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Nonlinear PD Controllers with Gravity Compensation for Robot Manipulators

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Abstract: *A Nonlinear Proportional-Derivative (NPD) controller with gravity compensation is proposed and applied to robot manipulators in this paper. The proportional and derivative gains are changed by the nonlinear function of errors in the NPD controller. The closed-loop system, composed of nonlinear robot dynamics and NPD controllers, is globally asymptotically stable in position control of robot manipulators. The comparison of the simulation experiments in the position control (the step response) of a robot manipulator with two degrees of freedom is also presented to illustrate that the NPD controller is superior to the conventional PD controller in a position control system. The experimental results show that the NPD controller can obtain a faster response velocity and higher position accuracy than the conventional PD controller in the position control of robot manipulators because the proportional and derivative gains of the NPD controller can be changed by the nonlinear function of errors. The NPD controller provides a novel approach for robot control systems.*

Keywords: *Nonlinear proportional-derivative controller, proportional-derivative controller, robot manipulator, position control.*

1. Introduction

Robot dynamics are highly nonlinear because of the coupling between joints. Due to the parametric uncertainties in the system dynamics, it is difficult to derive the exact description of the system. The position control (also called a regulation problem) is one of the most relevant issues in the operation of robot manipulators. This is a particular case of the motion control or trajectory control. The primary goal of the motion control in the points space is to make the robot joints track a given time-varying desired joint position. Takegaki and Arimoto [1], Arimoto and Miyazaki [2] showed that simple controllers, such as the Proportional-Derivative (PD) controller and Proportional-Integral-Derivative (PID) feedback controller are efficient for general control, despite the nonlinearity and uncertainty of the robot dynamics. In recent years, various linear PD- or PID-type control schemes have been extended to a nonlinear PID control strategy. A PD controller with stability robustness in the presence of parametric uncertainty in the gravitational torque vector was presented by Hsia [3]. A class of nonlinear PD-type controllers for robot manipulators was proposed by Kelly and Carelli [4]. Seraji [5] presented the analysis and design of a nonlinear PID control with an extension to tracking. Bucklaew and Liu [6] also proposed a nonlinear gain structure for PD-type controllers in robotic applications. Furthermore, Reyes and Rosado [7] proposed a polynomial family of PD-type controllers for robot manipulators. However, these PID controllers are difficult to determine the appropriate PID gains in case of nonlinear and unknown controlled plants, and then the PID controller with fixed parameters may usually deteriorate the control performance. Therefore, various types of PID control have been developed by means of neural networks [8-12]. However, neural networks may be difficult to reach the real-time control of robot systems due to quick learning problems on line.

The strategy of PID control has been one of the most sophisticated and most frequently used methods in industry. This is because the PID controller has a simple form and strong robustness under broad operating conditions. However, the conventional PID controller may usually deteriorate the control performance in nonlinear control systems. The nonlinear PID-type controllers have been proved to be a promising approach to solve nonlinear control problems and are adapted to the control of robot manipulators, because the nonlinear PID controllers have the nonlinear characteristics and advantages of PID controllers. Hence, the aim of this paper is to propose a Nonlinear Proportional-Derivative (NPD) controller with gravity compensation to control robot manipulators, which leads to global asymptotic stability of the closed-loop system (dynamics model of a robot manipulator plus controllers). This paper is organized as follows. Section 2 discusses the robot dynamics. In Section 3 a NPD controller with gravity compensation is presented based on variable proportional and derivative gains corresponding to the error. Section 4 contains the simulation experimental comparison between the NPD controller and the conventional PD controller on a robot arm with two degrees of freedom. The discussion of the experimental results is given in Section 5. Finally, some conclusions and future work are offered in Section 6.

