



Nonlinear multi-objective optimization of machining processes parameters

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Abstract

Multi-objective optimization applications in manufacturing of structural components have been mostly in the area of scheduling and process planning. Until now most well developed multi-objective optimization codes are linear. Due to the high level of nonlinearity, few applications have been reported in the area of multi-objective optimization of manufacturing processes parameters. This paper presents a general nonlinear multi-objective optimization model for machining processes parameters. The developed nonlinear multi-objective optimization model allows different priority level for each manufacturing objective. The resulting nonlinear optimization problem is solved using unconstrained optimization techniques. This reduces the effort required to model the manufacturing process problem since linearization is not required. The optimum solution can also be achieved from any starting point, since no feasibility conditions are required. The application and efficiency of the developed model is studied using a machining process test case with different manufacturing priority.

1. Introduction

In real-life optimization problems, the major difficulty is to define the problem in a mathematical programming form in order to find a solution. This is due to the fact that most linear and non-linear optimization techniques deal with a single objective. Most common industrial and engineering optimization problems, However, have multiple and often conflicting objectives. These objectives can be linear or non-linear in nature. In a manufacturing engineering problem, for example, manufacturing goals may be ranked in some specific order of preference or weighted, but it is not easy to select one as the objective function and to form the rest as constraints. Also, as stated before, some goals may be conflicting in nature. Therefore, under these circumstances, traditional optimization techniques cannot be applied to such problems to obtain the optimum solution.

Goal programming is a technique capable of handling weights and priority factors for conflicting multiple objectives or goals [1-6]. In a goal programming (GP) formulation, the goals are defined weights and priority factors are assigned to each. The algorithm then attempts to find the optimum solution within the priority

structure starting with the highest priority goal. This eliminates the difficulty of defining the optimization problem with one objective function and constraints, which makes goal programming an ideal real life optimization tool.

Most development and application of goal programming was directed to linear goal programming and decision making application [7-18]. Due to the high non-linearity of manufacturing process equations, few attempts were made previously to model these processes using goal programming. In this paper a general multi-objective optimization model using non-linear goal programming is developed and applied to machining. The developed model allows different priority levels for each objective. The resulting optimization problem can be solved using unconstrained nonlinear technique with no need for linearization. The developed model results are compared with the results of fuzzy optimization [19].

2. Machining Cost Optimization Model

The machining cost per component is made up of a number of different costs. The single pass case consists of the nonproductive cost per component, the cost of machining time, the tool-changing time cost, and the tool cost per component. For a single pass case with a D_w diameter and l_w length workpiece, the machining cost per component can be written as [19]:

$$C_{pr} = C_m t_h + C_m [t_i + t_m + t_m (t_c + C_e / C_m) / t_s] + C_b / N \quad (1)$$

$$t_m = l_w / (C_v f v / \pi D_w) \quad (2)$$

$$t_s = (\pi D_w l_w) (C_m t_c + C_e) / C_v C_s (v^\phi f^\varphi) \quad (3)$$

where,

C_b / N_b	unit set up and preparation cost for each workpiece
C_m	total machine and operator rate
t_h	handling time for each workpiece
t_i	idle time for each workpiece
t_m	machining time for each workpiece
t_c	tool change time
t_s	tool life
C_c	cost of each tool cutting edge
$C_m t_m t_c / t_s$	unit tool change for each workpiece
$C_m t_m C_c / t_s$	unit cost of a sharp tool edge for each workpiece
$C_s \cdot \phi$ and φ	are cutting operation constants

The power, cutting force, velocity and surface-roughness can be written, for a D_w diameter and l_w length workpiece, as:

$$P_m = \frac{C_p \cdot F_c \cdot v}{\eta_m} \quad (4)$$

$$F_c = C_F \cdot v^\alpha \cdot f^\beta \cdot a \quad (5)$$

$$v = \frac{\pi \cdot D_w \cdot n}{C_v} \quad (6)$$

$$R_a = C_R \cdot v^\gamma \cdot f \quad (7)$$



where,

P_m = the mechanical power

F_c = the cutting force

v = the cutting velocity

η_m = the mechanical efficiency

f = the cutting feed

a = the depth of cut

n = the spindle rotating velocity

$C_p, C_F, C_v, C_R, \alpha, \beta$ and γ are cutting operation constants

The total unit cost for a workpiece must be at least less than C_o , a predetermined aspiration level selected by the job shop manager:

$$C_{pr} \leq C_o$$

subject to:

