f5 considered as unknowns, two equations have to be fulfilled

$$p_{t,3}(\varepsilon_3) = p_{t,4}(\varepsilon_5) \tag{8}$$

and $$\alpha_{v,3}(\varepsilon_3) = \alpha_{v,4}(\varepsilon_5) \tag{9}$$

where $\alpha_{v,j}$ is the angle between $\underline{v}_p$ and the Y-axis in sector j. A solution involving a rarefaction wave R_f3 is not possible, because it would require $\alpha_{v,3} < \alpha_{v,4}$. Because of $p_{t,5} > p_{t,1}$, equation 8 and $p_{t,1} = p_{t,2}$ we get $p_{t,5} > p_{t,4}$, which requires a wave R_f5. Since $\alpha_{v,1} = \alpha_{v,2}$ and $\alpha_{v,4} > \alpha_{v,5} > \alpha_{v,1}$ and $\alpha_{v,3} < \alpha_{v,2}$ we obtain $\alpha_{v,3} < \alpha_{v,4}$ in contradiction to equation 9.

The condition, that S_f1 is an "incoming" wave, leads to a limitation of α_2:

$$\alpha_{M_f} < \alpha_2 < \alpha_{crit} < 90^\circ \qquad (10)$$

where α_{M_f} is the Mach angle of the fast wave i.e., the angle α_2 at which the fast Mach number equals one for given s_1, $\alpha_{iB,1}$ and $A_{n,1}$. The angle α_{crit} is given by solving equation

$$A^2_{n,5} = \frac{1}{2}\,\frac{s_5 + 1}{\cos^2\alpha_{iB,5}} + \left(\left(\frac{1}{2}\,\frac{s_5 + 1}{\cos^2\alpha_{iB,5}}\right)^2 - \frac{s_5}{\cos^2\alpha_{iB,5}}\right)^{1/2} \qquad (11)$$

for $\alpha_2 = \alpha_{crit}$, where $A_{n,5}$ and s_5 are functions of s_1, α_2, $\alpha_{iB,1}$ and $A_{n,1}$ and where s_1, $\alpha_{iB,1}$ and $A_{n,1}$ are considered to be given. s_5 is defined analogously to s_2 in equation 7. $\alpha_{iB,1}$ is the inclination of the magnetic field in sector 1 with respect to the XY-plane with $\alpha_{iB,1} = 0$ for $B_{Z,1} = 0$ and $B_{y,1} > 0$.

III. Similarity Laws

We rewrite equations 8 and 9 using $p_{t,1} = p_{t,2}$ and $\alpha_{v,1} = \alpha_{v,2}$:

$$\frac{p_{t,3}}{p_{t,2}} = \frac{p_{t,4}}{p_{t,5}} \cdot \frac{p_{t,5}}{p_{5,1}}$$

or

$$Y_{t,f}(s_2, \alpha_{iB,2}, A_{n,2}, \alpha_3 - \alpha_{v,2}) = Y_{t,f}(s_5, \alpha_{iB,5}, A_{n,5}, \alpha_5 + \alpha_{v,5}) \cdot$$

$$\cdot\ Y_{t,f}(s_1, \alpha_{iB,1}, A_{n,1}, \alpha_2) \tag{12}$$

and $$\alpha_{v,3} - \alpha_{v,2} = \alpha_{v,4} - \alpha_{v,5} + \alpha_{v,5} - \alpha_{v,1}$$

or

$$\delta_f(s_2, \alpha_{iB,2}, A_{n,2}, \alpha_3 - \alpha_{v,2}) = -\delta_f(s_5, \alpha_{iB,5}, A_{n,5}, \alpha_5 + \alpha_{v,5})$$

$$+\ \delta_f(s_1, \alpha_{iB,1}, A_{n,1}, \alpha_2) \tag{13}$$

Equations 12 and 13 have to be solved for the unknowns α_3 and α_5. The angles α_3 and α_5 are the angles between the discontinuities S_f3 and S_f5 and the Y-axis as shown in Fig. 1. R_f5 is characterized by the angle between the weak discontinuity at its downstream side and the Y-axis. The functions $Y_{t,f}$ and δ_f are derived from the general jump relations in the appendix for shocks. The appendix also contains the functional dependence of s_5, $\alpha_{iB,5}$, $A_{n,5}$ and $\alpha_{v,5}$ on s_1, α_2, $\alpha_{iB,1}$, $A_{n,1}$.

Inspection of equations 12 and 13 shows, that the initial conditions define seven dimensionless parameters s_1, α_2, $\alpha_{iB,1}$, $A_{n,1}$, s_2, $\alpha_{iB,2}$ and $A_{n,2}$, which characterize the complete problem.

We have started with 17 quantities α_2, ρ_1, p_1, $\underline{v}_1$, $\underline{B}_1$, ρ_2, p_2, $\underline{v}_2$, $\underline{B}_2$ or 14 quantities, if the jump relations across T2 are taken into account. This reduction in parameters to seven dimensionless parameters is very convenient for a further discussion and leads to what can be referred to as similarity laws. The problem of the uniqueness of the solutions of equations 12 and 13 will be treated in Section IV.

The solution determines the dimensionless quantities characterizing each sector and each discontinuity or rarefaction wave, i.e., α_j, s_j, $\alpha_{iB,j}$, $\alpha_{v,j}$, $A_{n,j}$, $r_{B,j}$ with j = 3, 4, 5 and the fast Mach numbers M_f for S_f3 and S_f5 or R_f5, where $r_{B,j}$ is the magnetic field magnitude in sector j measured in units of $|\underline{B}_1|$.

The parameters entering equations 12 and 13 are independent of the velocity component in the Z-direction. Introducing $\alpha_{iv,j}$ the inclination of the velocity vector in sector j the quantities $\alpha_{iv,5}$ and $\alpha_{iv,4}$ are determined by the choice $\alpha_{iv,1} = 0^{\circ}$. To determine $\alpha_{iv,3}$ it is necessary to specify an eigth initial parameter $\alpha_{iv,2}$. $\alpha_{iv,2}$ only influences $\alpha_{iv,3}$, however.

Obviously, the solution for given physical parameters α_2, ρ_1, p_1, $\underline{v}_1$, $\underline{B}_1$, ρ_2, p_2 $\underline{v}_2$, $\underline{B}_2$ can easily be obtained by using the results of the dimensionless problem. The initial conditions must be compatible with a tangential discontinuity connecting sector 1 and sector 2.

Symmetries in the discontinuity jump relations immediately lead to a number of simple results. Replacing $\alpha_{iB,1}$ by $-\alpha_{iB,1}$ leads to $\alpha_{iB,5} \rightarrow -\alpha_{iB,5}$, $\alpha_{iB,4} \rightarrow -\alpha_{iB,4}$. $\alpha_{iB,2} \rightarrow -\alpha_{iB,2}$ in sector 2 leads to $\alpha_{iB,3} \rightarrow -\alpha_{iB,3}$ in sector 3, etc.

Jaggi and Wolf[9] consider the generation of fast magneto-acoustic waves (in our notation S_f5 or R_f5) by weak tangential discontinuities: They show results for all five linearly independent types of discontinuities. As our dimensionless parameters show, there are only three parameters s, α_{iB} and A_n, the changes of which characterize the kind of interaction generating S_f5 or R_f5. Their Figures 5, 7, 8 can therefore be simply related by comparing the corresponding changes in A_n from sector 1 to sector 2.

The similarity laws included in the seven or eight dimensionless parameters can be used to immediately derive some important results. Suppose $\alpha_2 < \alpha_{crit}$ to be given together with ρ_1, p_1, $\underline{v}_1$, $\underline{B}_1$ and $\rho_2 = \rho_1$, $p_2 = p_1$, $\underline{v}_2 = \underline{v}_1$ and $\underline{B}_2 = \underline{B}_1$. Since T2 is a zero-strength discontinuity we would expect no interaction and no change in the shock flow pattern at all, i.e. S_f3 would be identical to S_f1 and there would be no S_f5 or R_f5. There would be a zero-strength discontinuity T4 with an angle $\alpha_4 < \alpha_2$ because of the bending of the stream-lines as they pass through S_f1. We also have $s_2 = s_1$, $\alpha_{iB,2} = \alpha_{iB,1}$, $A_{n,2} = A_{n,1}$ and $\alpha_{iv,2} = \alpha_{iv,1}$. If we introduce a velocity shear in the Z-direction by choosing $v_{Z,2} \neq 0$, none of the seven characteristic parameters, and therefore also the solution just described, change except for $v_{Z,3} = v_{Z,2} + v_{Z,5}$. Consequently, a shock S_f1 interacting with a tangential discontinuity with a velocity shear in the direction of the interaction line only will not be modified, i.e., $\underline{n}_{S_f3} = \underline{n}_{S_f1}$, and all properties of the shock S_f1 are identical to those of shock S_f3. Only the Z-components of the velocity vector will be different for both shocks. The angle α_4 will be less than α_2 and T4 is characterised by the same velocity shear as T2 and no change in any other physical quantity. There will be no generated or reflected wave S_f5 or R_f5. The same nondimensional solution applies for T2 having a velocity shear in the XY-plane, if it is compensated by a density change such that $A_{n,1} = A_{n,2}$ and $s_2 = s_1$ as well as $\alpha_{iB,2} = \alpha_{iB,1}$. For the density ratios we must have

$$\frac{\rho_2}{\rho_1} = \frac{v_{p,1}^2}{v_{p,1}^2} = \frac{\rho_3}{\rho_4} = \frac{v_{p,4}^2}{v_{p,3}^2} .$$

These considerations show that a small change in the dimensionless quantities s, A_n and α_{iB} across T2 is a sufficient condition for the interaction to be weak, i.e., for the change $S_f1 \rightarrow S_f3$ to be small and the strength of S_f5 or R_f5 to be weak. This condition is evidently much less restrictive than the condition of a small change in the dimensional quantities p, ρ, $\underline{v}$, $\underline{B}$.

