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NONLINEAR PENETRATION OF A LANGMUIR WAVE INTO A PLASMA DENSITY GRADIENT

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ABSTRACT

The nonlinear Schrödinger equation, descriptive of an intense Langmuir wave near critical density, is solved for the steady state of reflection from a linear unperturbed density profile.

Recent studies[1] of the behavior of nonlinear electron plasma waves have shown that the "nonlinear Schrödinger equation", with suitable modifications when necessary, can describe the mutual effect of density perturbations and amplitude modulations. The terms used to describe the phenomena vary: solitons, oscillating two-stream instability, modulational instability, self-trapping, four-wave interaction; but the fundamental physics is the same.

Since one characteristic of a soliton is its density depression, allowing propagation through a slightly overdense plasma, the question arises as to its behavior in a density gradient. In this paper, we present a particularly simple solution of the nonlinear Schrödinger equation in a one-dimensional density gradient, showing that the density depression associated with the wave penetrates into the overdense region a distance proportional to the wave energy density.

In the field of the Langmuir wave $\mathrm{Re}\, E(x,t)e^{-i\omega_0 t}$, the oscillation-center of an electron experiences an effective low-frequency ponderomotive potential energy $\psi_e(x,t) \equiv \langle \frac{1}{2} m_e v^2 \rangle = e^2|E|^2/4m_e\omega_0{}^2$.

In (low-frequency) thermal equilibrium, the electron density is $n_e(x,t) = n_0 \exp\left[-\beta_e(-e\phi + \psi_e)\right]$, where ϕ is the low-frequency potential. For the special case of a stationary density profile, the ion density may also be taken as a Boltzmann distribution: $n_i(x) = n_0 \exp\left[-\beta_i(e\phi + \psi_i)\right]$, where ψ_i is a pseudopotential responsible for the unperturbed $(\psi_e = 0)$ density profile. If the densities vary little over a Debye length, we may equate $n_i(x) = n_e(x)$, and solve for $e\phi(x) = (\beta_e\psi_e - \beta_i\psi_i)(\beta_e + \beta_i)^{-1}$, finally obtaining for the common density

$$n(x) = n_0(x) \exp(-\psi_e/T) \quad , \tag{1}$$

where $T \equiv \beta_e^{-1} + \beta_i^{-1}$, and $n_0(x) \equiv n_0 \exp(-\psi_i/T)$ is the unperturbed profile.

The nonlinear equation for the (stationary) Langmuir amplitude $E(x)$ is most expeditiously obtained from the linear dispersion relation

$$\omega^2 = \omega_p^2 + 3\, k^2 v_e^2 \quad , \tag{2}$$

by setting $\omega \equiv \omega_0$, $\omega_p^2 = 4\pi n(x)e^2/m$ (using (1)), $k^2 \equiv -d^2/dx^2$, and treating (2) as an operator on $E(x)$. Then (2) becomes

$$3v_e^2 \frac{d^2E}{dx^2} + \omega_0^2\, E(x) = \left[4\pi n_0(x)e^2/m\right] E(x) \exp(-e^2|E|^2/4m_e\omega_0^2 T) \quad . \tag{3}$$

We choose a linear unperturbed profile: $n_0(x) = n_0(1 + x/L)$, with $\omega_0^2 \equiv 4\pi n_0 e^2/m$, so that $x = 0$ is the critical surface. If the exponent is small (as discussed below), it can be expanded to first order. Setting $\lambda_0^2 \equiv 3v_e^2/\omega_0^2$, we then obtain

$$\lambda_0{}^2 d^2E/dx^2 = \left[(x/L) - (e^2|E|^2/4\, m_e\omega_0{}^2\, T)\right] E \quad . \tag{4}$$

To make all three terms of the same order, we choose dimensionless variables of order unity:

$$\xi \equiv x/x_0 \; , \qquad F \equiv E/E_0 \; , \tag{5}$$

where $x_0 \equiv \lambda_0^{2/3}\, L^{1/3}$,

$$E_0{}^2 \equiv 16\pi n_0 T(\lambda_0/L)^{2/3} \quad . \tag{6}$$

Equation (4) then reads

$$d^2F/d\xi^2 = (\xi - F^2)F \; , \tag{7}$$

with F assumed real, and the boundary condition $F(+\infty) \to 0$. With the change of variables (5), the exponent in Eq. (3) is $(\lambda_0/L)^{2/3}\, F^2$. Thus the assumption $\lambda_0 << L$, which is necessary for quasi-neutrality, as well as for the use of (2) as an operator equation, assures us that the exponent is indeed small for F of order unity.