2. Robot dynamics

The dynamics of a serial *n*-link rigid robot can be written as [7]:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{F}(\dot{\boldsymbol{q}}) = \tau, \tag{1}$$

where $q, \dot{q}, \ddot{q}$ are the $n\times1$ vectors of the joint displacement, velocity, and acceleration; $\boldsymbol{\tau}$ is the $n\times1$ vector of input torques; $\boldsymbol{M}(\boldsymbol{q})$ is the $n\times n$ symmetric positive definite manipulator inertia matrix; $\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is the $n\times n$ matrix of centripetal and Coriolis torques; $\boldsymbol{G}(\boldsymbol{q})$ is the $n\times1$ vector of gravitational torques obtained as the gradient of the robot potential energy due to gravity; $\boldsymbol{F}(\dot{\boldsymbol{q}})$ is the $n\times1$ vector for the friction torques. The matrix $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ and the time derivative $\dot{\boldsymbol{M}}(\boldsymbol{q})$ of the inertia matrix satisfy:

$$\dot{\boldsymbol{q}}^{\mathrm{T}}\left[\frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) - \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\right]\dot{\boldsymbol{q}} = 0. \tag{2}$$

3. NPD controllers

We introduce the design ideas for a NPD controller in the dynamic process of a control system, described as follows.

For the proportional gain k_p, when the control error is increased, k_p is increased under keeping the response velocity without the overshoot; while the control error is decreased, k_p is decreased to decrease the overshoot and to rapidly reach a stable point under an adequate k_p. According to the requirements, we select the shape of the gain k_p with respect to the change of the control error e as shown in Fig. 1.

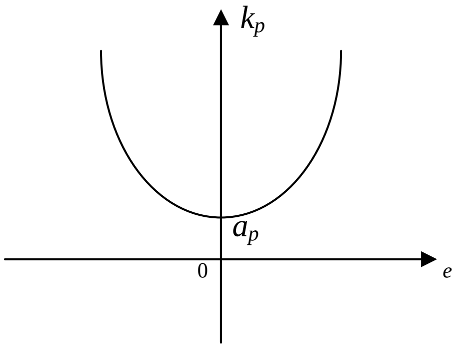

Fig. 1. Variable curve of the proportional gain k_p corresponding to the control error e

Based on Fig. 1, we present the following nonlinear function:

$$k_p(e) = a_p + b_p e_p^2, \tag{3}$$

where a_p and b_p are parameters, which can change the k_p curve.

For the derivative gain k_d, when the control error is increased, k_d is decreased to keep the response velocity without the overshoot; while the control error is decreased, k_d is increased to decrease the overshoot. According to the requirements, we select the shape of the gain k_d with respect to the change of the control error e, as shown in Fig. 2.

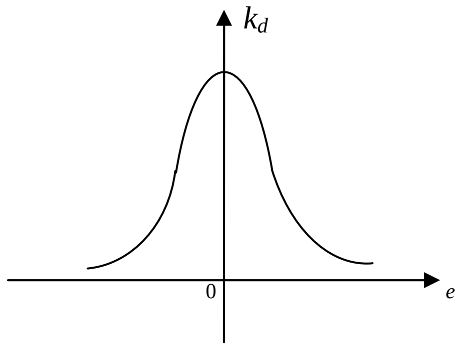

Fig. 2. Variable curve of the derivative gain k_d corresponding to the error e

Based on Fig. 2, we present the following nonlinear function:

$$k_d(e) = \frac{a_d}{0.001 + e^2}, \quad (4)$$

where a_d is a parameter. The denominator takes the smaller value 0.001,when $e = 0$ in (4) to overcome the undefined function.

The NPD controller for n degrees of freedom for the robot arm is presented by the following control scheme with gravity compensation:

$$\boldsymbol{\tau} = \boldsymbol{K}_p(\boldsymbol{e})\boldsymbol{e} - \boldsymbol{K}_d(\boldsymbol{e})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{F}(\dot{\boldsymbol{q}}), \quad (5)$$

where $\boldsymbol{e} \in R^{n\times 1}$ is the position error vector which is defined as $\boldsymbol{e} = \boldsymbol{q_d} - \boldsymbol{q}$, $\boldsymbol{q_d} \in R^{n\times 1}$ representing the desired joint position, $\boldsymbol{K_p} \in R^{n\times n}$ is the proportional gain which is diagonal matrix, and $\boldsymbol{K_d} \in R^{n\times n}$ is the derivative gain which is diagonal matrix.

For the nth joint of the robot arm, the following controller can be obtained from (5):

$$\begin{aligned} \tau_{\mathrm{NPD}n} &= k_{pn}(e_n)e_n - k_{dn}(e_n)\dot{q}_n + g_n(q) + f_n(\dot{q}) = \\ &= (a_{pn} + b_{pn}e_n^2)e_n - \frac{a_{dn}}{0.001 + e_n^2}\dot{q}_n + g_n(q) + f_n(\dot{q}), \end{aligned} \quad (6)$$

where $\tau_{\mathrm{NPD}n}$ is the output torque of the nth NPD controller, which drives the nth joint in the robot arm.