IV. Uniqueness of Solution and Boundaries in Parameter Space

The uniqueness of the solutions α_3 and α_5 can be discussed with the help of the graphical representation in Figure 3 in the α_v, $p_t/p_{t,1}$ plane. We consider the seven initial parameters s_1, α_2, $\alpha_{iB,1}$, $A_{n,1}$, s_2, $\alpha_{iB,2}$, $A_{n,2}$ to be given. The point Se 1,2 then gives the state in sectors 1 and 2. The curve labelled S_f1 is the locus of all points $(\alpha_{v,5}, p_{t,5}/p_{t,1})$ obtained by keeping s_1, $\alpha_{iB,1}$ and $A_{n,1}$ in sector 1 constant and varying α_2 between α_{M_f} $(s_1, \alpha_{iB,1}, A_{n,1})$ and α_{crit} $(s_1, \alpha_{iB,1}, A_{n,1})$. Beyond the point B_1, corresponding to $\alpha_2 = \alpha_{crit}$, the shock wave S_f1 would be outgoing. The initial value of α_2 chosen in the initial parameter set leads to point Se5, which is also the upstream state for S_f5 or R_f5. The curve labelled S_f5 is the locus of all points $(\alpha_{v,4}, p_{t,4}/p_{t,1})$ obtained by varying α_5 such that a shock S_f5 is given with the upstream state s_5, $\alpha_{iB,5}$, $A_{n,5}$. It contains outgoing and incoming solutions. The outgoing and therefore allowed part of the curve S_f5 starts at Se5, passes through Se4, T and Se'4 and ends shortly thereafter. The curve S_f5 is connected continuously up to second order derivatives with the rarefaction wave branches R_f5, which are not shown completely. Similarly, the curve S_f3 is the locus of the S_f3 shocks obtained by varying α_3. It depends on s_2, $\alpha_{iB,2}$ and $A_{n,2}$. The intersections of curve S_f3 with curve S_f5,R_f5 give the solutions to equations 8 and 9 or 12 and 13. Every point on these curves is uniquely given by an angle α_3 or α_5, respectively. The case shown permits two solutions given by the points Se4 and Se'4, in which S_f3 and S_f5 are both "outgoing". However, the curves S_f3

and S_f5 could also include switch-on shock waves for $s_2 < 1$ and/or $s_5 < 1$, respectively. For example, the curve S_f5 then ends in a switch-on shock as a limiting case, before it reaches the line $\alpha_{v,4} = \alpha_{v,5}$ with infinite slope. In such cases there is the possibility of one solution only. If S_f3 misses the allowed part of R_f5, S_f5, no solution is possible. One invariable feature of these curves is that the curve labelled S_f5, R_f5 is symmetric to the horizontal line $\alpha_{v,4} = \alpha_{v,5}$, if for a moment we do not distinguish between "incoming" and "outgoing" waves. In addition, on each of the symmetric parts the second derivative does not change sign.

If more than one solution is possible, we pick the admissible one by the following postulate: The admissible solution Se4 must continuously tend to the point Se5 without S_f3 ever being tangent to S_f5 or R_f5 during the approach, when s_2, $\alpha_{iB,2}$ and $A_{n,2}$ tend to s_1, $\alpha_{iB,1}$, $A_{n,1}$, respectively, in a continuous manner. Although this device works remarkably well in gasdynamics, a more physical selection of the correct solution would be desirable. It could possibly be provided by a stability analysis of the type carried through for a plane shock by Gardner and Kruskal[14]. The solution of this problem is outside the scope of this work. It would be interesting to investigate experimentally the question whether under certain conditions a flipping from solution Se4 to Se'4 is possible as a result of an instability of solution Se4. Solutions of the type Se'4 could also occur as part of a more complicated interaction pattern outside the range of the initial parameters making a steady flow pattern possible. The numerical results in Section V will only contain the admissible solutions Se4. Finally we note a necessary condition for Se4 to be admissible. It may be admissible only, if it lies on the branch from Se5 to the point T in Fig. 3, which is obtained by drawing the tangent from Se1,2 at curve S_f5.

An important problem is to find the region in the seven-dimensional space of our initial parameters, in which the steady-state solution is possible. If the orientation of both discontinuities and the physical parameters in front of S_f1 are given, s_1, α_2, $\alpha_{iB,1}$, $A_{n,1}$ are determined. From Figure 3 and the previous discussion we see that three limiting cases exist for the basic steady flow pattern S_f1, T2, S_f3, T4, S_f5. Firstly the shock S_f5 can tend to zero strength, i.e., the fast Mach number $M_{f,5} = 1$. Secondly, the curves S_f5 and S_f3 can become tangent. After a further change of the parameters s_2, $\alpha_{iB,2}$, $A_{n,2}$ in the right direction, no solution could exist any more. Thirdly, the parameter range is limited by the case of Se4 tending to a switch-on shock. A similar situation is given for the basic pattern S_f1, T2, S3, T4, R_f5. In the following we restrict ourselves to the solution involving S_f5.

Each of the three limiting cases involves a relationship between the seven initial parameters. As an example we consider the relationship between $A_{n,2}$ and s_2 for a given set of s_1, α_2, $\alpha_{iB,1}$, $A_{n,1}$ and $\alpha_{iB,2}$ as shown in Figure 4. In the case $A_{n,1}= 8$ shown in Fig. 4a the limiting curve corresponding to the fast Mach number of shock S_f5 being $M_{f,5} = 1$ extends from $s_2 = 0$ to ∞. The same is true for the limiting curve describing the tangency condition. Between these curves the solution S_f1, T2, S_f3, T4, S5 is possible. Below the curve $S_f5 = 0$ the solution is S_f1, T2, S_f3, T4, R_f5. The latter type of solution is limited by another curve below the curve $M_{f,5} = 1$ expressing the tangency condition for the rarefaction wave. This curve has not been computed, however. In the other case $A_{n,1} = 4$ shown in Fig. 4b there must always be a reflected wave called generated wave by Jaggi and Wolf[9] above $s_2 \sim 56$ for a steady-state solution to exist in our frame of reference. Again an upper limit exists for the Alfvên Mach number $A_{n,2}$ as a function of s. The curve expressing the tangency condition extends continuously beyond the point, where it meets the curve $M_{f,5} = 1$, thereby limiting the solutions of type S_f1, T2, S_f3, T4, R_f5.

Inspection of Figure 3 shows, that an upper limit to $A_{n,2}$ due to the tangency limit can only be found if the minimum of curve S_f5 fulfills the inequality $\alpha_{v,4} > -\alpha_2$, since the curve S_f3 is stretched progressively with increasing Mach number $A_{n,2}$, when all other parameters are kept constant. The marginal case can be expressed as a condition on s_1, α_2, $\alpha_{iB,1}$, and $A_{n,1}$.

In addition to the cases illustrated in Figure 4 a large number of other possibilities exists. It is for example also possible, that only above a minimum value of s_2 $M_{f,5}$ can be equal to one.

The disappearance of S_f5 from the flow pattern requires one condition on the seven initial parameters to be fulfilled. S_f3 cannot disappear by the argument in section II, item 5 proving the impossibility of a wave R_f3. T4 can disappear, however. For this to happen three conditions must be fulfilled by the seven initial parameters leading to $s_3 = s_4$, $\alpha_{iB,3} = \alpha_{iB,4}$ and $A_{n,3} = A_{n,4}$. The condition $\alpha_{iv,3} = \alpha_{iv,4}$ can be enforced separately by the correct choice of $\alpha_{iv,2}$. A trivial case is apparently given by the parameter set

s_1, α_2, $\alpha_{iB,1}$, $A_{n,1}$, $s_2 = s_1$, $\alpha_{iB,2} = \alpha_{iB,1}$ and $A_{n,2} = A_{n,1}$. Since the disappearance of T4 is a very special case due to the three required conditions on the initial parameters the possible existence of nontrivial solutions will not be considered.

In closing this section we make two more points. In section II, item 4 the assumption has been made that no switch-on or switch-off waves occur in the general solution. The development of the theory in this section is consistent with this assumption, since switch-on shocks occur as limiting cases only. Switch-off waves are not necessary at all to construct a solution, since they are of the S_s-type.

The second point concerns the poles in the generation coefficients of the paper by Jaggi and Wolf[9]. These poles do not occur in the nonlinear analysis. They are due to the linear approximations in the vicinity of points Se5 and Se4.

V. Parametric Study of the Solutions of the Interaction Problem

In this section we present a limited parametric study of our interaction problem. Instead of attempting a complete presentation of the characteristics of the physical solution, we consider a few cases of geoastrophysical interest. We shall exclude from consideration results for large s_1 and s_2 and large A_1 and A_2 since these are well known in the gasdynamic literature. Although the magnetic field does not greatly influence the results in these cases, it may be very important as a diagnostic tool. Here we introduce a new Alfvén Mach number

$$A = A_n \cos \alpha_{iB} = \frac{|\underline{v}_p| \sqrt{4\pi\rho}}{|\underline{B}|} \tag{14}$$

which is often more convenient than A_n for applications. For example,

$$A_1 = \frac{|\underline{v}_1 \cdot \underline{n}\ s_f 1|}{\sin \alpha_2} \frac{\sqrt{4\pi\rho_1}}{|\underline{B}_1|} .$$

We note that the value of $s = 1$ is typical for the solar wind in the vicinity of the earth. Typical values for A_1 are 4 and 12 for interplanetary shocks and the earth's bow shock near the subsolar point, respectively. These values will not be chosen exclusively, however.