The family of solutions of (7) is shown in Fig. 1. The small amplitude limit ($|F| << 1$) is the Airy function, as seen from (7) and the Figure. As the amplitude increases (from one solution to another), the ponderomotive force depresses the density, allowing penetration of the wave into the overdense region of the unperturbed profile. The depth of penetration can be characterized by the last inflection point: $\xi = F^2$, or in dimensional terms: $x = (|E|^2/16\pi n_0 T)L$.

The density $n(x) = n_0\left[1 + (\xi - F^2)(\lambda_0/L)^{2/3}\right]$ is shown in Fig. 2 for the family of solutions of Eq. (7). Similar density depressions have been observed in simulation of resonance absorption of obliquely incident electromagnetic radiation[2,3], in laboratory experiments[4], and in cold-plasma two-species analytic theory[5].

While the discussion here referred to a Langmuir wave, the same analysis applies to a normally incident electromagnetic wave, with the new definition $\lambda_0 = c/\omega_0$.

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FIGURE CAPTIONS

Fig. 1. The family of solutions of Eq. (7), representing a standing wave, i.e., an incident wave and its reflection from the self-consistent density profile.

Fig. 2. The family of density profiles, perturbed by the ponderomotive force of the radiation reflected from it.

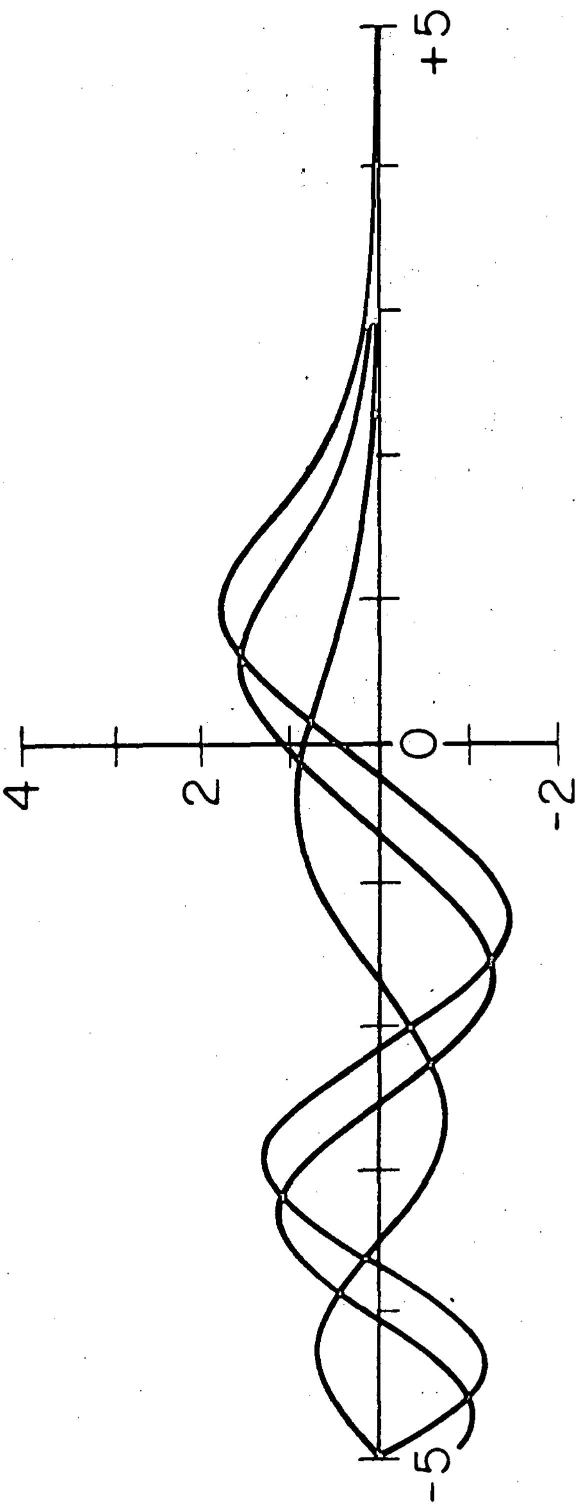

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Fig. 1.