4. Simulation example

In this section, to verify the control efficiency of the proposed NPD controllers for the robot arm, the proposed NPD controllers are employed in the position control of a two-link robot as shown in Fig. 3.

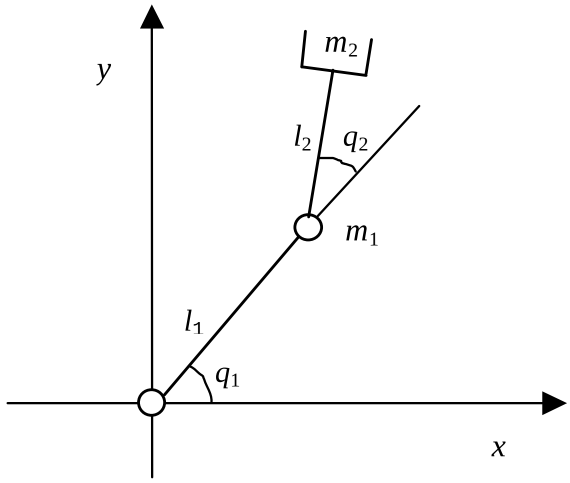

Fig. 3. A two-link robot

In Fig. 2 m_1 and m_2 are masses of arm 1 and arm 2, respectively; l_1 and l_2 are lengths of arm 1 and arm 2; τ_1 and τ_2 are driven torques on arm 1 and arm 2; q_1 and q_2 are positions of arm 1 and arm 2. The dynamics model of the two-link robot is the same as (1).

Let

$$\boldsymbol{q} = [q_1, q_2]^{\mathrm{T}}, \quad \dot{\boldsymbol{q}} = [\dot{q}_1, \dot{q}_2]^{\mathrm{T}}, \quad \ddot{\boldsymbol{q}} = [\ddot{q}_1, \ddot{q}_2]^{\mathrm{T}}, \quad \boldsymbol{\tau} = [\tau_1, \tau_2]^{\mathrm{T}},$$

$$c_i = \cos(q_i),\ s_i = \sin(q_i),\ c_{ij} = \cos(q_i + q_j),\ s_{ij} = \sin(q_i + q_j),$$

then $\boldsymbol{M}$, $\boldsymbol{V}$, $\boldsymbol{G}$ in (1) can be described as

$$\boldsymbol{M}(\boldsymbol{q}) = \begin{bmatrix} m_1 l_1^2 + m_2(l_1^2 + l_2^2 + 2l_1 l_2 c_2) & m_2 l_2^2 + m_2 l_1 l_2 c_2 \\ m_2 l_2^2 + m_2 l_1 l_2 c_2 & m_2 l_2^2 \end{bmatrix},$$

$$\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \begin{bmatrix} -m_2 l_1 l_2 s_2 \dot{q}_2^2 - 2m_2 l_1 l_2 s_2 \dot{q}_1 \dot{q}_2 \\ m_2 l_1 l_2 s_2 \dot{q}_1^2 \end{bmatrix},$$

$$\boldsymbol{G}(\boldsymbol{q}) = \begin{bmatrix} m_2 l_2 g c_{12} + (m_1 + m_2) l_1 g c_1 \\ m_2 l_2 g c_{12} \end{bmatrix}.$$

In this case the parameters of the two-link robot are m_1 = 10 kg, m_2 = 3 kg and l_1 = 1.1 m, l_2 = 0.8 m.

To support the theoretical developments, this section presents an experimental comparison of the two position controllers on a two-degree-of-freedom direct drive robot manipulator, where the servo motors directly drive the joints without gear reduction. The advantages of this type of a direct-drive actuator include freedom from a backlash and significantly lower joint friction compared to the actuators composed by gear drives [7]. To investigate the performance between controllers, they are classified as τ_{NPD} for NPD controller and τ_{PD} for the conventional PD controller. The applied torques of the actuators for joints 1 and 2 are chosen so that $\tau_{1\max} \le 600$ N.m and $\tau_{2\max} \le 200$ N.m, respectively, by practical considerations, because it can also produce torque saturation of the actuators.