The first case we consider is the interaction of s_f1 with T2, where T2 is characterized by a change in magnetic field direction only i.e., $\rho_1 = \rho_2$, $p_1 = p_2$, $\underline{v}_1 = \underline{v}_2$ or $s_1 = s_2$, $A_1 = A_{n,1} \cos \alpha_{iB,1} = A_2 = A_{n,2} \cdot \cos \alpha_{iB,2}$ but $\alpha_{iB,1}$ generally different from $\alpha_{iB,2}$. This type of discontinuity, corresponding to a type (a) discontinuity in Jaggi and Wolf's[8] treatment of weak discontinuities, is most commonly found in interplanetary space[2,8]. It illustrates most clearly the influence of the magnetic field. For the typical interplanetary values of $s_1 = s_2 = 1$ and $A_1 = A_2 = 4$ and $\alpha_2 = 45^{\circ}$ Figures 5a, b illustrate various interesting quantities as a function of $\alpha_{iB,1}$ and $\alpha_{iB,2}$. Figure 5a shows $\Delta\omega = \omega_{3,4} - \omega_{2,1}$ i.e., the change of the angle between the B-vectors on both sides of the tangential discontinuity during the interaction. We have defined $\omega_{k,e} = \alpha_{iB,k} - \alpha_{iB,e}$. The curves can be continued to values of $\alpha_{iB,2} > 90^{\circ}$ by use of some of the symmetry properties of the solutions. Thus if $\alpha_{iB,3}$ and $\alpha_{iB,4}$ and therefore $\omega_{3,4}$ are solutions for s_1, α_2, $\alpha_{iB,1}$, $A_{n,1}$, s_2, $\alpha_{iB,2}$, $A_{n,2}$ given with $\alpha_{iB,2} < 90^{\circ}$, say, the transformation $\alpha_{iB,2} \rightarrow 180^{\circ} - \alpha_{iB,2}$ yields $\alpha_{iB,3} \rightarrow 180^{\circ} - \alpha_{iB,3}$, $\alpha_{iB,4} \rightarrow \alpha_{iB,4}$ and therefore $\omega_{3,4} \rightarrow 180^{\circ} - \alpha_{iB,3} - \alpha_{iB,4}$ and finally $\Delta\omega \rightarrow 2\,\Delta\omega\,(\alpha_{iB,2} = 90^{\circ}) - \Delta\omega$, i.e., the curves $\Delta\omega\,(\alpha_{iB,2})$ can be continued to $\alpha_{iB,2} > 90^{\circ}$ by reflection with respect to their points of intersection with the line $\alpha_{iB,2} = 90^{\circ}$. Only in the case $\alpha_{iB,1} = 0^{\circ}$ an S_f5 shock occurs all the time. Increasing $A_1 = A_2$ with $s_1 = s_2$ and α_2 kept constant leads to a further limited increase of $\Delta\omega$.

If the magnetic field could be neglected in the dynamics of the problem, we would expect $\alpha_3 = 0$ in the case considered above with $s_1 = s_2$ and $A_1 = A_2$. That the magnetic field influences the dynamics is shown in Figure 5b, where angles of α_3 up to 4° are shown. The curves can be reflected at the line $\alpha_{iB,2} = 90^{\circ}$ to obtain values for $\alpha_{iB,2} > 90^{\circ}$. Increasing the Mach numbers $A_1 = A_2$ leads to lower values of α_3, since the pressure forces become more and more important compared with the magnetic forces. The shocks S_f5 are very weak in all these cases. The largest value of $M_{f,5} = 1.04$ is attained for $\alpha_{iB,1} = 0^o$ and $\alpha_{iB,2} = 90^o$. An interesting feature of these solutions is, that whereas T2 shows a change in direction of $\underline{B}$ (given by $\omega_{2,1}$) only, the modified tangential discontinuity can display additional changes e.g. in $|\underline{B}|$. In contrast to $|\underline{B}_1| = |\underline{B}_2|$ we have a maximum ratio of $|\underline{B}_3|/|\underline{B}_4| = 1.12$ at $\alpha_{iB,1} = 0^o$ and $\alpha_{iB,2} = 90^o$, and a minimum density ratio of ~ 0.9 across T4 for the same parameters compared with a value of one at T2. Increasing $A_1 = A_2$ to a value of 8 leads to a maximum $|\underline{B}_3|/|\underline{B}_4|$ of 1.24 at the same $\alpha_{iB,1}$ and $\alpha_{iB,2}$.

The next case we consider involves density changes across T2 or equivalently a velocity shear. The results for α_3 are shown in Figure 6, where α_3 is plotted as a function of A_2 with $s_1 = s_2 = 1$, $\alpha_2 = + 45^o$, $\alpha_{iB,1} = \alpha_{iB,2} = 0^o$ and $A_2 = 4$ and 12. It is expected that the change in direction of the shock normal is much more pronounced in this case. An increase in density across T2 by a factor of 2, corresponding to a ratio $A_2/A_1 = \sqrt{2}$, leads to $\alpha_3 = - 10.9^o$ and a fast Mach number $M_{f,5} = 1.1$ for $A_1 = 4$ and $\alpha_3 = - 9.3^o$ and $M_{f,5} = 1.13$ for $A_1 = 12$. The density ratio across T4 is 1.84 and 1.68 for $A_1 = 4$ and 12, respectively, compared with two across T2. On the other hand the density ratios across S_f5 are 1.14 and 1.21 and magnetic field magnitude ratios 1.1 and 1.14, respectively.

The last example illustrates the influence of a change in s across T2 with $\alpha_{iB,1} = 0^o$, $s_1 = 1$, $A_1 = A_2 = 8$ on the relationship $\Delta\omega$ $(\alpha_{iB,2})$. Since a change in s across T2 implies $|\underline{B}_1| \neq |\underline{B}_2|$ $A_1 = A_2$ implies an additional change in density or velocity $\underline{v}_p$ across T2. The results are shown in Figure 7.

We see, that an increase in $|\underline{B}_2|/|\underline{B}_1|$ implied by a decrease in s_2/s_1 leads to progressively smaller changes in $\Delta\omega = \omega_{3,4} - \omega_{2,1}$.

VI. Conclusions

In conclusion we have shown that for a range of initial conditions the interaction between a fast shock S_f1 and a tangential discontinuty T2 can lead to a steady flow pattern in the frame of reference in which both discontinuities are at rest. For $A_{n,2} \geq \max\{1, \sqrt{s_2}\}$ two solutions are possible, S_f1, T2, S_f3, T4, S_f5 and S_f1, T2, S_f3, T4, R_f5, which are adjacent in the parameter space s_1, α_2, $\alpha_{iB,1}$, $A_{n,1}$, s_2, $\alpha_{iB,2}$, $A_{n,2}$. Uniqueness of the solutions has been enforced by the requirement of continuous connection to the limiting cases of linear interaction, a device, which works remarkably well in gasdynamics. Further work is necessary in this area concerning the stability of the solutions obtained. Also laboratory experiments and space observations could be used to finally decide the question of uniqueness.

The set of seven dimensionless parameters turns out to be very economical in ordering the large number of solutions. The similarity laws defined thereby show that a velocity shear parallel to the interaction line constitutes a very weak interaction leading to S_f3 essentially identical to S_f1 and no wave S_f5 or R_f5. These laws also show that velocity shears perpendicular to the interaction line and density changes across T2 are equivalent. The computations for individual cases show that a tangential discontinuity T2 with a change in direction of the magnetic field only and $s_1 = s_2 = 1$ produces a very weak reflected or generated fast wave, whereas the tangential discontinuity T4 has more complicated characteristics than T2, i.e., a change in magnetic field magnitude, density, temperature and velocity shear. Because of the deviation of the shock normal to up to 4° for $s_1 = s_2 = 1$, $\alpha_2 = 45^{\circ}$ and $A_1 = A_2 = 4$ the propagation of a shock through an ensemble of tangential discontinuities of this type like in the solar wind can lead to an appreciable random walk of the normal direction and other shock properties.

An initial tangential discontinuity with a density jump leading to a sudden impulse in the geomagnetic field accordingto Burlaga[15] splits into two disturbances after interacting with the earth's bow shock which due to their different propagation velocities would have different travel times. Since in this case S_f5 or R_f5 are not weak the initiating signal at the magnetopause is a broad signal and not a one step signal as often assumed in the past.

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APPENDIX

In this appendix we shall collect the functional relationships needed in equations 12 and 13. Figure 8 shows the geometry of the problem. The upstream region is denoted by the subscript zero and the downstream region by one. The upstream region is then characterized by s_o, $\alpha_{iB,o}$ and $A_{n,o}$ and the shock or rarefaction wave by the angle α between the <u>downstream</u> edge of the S_f or R_f-disturbance and the projection of the streamline on the x,y-plane. This definition of α is useful for both S_f and R_f. For a shock the downstream and upstream edges are parallel trivially. For the rarefaction wave the direction of the upstream weak discontinuity is given by the initial conditions already, whereas the direction of the downstream weak discontinuity can be used to characterize its strength.

In the following we restrict ourselves to the shock case. We use the formulas derived by Bazer and Ericson[13] and also given by Jeffrey and Taniuti[16] with the evolutionarity conditions imposed by them. Note, that in our case $\underline{n}$ is pointing in the direction of propagation. The angle between $\underline{n}$ and $\underline{B}_o$ is called θ_o and without loss of generality we can write $0 \leq \theta_o < 180^o$ or $\sin \theta_o \geq 0$. We obtain

$$\cos \theta_o = - \sin \alpha \cos\alpha_{iB,o} \qquad \text{(A1)}$$

A shock is then possible for the angular range

$$\alpha_{M_f} \leq \alpha \leq 180^o - \alpha_{M_f} \qquad \text{(A2)}$$

where α_{M_f} is given by

$$\sin \alpha_{M_f} = \frac{1}{|\cos \alpha_{iB,o}| A^2_{n,o}} (-s_o + (s_o + 1) A^2_{n,o})^{1/2} \qquad \text{(A3)}$$

with $0 < \alpha_{M_f} \leq 90^o$ and $A_{n,o} \geq \max (1, \sqrt{s_o})$.