1. The restriction of the cutting speed.

$$v_{min} \leq v \leq v_{max}$$

2. Consideration of the allowable feed rates.

$$f_{min} \leq f \leq f_{max}$$

3. The constraints of the maximum cutting force.

$$F_c \leq F_c \max$$

4. The constraint of necessary machining power.

$$P_m \leq P \max$$

5. The limited surface-roughness level.

$$R_a \leq R_a \max$$

6. The relevant constraint for depth of cut and feed rate.

$$(a/f)_{min} \leq a/f \leq (a/f)_{max}$$

7. Nonnegativity requirements.

$$f, v \geq 0$$

A general multi-objective optimization model for machining, using nonlinear goal programming, can be written as :

Find the metal cutting process variables $x = (f, v)$ to;

$$\text{Minimize: } z = \begin{pmatrix} \text{weighted deviations of Priority 1} \\ \text{weighted deviations of Priority 2} \\ \vdots \\ \text{weighted deviations of Priority K} \end{pmatrix}$$

subject to:

$$C_{pr} - C_o = \text{deviation 1}$$

$$P_m - P \max = \text{deviation 3}$$

$$F_c - F_c \max = \text{deviation 2}$$

$$R_a - R_a \max = \text{deviation 4}$$

and side constraints,

$$v_{min} \leq v \leq v_{max}$$

$$f_{min} \leq f \leq f_{max}$$

$$(a/f)_{min} \leq a/f \leq (a/f)_{max}$$

$$v, f \geq 0$$

(8)



By using this model in a production environment, the process variables can be adjusted to meet new targets and priority changes. A variety of methods can be used to solve the resulting nonlinear optimization problems. The success of a method over other methods to find a solution may be relative to the particular problem. Due to the high level of nonlinearity, the most suitable approach for solving the nonlinear goal structural optimization problem of Equation (8) is to use a zero-order optimization method. One of the most reliable zero order methods is the Hooke-Jeeves pattern search method [20].

The NLGP algorithm first minimizes, as nearly as possible, the objectives with the highest priority level. It then proceeds to satisfy the objectives of the next priority level, as nearly as possible, without degrading the achievement of any objective in a higher priority level. This process is continued until all priority levels have been considered. The search procedure at each level follows the Hooke-Jeeves algorithm shown by the flow chart of Figure (1).

At each priority level the search is terminated when the difference between present and previous achievement function value becomes sufficiently small. The value of z will be equal to zero vector if all the objectives meet their aspiration levels. The value of z_k will be positive if one or more objectives in priority level k are not met.

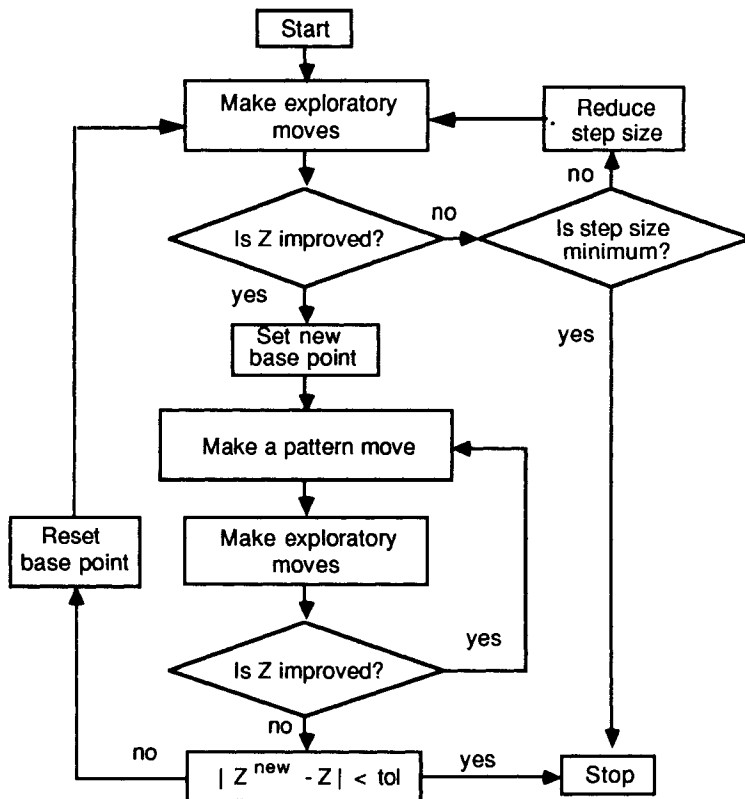


Figure (1) - Hooke-Jeeves Algorithm



3. Test Cases

Two test cases for two different multi-objective machining priorities were performed. The multi-objective machining test case studies were solved using the developed optimization code and the input data given by reference [19]. The data are presented in Tables (1)- (2).