An experiment of the position control is designed to compare the performance of the controllers in a direct-drive robot. The experiment consists of moving the end-effector from its initial position to a desired target (step response). For the present application the desired point positions are chosen as: $[q_{d1}, q_{d2}]^T = [1, 1.5]^T$ radians, the initial positions and velocities are set to zero (for example, at home position). The friction phenomena and disturbances are not modeled for compensation purposes. That is, all the controllers do not show any type of friction and disturbance compensations. Therefore, they consider the friction and disturbance as unmodelled dynamics. The friction forces of the joints and the disturbances are assumed (in N.m) as

$$\boldsymbol{F}(\dot{\boldsymbol{q}}) = \begin{bmatrix} 3\dot{q}_1 + 0.5\text{sign}(\dot{q}_1) \\ 2\dot{q}_2 + 0.5\text{sign}(\dot{q}_2) \end{bmatrix}, \quad \boldsymbol{T}_d(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \begin{bmatrix} 5\cos(q_1) \\ 5\cos(q_2) \end{bmatrix}.$$

Simulation experiments are carried out by using the NPD controller and the conventional PD controller to select their gains according to the method in [7], such that the best time response without an overshoot and a minimal steady-state position error are obtained without going into the saturation zone of the actuator's torques. The final values of all simulation parameters are shown in Table 1.

Table 1. Simulation parameters of the position control for a robot manipulator with two-joints

Controllers	Joint 1					Joint 2				
	a_{p1}	b_{p1}	a_{d1}	k_{p1}	k_{d1}	a_{p2}	b_{p2}	a_{d2}	k_{p2}	K_{d2}
NPD	90	560	14	/	/	20	950	0.6	/	/
PD	/	/	/	460	160	/	/	/	115	28

4.1. Simulation experiment of the NPD controllers

The position control of a two-degree-of-freedom direct drive robot manipulator uses the following NPD controllers:

$$\tau_{\text{NPD1}} = (a_{p1} + b_{p1}e_1^2)e_1 - \frac{a_{d1}}{0.001 + e^2}\dot{q}_1 + 23.52\cos(q_1 + q_2) + 140.14\cos(q_1), \tag{7}$$

$$\tau_{\text{NPD2}} = (a_{p2} + b_{p2}e_2^2)e_2 - \frac{a_{d2}}{0.001 + e_2^2}\dot{q}_2 + 23.52\cos(q_1 + q_2), \tag{8}$$

where τ_{NPD1} and τ_{NPD2} represent the applied torques for the two joints. The experimental parameters are shown in Table 1.

The experimental results of the step response of joint 1 and joint 2 are shown in Fig. 4 under NPD control (τ_{NPD1} and τ_{NPD2}) without the overshoot. On the other hand, the position errors of the NPD controllers (7) and (8), corresponding to two joints are depicted in Fig. 5, which demonstrates the convergence properties for each controller.