Introducing the quantity

$$h = \frac{B_{t,1} - B_{t,o}}{|\underline{B}_o|} \qquad \text{(A4)}$$

where B_t is the magnitude of the component of $\underline{B}$ in the shock-plane, we obtain the following cubic equation for h

$$h^3 + h^2 \frac{1}{3} \sin \theta_o (A^2_{n,o} + 11)$$

$$+ h \left[A^4_{n,o} \frac{2}{3} \cos^2 \theta_o + A^2_{n,o}(- \frac{10}{3} + 2s_o + \frac{17}{3} \sin^2\theta_o) + \frac{8}{3} - 2s_o\right]$$

$$+ 2\sin\theta_o \left[-A^4_{n,o}\cos^2\theta_o + A^2_{n,o}(1 + s_o) - s_o\right] = 0 \qquad \text{(A5)}$$

where we have used the ratio of specific heats $\gamma = 5/3$. The equation has one root fulfilling the requirement $0 \leq \bar{\eta}_f \leq 3$ necessary for a fast shock. The relative density change $\bar{\eta}_f$ is given by

$$\bar{\eta}_f = \frac{\rho_1}{\rho_o} - 1 = \frac{A^2_{n,o} - 1}{h + A^2_{n,o} \sin\theta_o} h \qquad \text{(A6)}$$

The relative pressure change $\bar{Y}_f$ is given by

$$\overline{Y}_f = \frac{p_1}{p_o} - 1 = \frac{5}{3s_o} \{- \frac{1}{2} h^2 + h \left(\frac{\overline{\eta}_f - h \sin\theta_o}{h - \overline{\eta}_f \sin\theta_o}\right)\} \tag{A7}$$

We are interested in the quantities s_1, $\alpha_{iB,1}$, $A_{n,1}$, $Y_{t,f}$, δ_f. Using $|\underline{B}_1|/|\underline{B}_o| = (1 + 2 h \sin \theta_o + h^2)^{1/2}$ we obtain

$$s_1 = \frac{1 + \overline{Y}_f}{1 + 2h \sin\theta_o + h^2} s_o \tag{A8}$$

$$Y_{t,f} = \frac{p_{t,1}}{p_{t,o}} = \frac{10\overline{\eta}_f \cos^2\theta_o}{h-\overline{\eta}_f \sin \theta_o} \frac{h}{6s_o + 5} + 1 \tag{A9}$$

$$A_{n,1} = A_{n,o} (1 + \overline{\eta}_f)^{-1/2} \tag{A10}$$

The quantities δ_f and $\alpha_{iB,1}$ are a little more difficult to obtain:

$$\sin \alpha_{iB,1} = \sin \alpha_{iB,o} \frac{\sin \theta_o + h}{\sin \theta_o (1+2h \sin \theta_o + h^2)^{1/2}}, \tag{A11}$$

where $\alpha_{iB,1}$ is in the same quadrant as $\alpha_{iB,o}$ from 0^o to 360^o and

$$\sin \delta_f = h \sin\alpha\cos\alpha \; (h^2\cos^2\alpha + \sin^2\theta_o + 2h\sin\theta_o\cos^2\alpha)^{-1/2}, \tag{A12}$$

where $|\delta_f| < 90^o$.

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Figure Captions

Figure 1 Magnetogasdynamic steady flow pattern as a result of the interaction of the fast shock S_f1 and the tangential discontinuity T2.

Figure 2 (a) Illustration of angular ranges of "outgoing" and "incoming" shocks for given upstream conditions.

(b) Illustration of "outgoing" and "incoming" fast rarefaction waves.

Figure 3 Graphical representation of the uniqueness problem in the mathematical solution of the interaction. Both points Se4 as well as Se'4 fulfill equations 8 and 9, i.e., $p_{t,3} = p_{t,4}$ and $\alpha_{v,3} = \alpha_{v,4}$.

Figure 4 (a) Limiting curves due to tangency condition and $M_{f,5} = 1$ for the steady solution S_f1, T2, S_f3, T4, S_f5. For the case of R_f5 instead of S_f5 only the limit of an infinitely weak R_f5 is shown i.e., the curve labeled $M_{f,5} = 1$. Both limiting curves are completely separated in this case.

(b) A more complicated case of limiting curves, in which tangency and $M_{f,5} = 1$ are possible at the same time.

Figure 5 (a) Change $\Delta\omega$ of the angle between magnetic field vectors on both sides of the tangential discontinuity during the interaction as a function of $\alpha_{iB,1}$ and $\alpha_{iB,2}$ with velocity, pressure and magnetic field magnitude as well as density continuous across T2 at the beginning.

(b) Deflection α_3 of the shock normal during the interaction.

Figure 6 Deflection α_3 of the shock normal and fast magnetoacoustic Mach number of the reflected wave S_f5 for tangential discontinuities with density change and/or velocity shear for $\Lambda_1 = 4$ and 12. Note the large deflections compared with Figure 5a.

Figure 7 $\Delta\omega$ as a function of $\alpha_{iB,2}$ for various values of s_2 expressing magnetic field increases and plasma pressure decreases. T2 is of a mixed type, since $|\underline{B}|$ changes as well as $|\underline{v}_p|^2 \rho$.

Figure 8 Geometry of a stationary oblique shock.

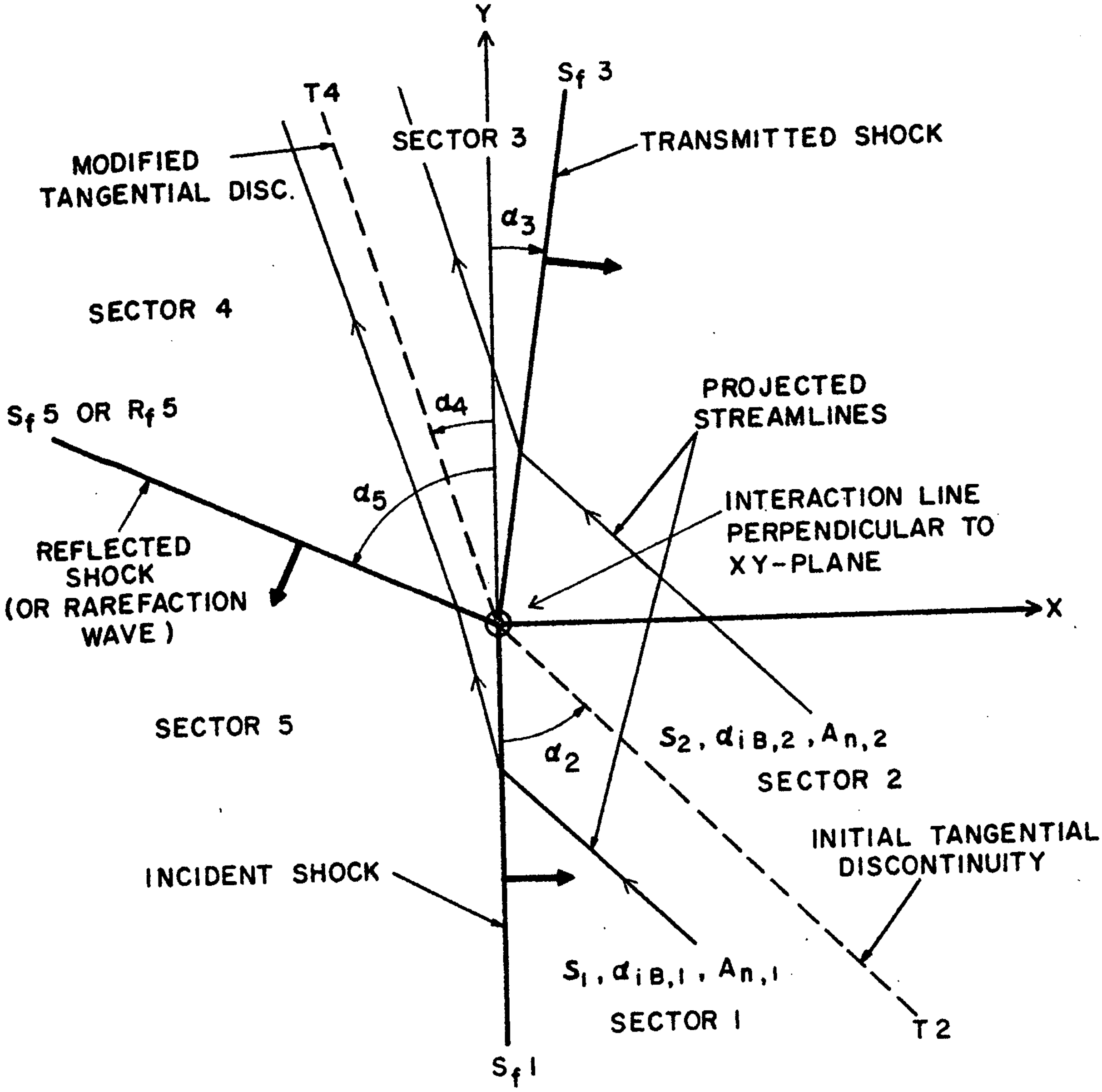

Y
S_f 3
T4
SECTOR 3
TRANSMITTED SHOCK
MODIFIED
TANGENTIAL DISC.
α_3
SECTOR 4
α_4
PROJECTED
STREAMLINES
S_f 5 OR R_f 5
α_5
INTERACTION LINE
PERPENDICULAR TO
XY-PLANE
REFLECTED
SHOCK
(OR RAREFACTION
WAVE)
X
SECTOR 5
S_2, $\alpha_{iB,2}$, $A_{n,2}$
α_2
SECTOR 2
INITIAL TANGENTIAL
DISCONTINUITY
INCIDENT SHOCK
S_1, $\alpha_{iB,1}$, $A_{n,1}$
SECTOR 1
T2
S_f 1