Table (1) - Specifications

Tool: ISO SNMA 120xxx-P20
Holder: ISO PBSNR 2525
Material: SAE 1045 Cd
Machine: ENGINE LATHE
Feed: 0.05-2.5 mm/rev
Speed: 20-1600 rpm
Power: 7.5 kw
at 80% efficiency

Table (2) - Input Parameters

C _m	0.25/min.
C _e	0.50/edge
C _b	7.2/batch
D _w	100 mm
e	55 mm
P	0.1
t _h	1.35 min/pc
t _i	0.2 min/pass
t _c	1.0 min/edge
l _w	250 mm
N _b	25 parts/batch
a	3mm

Case 1:

The first case, for which fuzzy formulation optimization results are available [19]. Using Equation (8), the multi-objective optimization model for this case can be written as:

Find the metal cutting process variables $x = (f, v)$ to;

Minimize:

$$z = (\text{deviation 1} + \text{deviation 2} + \text{deviation 3} + \text{deviation 4})$$

subject to:

$$\begin{array}{rcll} C_{pr} & - & 1.50 & = \text{deviation 1} \\ P_m & - & 6.00 & = \text{deviation 2} \\ F_c & - & 170 & = \text{deviation 3} \\ R_a & - & 1.50 & = \text{deviation 4} \end{array}$$

and side constraints,

$$\begin{array}{rcll} 6.28 & \leq v & \leq & 502.40 \\ 0.05 & \leq f & \leq & 2.50 \\ 1.00 & \leq a/f & \leq & 20.00 \\ v, f & \geq & & 0 \end{array} \quad (9)$$

The nonlinear goal programming problem was solved with one priority level for all objectives in order to compare the optimum solution with the values of the fuzzy formulation of reference [19].

Case 2:

This case study has unit cost as the first priority. This situation represents minimizing the unit cost as the most important parameter in the cutting process. Keeping the cutting power, the cutting force, and roughness as close as possible to their targets is a second priority. Using Equation (8), the multi-objective optimization model for unit cost as the highest priority can be written as:

Find the metal cutting process variables $x = (f, v)$ to;

Minimize:

$$z = \left(\begin{array}{c} \text{deviation 1} \\ \text{deviation 2 and deviation 3 and deviation 4} \end{array} \right)$$

subject to:

$$\begin{array}{rcll} C_{pr} & - & 1.50 & = \text{deviation 1} \\ P_m & - & 6.00 & = \text{deviation 2} \\ F_c & - & 170 & = \text{deviation 3} \\ R_a & - & 1.50 & = \text{deviation 4} \end{array}$$

and side constraints,

$$\begin{array}{rcll} 6.28 & \leq v & \leq & 502.40 \\ 0.05 & \leq f & \leq & 2.50 \\ 1.00 & \leq a/f & \leq & 20.00 \\ v, f & \geq & & 0 \end{array} \quad (10)$$

The optimum results of the two test cases, with the results of the fuzzy formulation of reference [19], are presented in Table (3).

Table (3) - Multi-Objective Results Compared with Fuzzy Optimization

Method	Feed mm/rev.	Velocity m. /min.	Unit Cost dollars
Fuzzy Optimum	0.207	316.68	1.65
Multi- Objective 1-Priority	0.350	332.56	1.63
Multi- Objective 2-Priorities	0.500	331.69	1.61

4. Conclusion

From Table (3), the comparison between the nonlinear goal programming and the fuzzy optimization results shows a slight decrease in unit cost due to slight increase in both cutting speed and feed. The slight difference between the two solutions is due to the replacement of the fuzzy constraints by the crisp definition of the objectives in the multi-objective formulation. The fuzzy optimization solution can be considered as the fuzzy alternative or the approximation of the goal programming solution. Further reduction of cost is achieved in comparison with the single objective solution, when unit cost is considered as the highest priority. In the machine shop this reduction could be achieved by increasing the cutting feed with less than 1 m./min. decrease in the cutting velocity. The advantages of goal programming over fuzzy optimization can be summarized as:

1. Goal programming allows for different priority levels while fuzzy optimization allows only one priority level.
2. Due to the ability of goal programming to reach to the targets, in each objective, as close as possible fuzzy definitions may not be necessary.
3. By using unconstrained optimization for solving the goal programming problem a solution can be achieved from any starting point. The fuzzy optimization algorithm of reference [19], is based on constrained optimization for which an optimum solution is not always guaranteed.

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