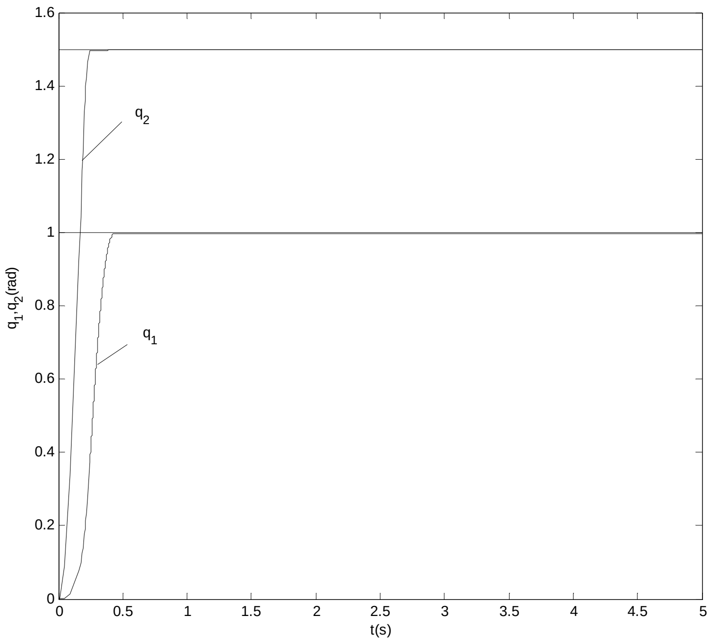

Fig. 4. Step response of the NPD control

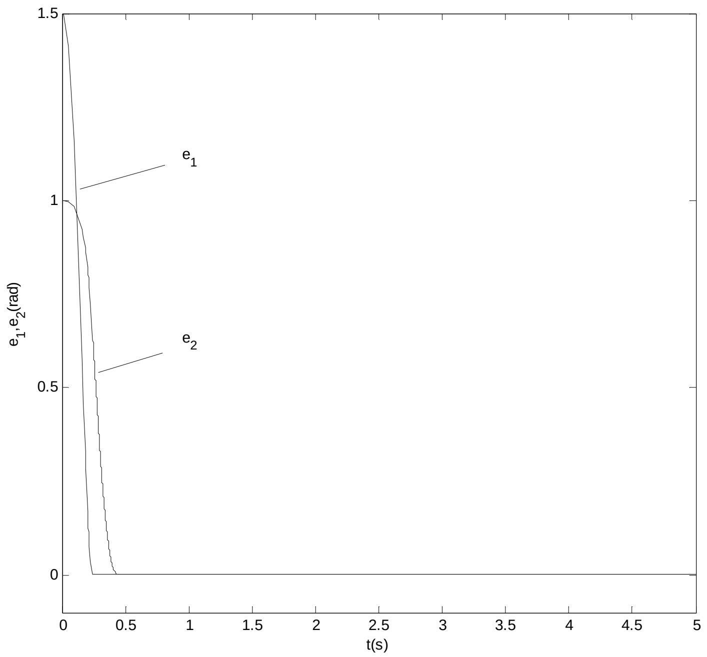

Fig. 5. Position errors of the NPD control

4.2. Simulation experiment of the conventional PD controllers

For the simulation experiment of conventional PD controllers, the desired position and initial conditions are the same as in the previous experiment. The PD controllers for the robot arm (n = 2) are represented by the following equations:

$$\tau_{\text{PD1}} = k_{p1}e_1 - k_{d1}\dot{q}_1 + 23.52\cos(q_1 + q_2) + 140.14\cos(q_1)\,, \tag{9}$$

$$\tau_{\text{PD2}} = k_{p2}e_2 - k_{d2}\dot{q}_2 + 23.52\cos(q_1 + q_2)\,, \tag{10}$$

where τ_{PD1} and τ_{PD2} represent the applied torques for joints 1 and 2, respectively. The experimental parameters are shown in Table 1.

The step response for joints 1 and joint 2 is shown in Fig. 6 under PD control without the overshoot. Fig. 7 contains the experimental results of the position errors.