Figure 1

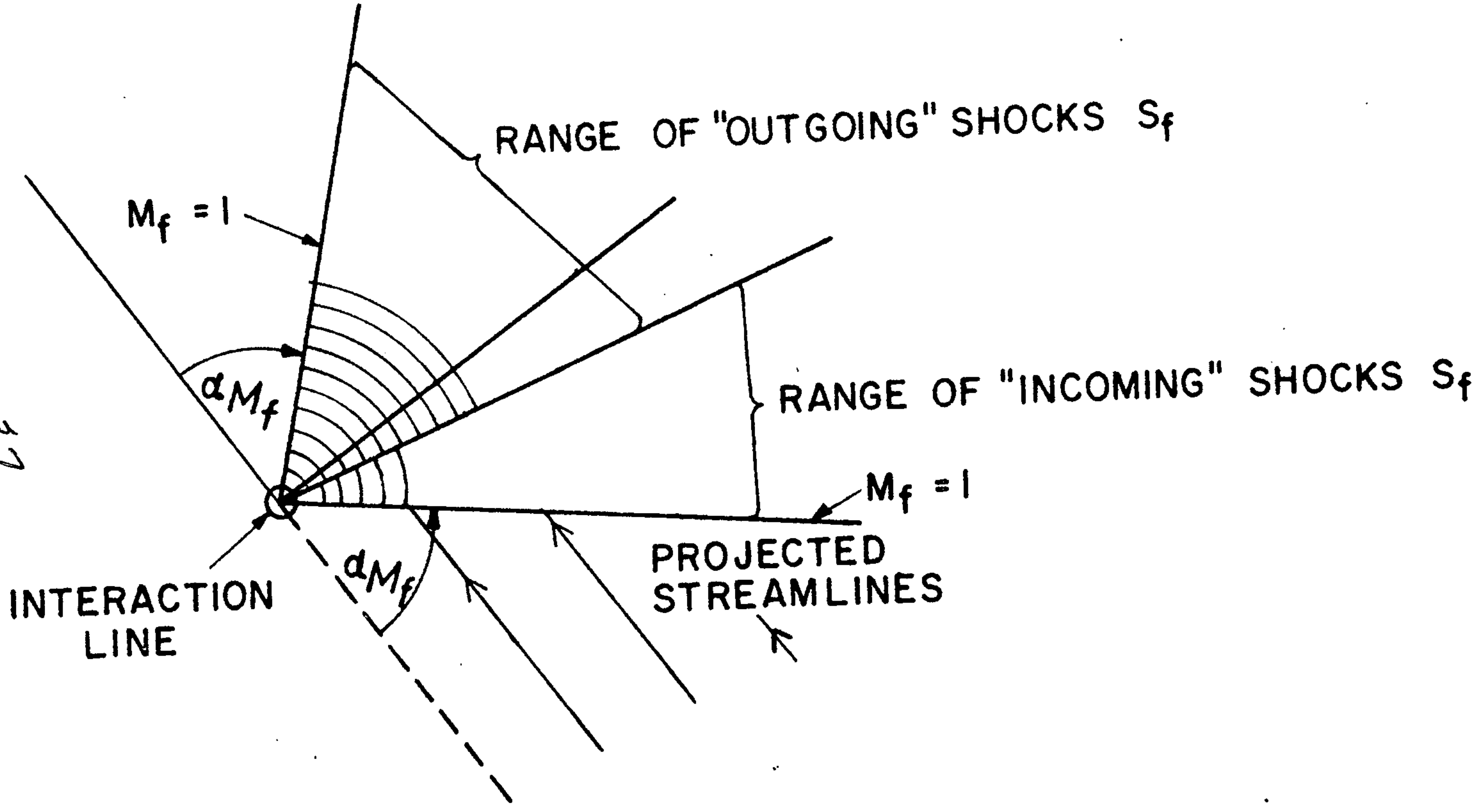
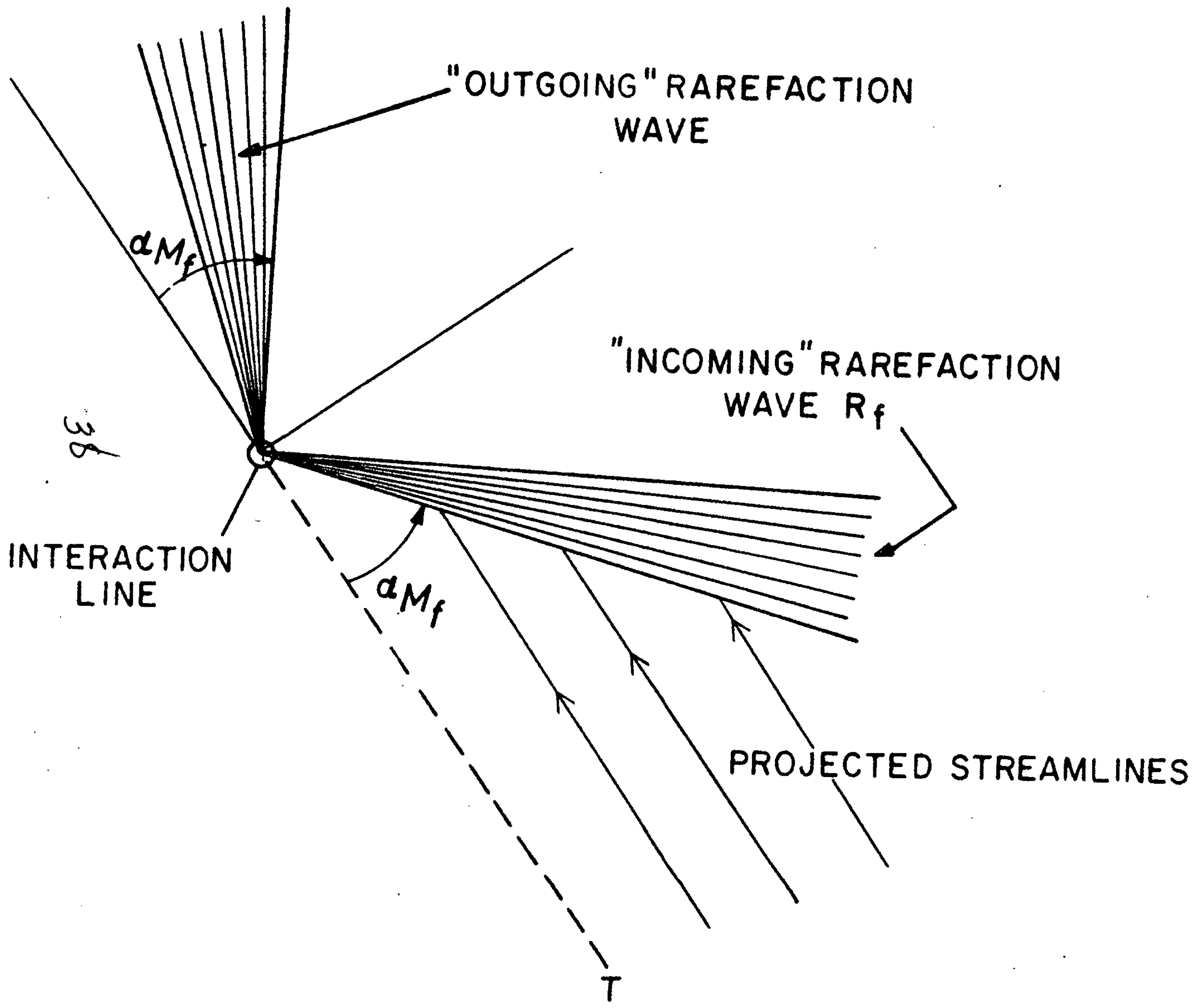
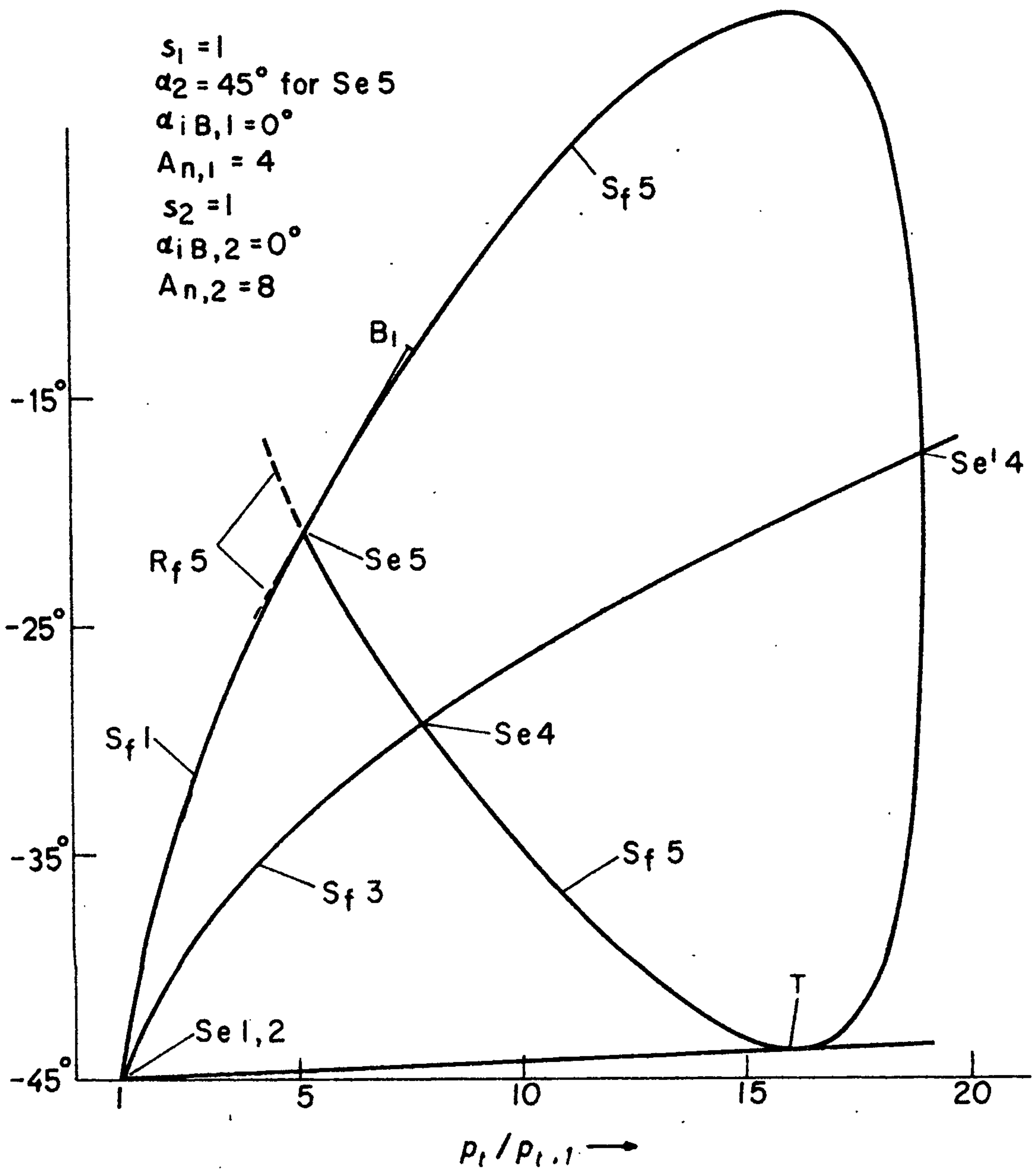