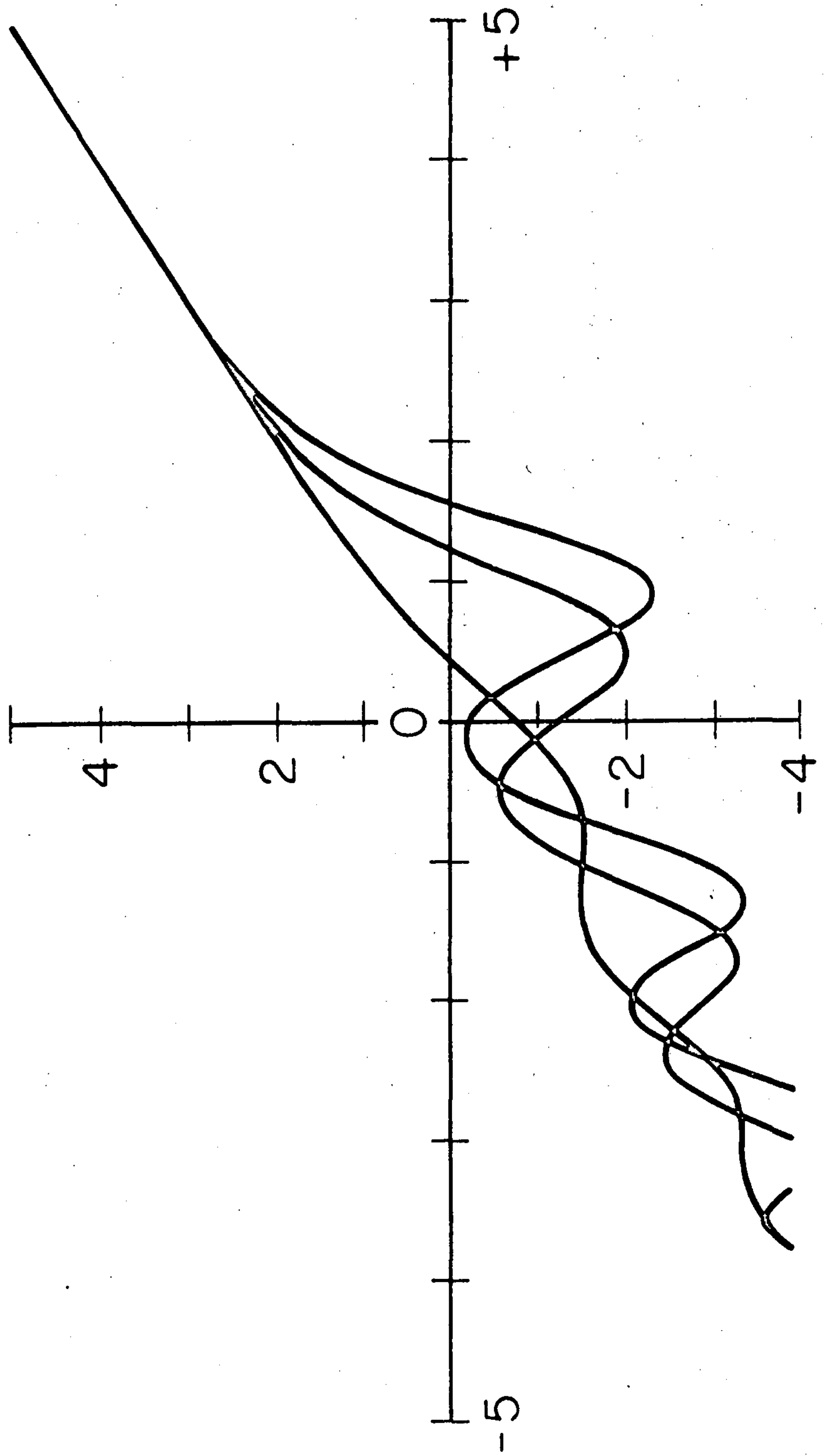

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Fig. 2.