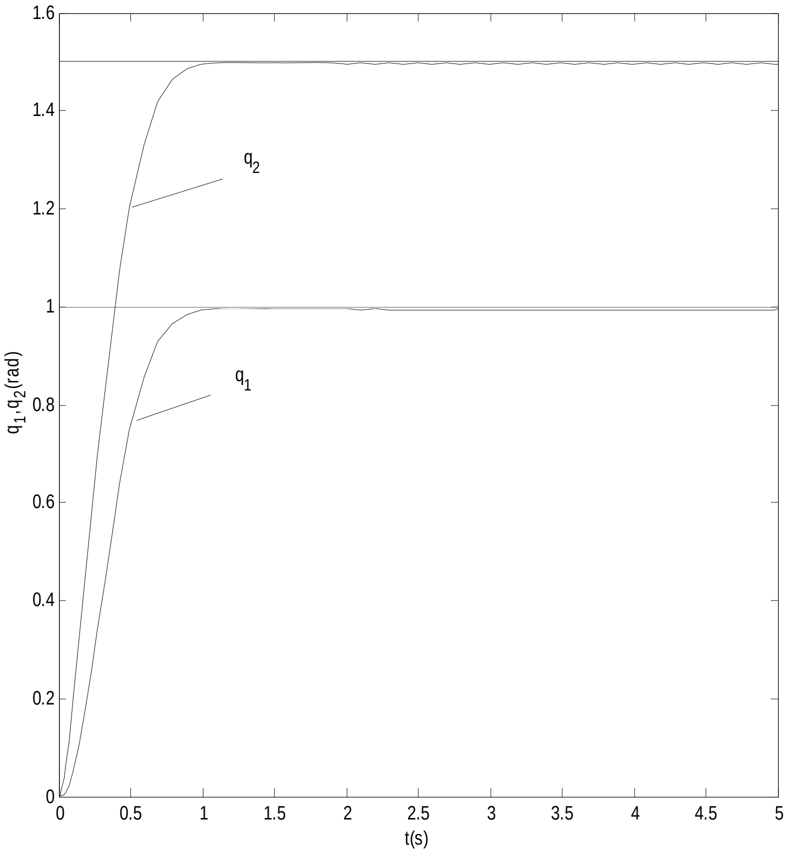

Fig. 6. Step response of the PD control

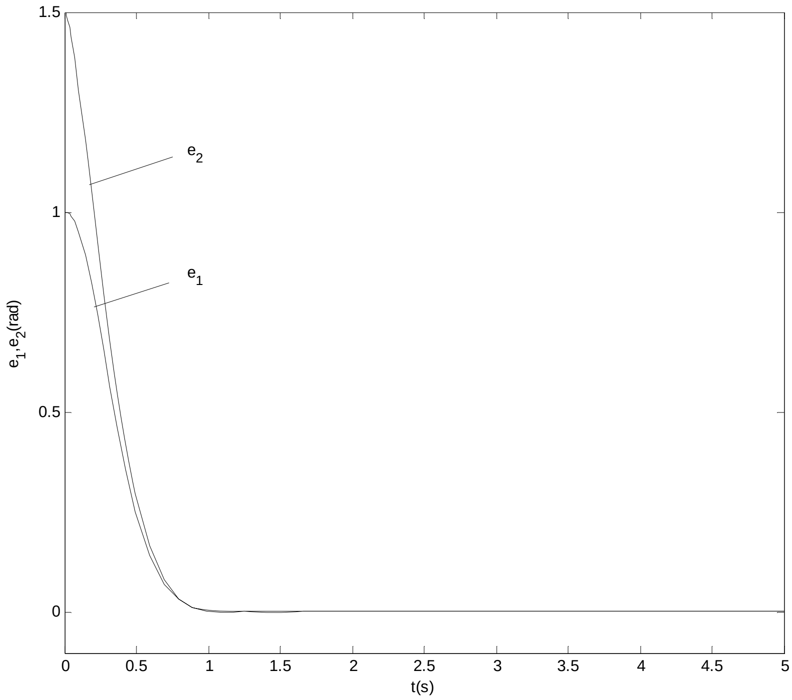

Fig. 7. Position errors of the PD control

5. Discussion

Through position control a two-degree-of-freedom direct drive robot manipulator is obtained by using the NPD controllers for the two joints. We can see from the experimental results of Figs 4 and 5 that the settling times of joint 1 and joint 2 are t_{s1} = 0.33 s and t_{s2} = 0.2 s, respectively, when the time required for the system to settle 2% of the step input amplitude and the final steady state errors are $[e_1, e_2]^T = [0.003, 0.001]^T$ radians. The steady-state position errors are presented due to the presence of frictions and disturbances at the joints and the lack of friction and disturbance compensations in the controllers. It is important to note that despite the presence of unmodelled friction and disturbance phenomena, these joint position errors are acceptably small.

Then, through position control a two-degree-of-freedom direct drive robot manipulator is carried out by using the conventional PD controllers for the two joints. We can see from the experimental results of Figs 6 and 7, that the settling times of joint 1 and 2 are t_{s1} = 0.53 s and t_{s2} = 0.52 s, respectively, when the time required for the system to settle 2% of the step input amplitude and the final steady state errors are $[e_1, e_2]^T = [0.0054, 0.0035]^T$ radians.

The above results show that PD controllers are relatively slower in the step response and have larger final errors than the NPD controllers. It is worth noticing that the response velocity of the proposed NPD controller is very fast. Therefore, the proposed NPD controllers can improve the control performance for the position control problem of robot manipulators because the gains of the NPD controller can be changed by the nonlinear function of control errors.

6. Conclusion

This paper proposed a NPD controller with gravity compensation for robot manipulators. The proportional and derivative gains of the NPD controller can vary as the error varies. The advantage is that the NPD controller has a faster response and smaller position errors compared to the conventional PD controllers in the position control of the robot arm. Therefore, the NPD controller is superior to the conventional PD controller in the position control system and provides a novel approach for robot control systems. In future, our further work will investigate the control performance of the NPD controller in various nonlinear control systems.

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