RANGE OF "OUTGOING" SHOCKS S_f
$M_f = 1$
α_{M_f}
RANGE OF "INCOMING" SHOCKS S_f
$M_f = 1$
INTERACTION LINE
α_{M_f}
PROJECTED STREAMLINES

Figure 2a



"OUTGOING" RAREFACTION WAVE
α_{M_f}
"INCOMING" RAREFACTION WAVE R_f
INTERACTION LINE
α_{M_f}
PROJECTED STREAMLINES
T

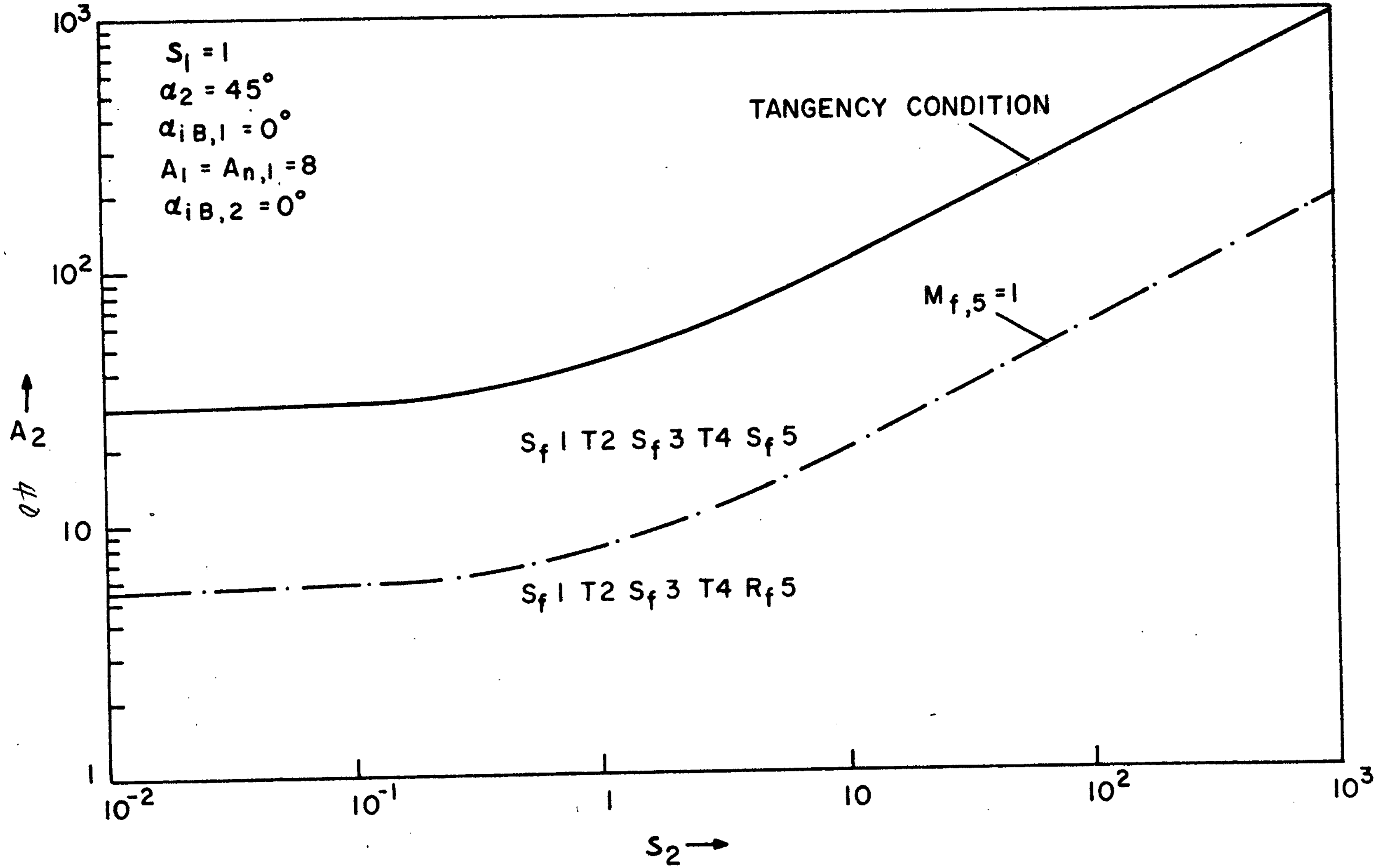
Figure 2b



s1 = 1
α2 = 45° for Se 5
αiB,1 = 0°
An,1 = 4
s2 = 1
αiB,2 = 0°
An,2 = 8
Sf 5
B1
-15°
Se' 4
Rf 5
Se 5
-25°
Se 4
Sf 1
-35°
Sf 3
Sf 5
T
Se 1,2
-45°
1
5
10
15
20
α
pt / pt,1

Figure 3

39



$S_1 = 1$
$\alpha_2 = 45°$
$\alpha_{iB,1} = 0°$
$A_1 = A_{n,1} = 8$
$\alpha_{iB,2} = 0°$
TANGENCY CONDITION
$M_{f,5} = 1$
$S_f 1\ T2\ S_f 3\ T4\ S_f 5$
$S_f 1\ T2\ S_f 3\ T4\ R_f 5$
A_2
S_2
10^3
10^2
10
1
10^{-2}
10^{-1}
1
10
10^2
10^3

Figure 4a

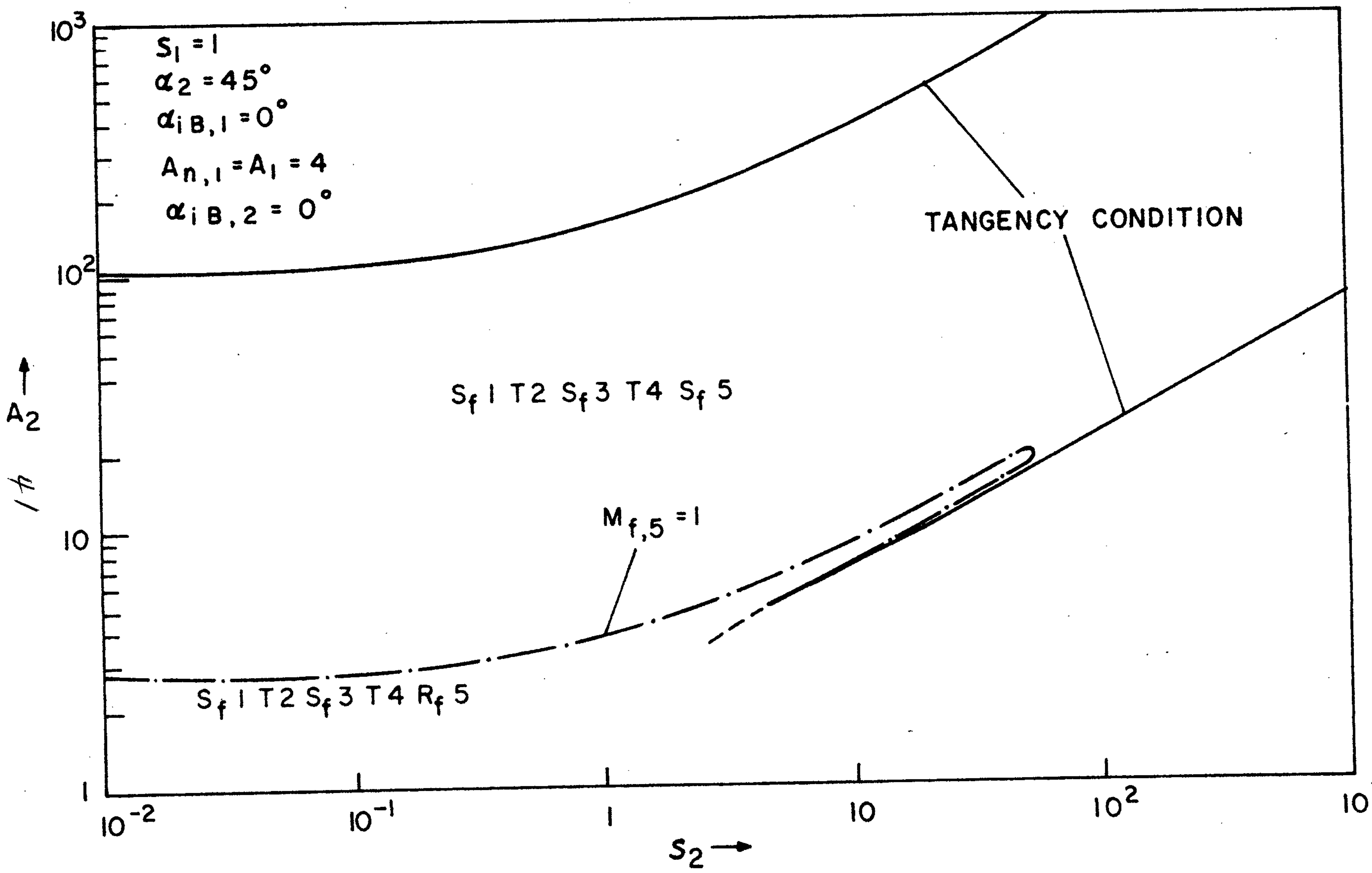

$S_1 = 1$
$\alpha_2 = 45°$
$\alpha_{iB,1} = 0°$
$A_{n,1} = A_1 = 4$
$\alpha_{iB,2} = 0°$
TANGENCY CONDITION
S_f 1 T2 S_f 3 T4 S_f 5
$M_{f,5} = 1$
S_f 1 T2 S_f 3 T4 R_f 5
$A_2 \rightarrow$
$s_2 \rightarrow$
10^3
10^2
10
1
10^{-2}
10^{-1}
1
10
10^2
10^3

