



ISNM
International Series of Numerical Mathematics
Vol. 135

Managing Editors:
K.-H. Hoffmann, München
D. Mittelmann, Tempe

Associate Editors:
R. E. Bank, La Jolla
H. Kawarada, Chiba
R. J. LeVeque, Seattle
C. Verdi, Milano

Honorary Editor:
J. Todd, Pasadena

Nonlinear Multiobjective Optimization

A Generalized Homotopy Approach

Claus Hillermeier

Author:

Claus Hillermeier
Siemens AG
ZT PP2
81730 München (Perlach)
Germany

until August 2001:
Chair of Applied Mathematics II
University of Erlangen-Nürnberg
Martensstr. 3
91058 Erlangen
Germany

2000 Mathematics Subject Classification 74P20, 58E17, 90C29, 65H20

A CIP catalogue record for this book is available from the Library of Congress, Washington D.C., USA

Deutsche Bibliothek Cataloging-in-Publication Data

Hillermeier, Claus:

Nonlinear multiobjective optimization : a generalized homotopy approach / Claus Hillermeier.

- Basel ; Boston ; Berlin : Birkhäuser, 2001

(International series of numerical mathematics ; Vol. 135)

ISBN 978-3-0348-9501-9 ISBN 978-3-0348-8280-4 (eBook)

DOI 10.1007/978-3-0348-8280-4

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, broadcasting, reproduction on microfilms or in other ways, and storage in data banks. For any kind of use whatsoever, permission from the copyright owner must be obtained.

© 2001 Springer Basel AG

Originally published by Birkhäuser Verlag, in 2001

Softcover reprint of the hardcover 1st edition 2001

Printed on acid-free paper produced of chlorine-free pulp. TCF ∞

ISBN 978-3-0348-9501-9

9 8 7 6 5 4 3 2 1

Dedicated to my parents

Preface

Real industrial systems are usually assessed by setting several objectives which are often competing with each other. Good compromise solutions are then looked for. The task of multiobjective optimization is to determine so-called efficient (or Pareto optimal) solutions which cannot be improved simultaneously with regard to all objectives.

The present book first gives a survey of the principles and classical methods of multiobjective optimization. Afterwards, the set of Pareto candidates is considered as a differentiable manifold, and a local chart is constructed which is fitted to the local geometry of this Pareto manifold. This opens up the possibility of generating new Pareto candidates by evaluating that local chart numerically. The generalized homotopy method thus developed has important advantages. It is capable of solving multiobjective optimization problems with an arbitrary number k of objectives, enables the generation of all types of Pareto optimal solutions and is able to produce a homogeneous discretization of the Pareto set.

In the theoretical part of the book, the homotopy method is put on a sound mathematical basis by providing a necessary and sufficient condition for the set of Pareto candidates to form a $(k - 1)$ -dimensional differentiable manifold. The theoretical discussion is followed by a description of the numerical details of the proposed homotopy algorithm. Finally, by solving three multiobjective sample problems we demonstrate how this algorithm works in practice. Two of these problems originate in optimization applications within the configuration of industrial systems.

Acknowledgements

First of all I wish to express my gratitude to Prof. Dr. Dr. h. c. Karl-Heinz Hoffmann for encouraging and supporting the piece of research presented here. I would like to thank Prof. Dr. Klaus Ritter and Prof. DDr. Stefan Schäffler for several fruitful discussions which were a pleasure and a great help. Special thanks also go to my colleagues at Siemens Corporate Technology and to our coach Prof. Dr. Albert Gilg for creating an enjoyable and stimulating working atmosphere. With gratitude I would like to mention the successful and pleasant collaboration with my colleagues at Siemens KWU. I wish to express my appreciation to Prof. Dr. Johannes Jahn for revising parts of the manuscript and providing valuable comments. Last, but not least, I am indebted to Rudolf Knop for his help with the English translation and to Dr. Michael Greiner for generously providing his TEX-expertise.

The work presented here has been supported by the German “Bundesministerium für Bildung und Forschung” in the framework of the project LEONET. This support is gratefully acknowledged.

Contents

1	Introduction	3
2	Vector Optimization in Industrial Applications	9
2.1	The Design of a Combined-Cycle Power Plant	10
2.2	The Optimal Operating Point of a Recovery-Boiler	12
3	Principles and Methods of Vector Optimization	15
3.1	The Concept of Pareto Optimality	15
3.2	Survey of Methods	19
3.3	A New Stochastic Method for Unconstrained Vector Optimization	30
3.3.1	A Curve of Dominated Points	31
3.3.2	Notions from Probability Theory	37
3.3.3	A Special Stochastic Differential Equation	39
3.3.4	A Stochastic Algorithm for Vector Optimization	42
4	The Connection with Scalar-Valued Optimization	45
4.1	The Karush-Kuhn-Tucker(KKT) Condition for Pareto Optimality	45
4.2	Differential-Topological Notations	47
4.3	The Geometrical Meaning of the Weight Vector	53
4.4	Classification of Efficient Points	59
5	The Manifold of Stationary Points	65
5.1	Karush-Kuhn-Tucker Points as a Differentiable Manifold M	66
5.2	Criteria for the Rank Condition	68
5.2.1	A Necessary and Sufficient Criterion	68
5.2.2	Interpretation in View of Optimization	71
5.2.3	Variability of the Weight Vector	75
5.3	A Special Class of Local Charts	79
6	Homotopy Strategies	87
6.1	Method I: Local Exploration of M	88
6.1.1	Method Principle	88

6.1.2	Comparison with the Classical Homotopy Method	89
6.1.3	Homogeneous Discretization of the Efficient Set	93
6.1.4	Numerical Algorithm	95
6.2	Method II: Purposeful Change of the Weights	99
6.2.1	Significance of the Weight Vector for the User	99
6.2.2	Principle of the Procedure	101
6.2.3	Numerical Algorithm	104
7	Numerical Results	109
7.1	Example 1 (academic)	109
7.2	Example 2: Design of a Combined-Cycle Power Plant	115
7.3	Example 3: The Optimal Operating Point of a Recovery-Boiler . .	123
	Bibliography	129
	Index	133