Figure 4b

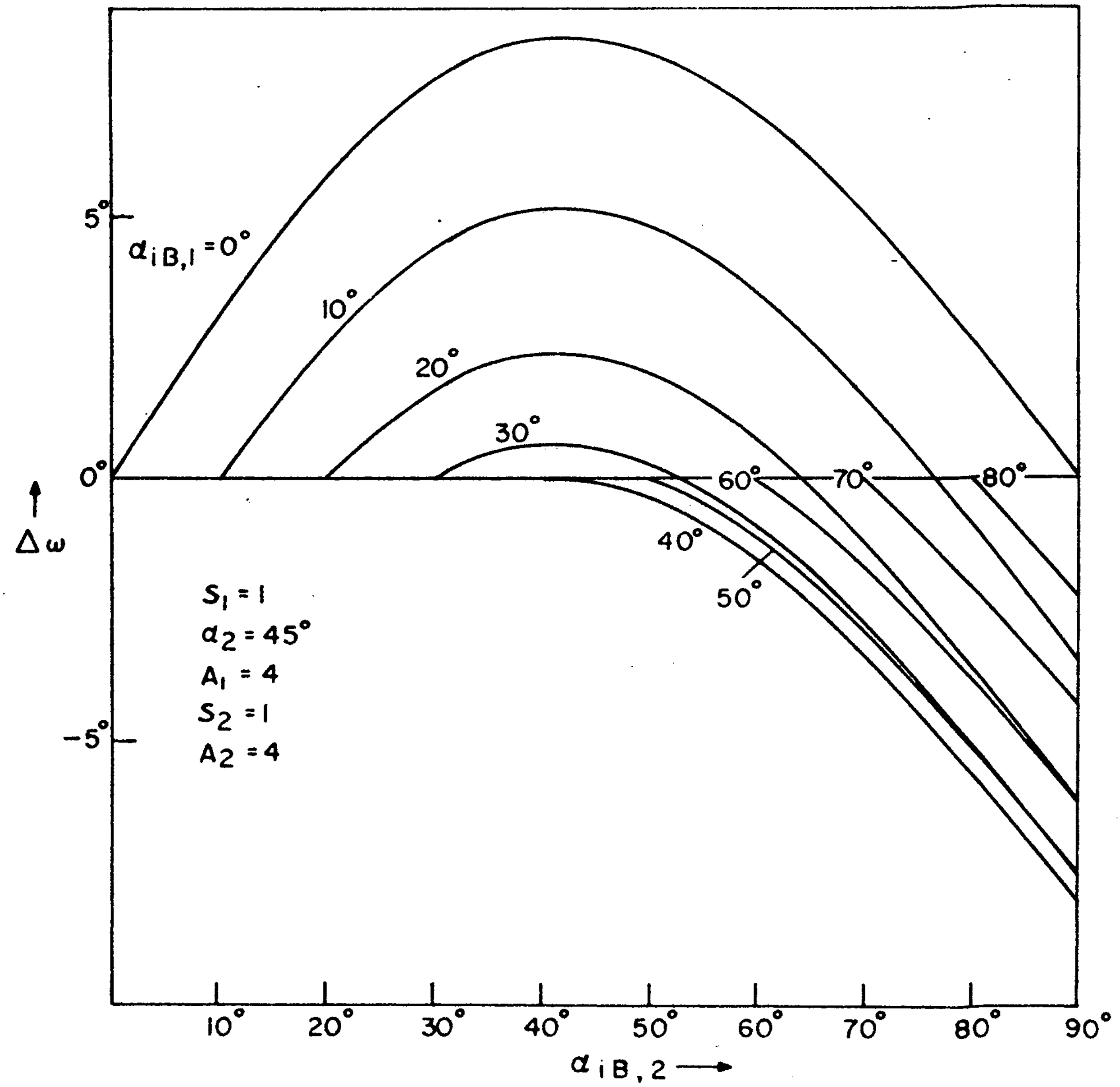
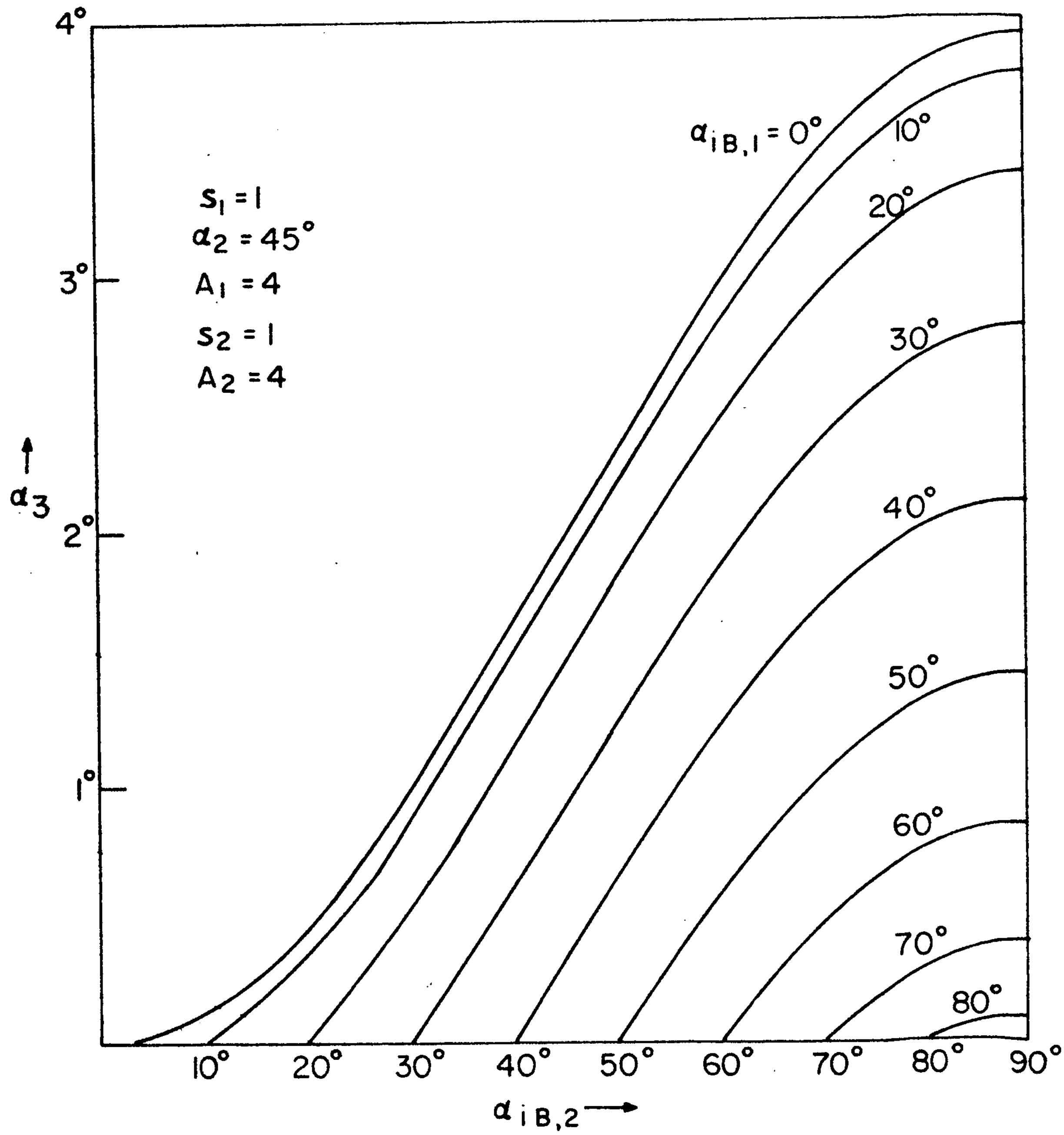
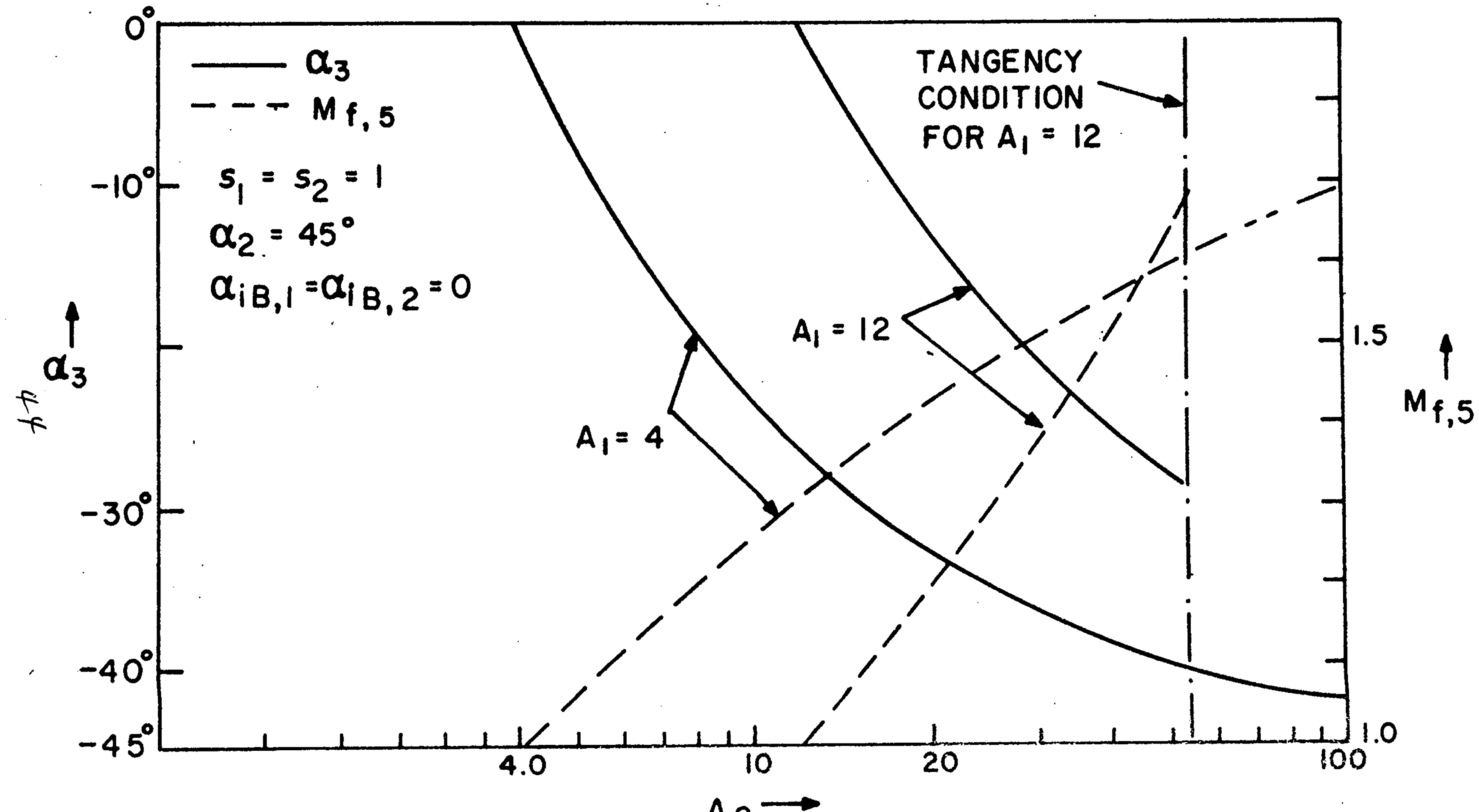

5°
0°
-5°
Δω
$\alpha_{iB,1} = 0°$
10°
20°
30°
40°
50°
60°
70°
80°
$S_1 = 1$
$\alpha_2 = 45°$
$A_1 = 4$
$S_2 = 1$
$A_2 = 4$
10°
20°
30°
40°
50°
60°
70°
80°
90°
$\alpha_{iB,2}$

Figure 5a



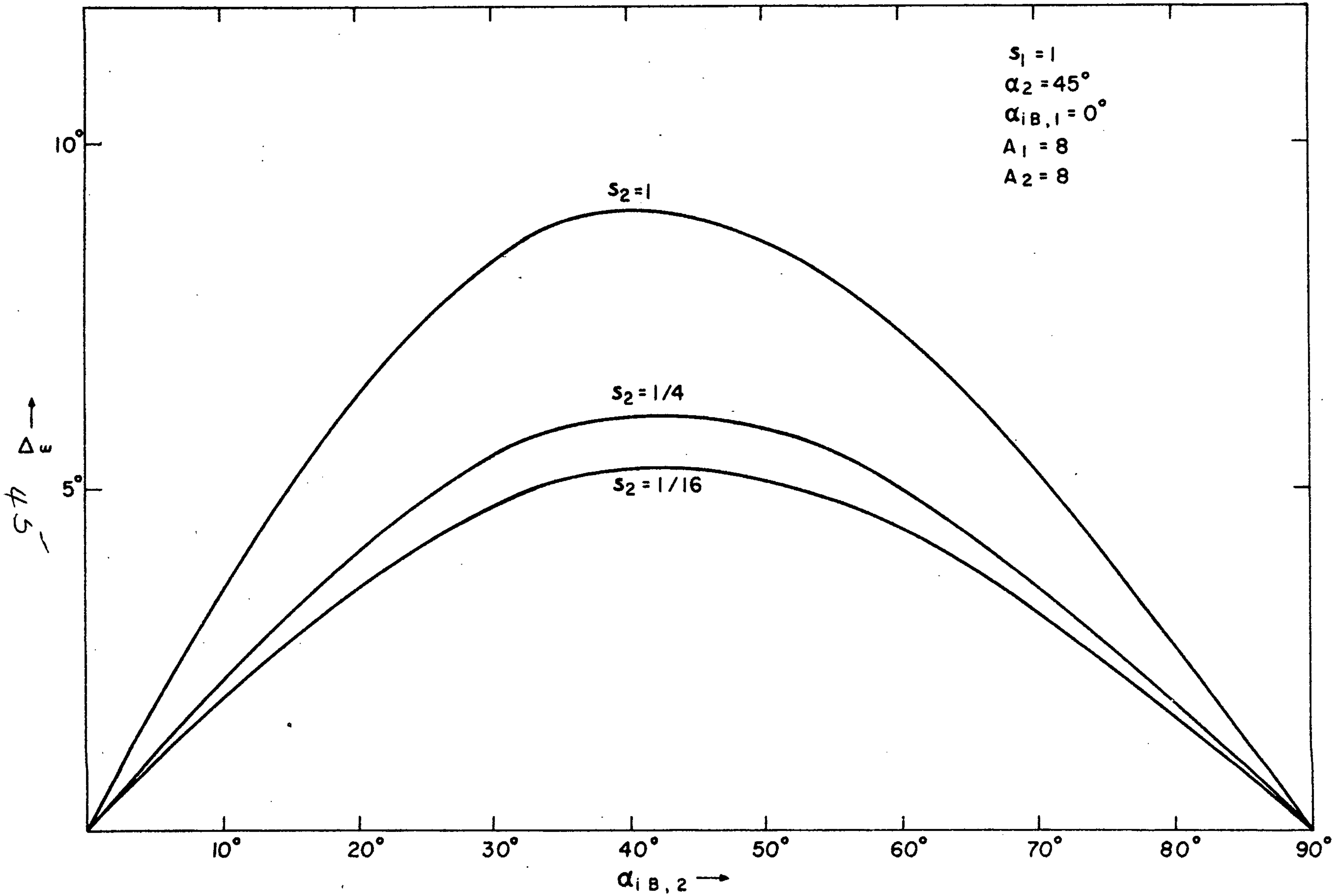
4°
3°
2°
1°
α_3
$s_1 = 1$
$\alpha_2 = 45°$
$A_1 = 4$
$s_2 = 1$
$A_2 = 4$
$\alpha_{iB,1} = 0°$
10°
20°
30°
40°
50°
60°
70°
80°
90°
$\alpha_{iB,2}$

Figure 5b



0°
-10°
-30°
-40°
-45°
α_3
$M_{f,5}$
$s_1 = s_2 = 1$
$\alpha_2 = 45°$
$\alpha_{iB,1} = \alpha_{iB,2} = 0$
α_3
TANGENCY CONDITION FOR $A_1 = 12$
$A_1 = 12$
$A_1 = 4$
1.5
1.0
$M_{f,5}$
4.0
10
20
100
A_2

Figure 6



$s_1 = 1$
$\alpha_2 = 45°$
$\alpha_{iB,1} = 0°$
$A_1 = 8$
$A_2 = 8$
$s_2 = 1$
$s_2 = 1/4$
$s_2 = 1/16$
10°
5°
Δ_ε
10° 20° 30° 40° 50° 60° 70° 80° 90°
$\alpha_{iB,2}$

Figure 7

45

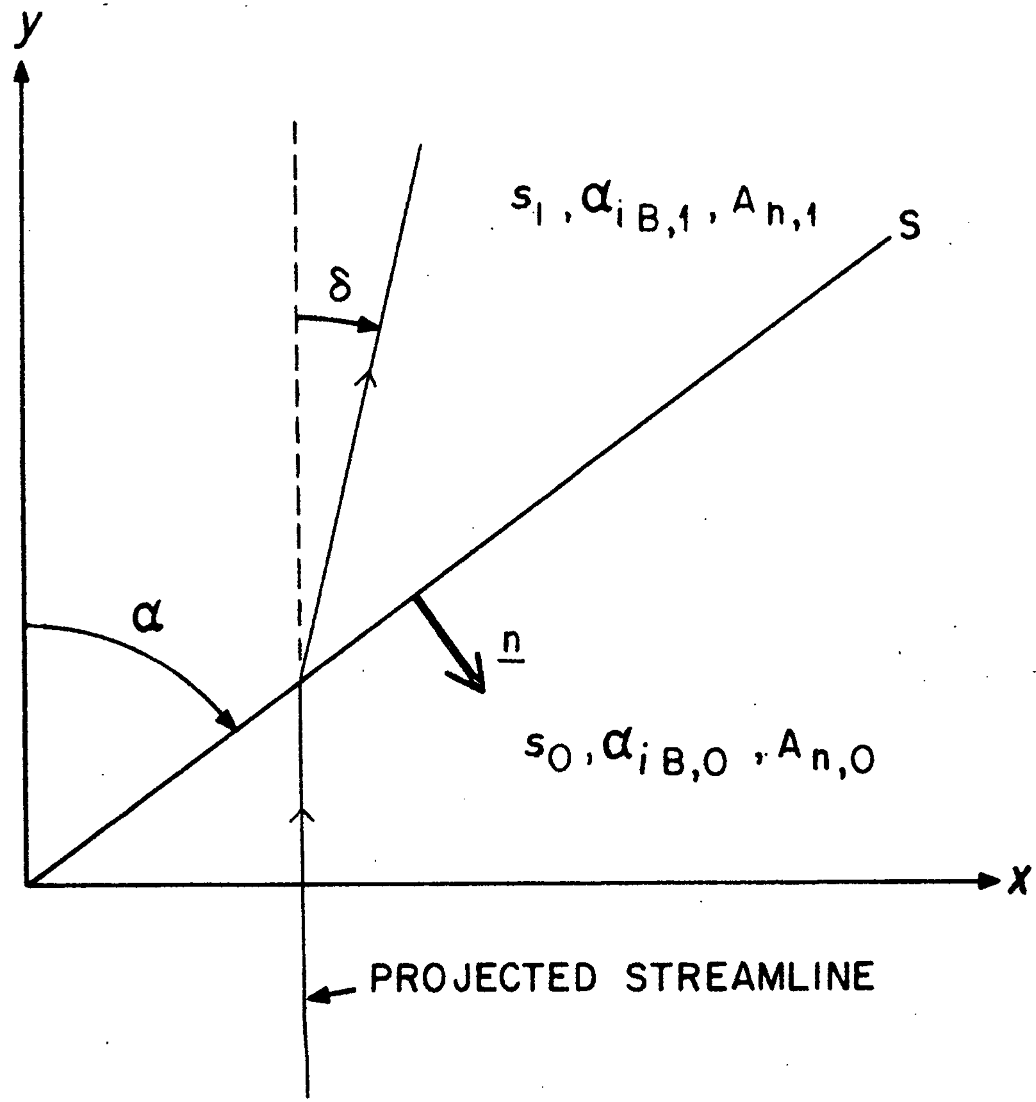


Figure 8