

# Nonlinear plasmonic slot waveguides

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**Abstract:** We study nonlinear modes in a subwavelength slot waveguide created by a nonlinear dielectric slab sandwiched between two metals. We present the dispersion diagrams of the families of nonlinear guided modes and reveal that the symmetric mode undergoes the symmetry-breaking bifurcation and becomes primarily localized near one of the interfaces. We also find that the antisymmetric mode may split into two branches giving birth to two families of nonlinear antisymmetric modes.

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## References and links

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## 1. Introduction

Recent progress in nanofabrication opens novel opportunities for engineering smaller optical devices. One of the rapidly emerging fields is the design of optical integrated circuits, which would allow increasing functionalities of basic operating elements in information processing.

However there exist some fundamental limits for scaling of optical elements and their overcoming is a challenging physical and engineering problem. However, it is believed that incorporating metals, compatible with nowadays electronics, into optical elements would allow overcoming the fundamental diffraction limits by exciting surface plasmons and squeezing light into subwavelength scales. Thus, the study of plasmonic waveguides and elements are of great interest these days.

Some basic linear plasmonic elements and devices have been proposed recently [1]. One of the simplest plasmonic waveguides is an interface between metal and insulator that supports plasmon polaritons; however, due to losses in metal excited plasmon propagates very short distances. Introducing three-layer system helps to increase substantially the propagation distances due to the coupling of plasmons at the neighboring interfaces and the field concentration in dielectric rather than metal [2]. Thus, two possible geometries seem interesting for guiding of plasmons, namely, insulator-metal-insulator and metal-insulator-metal. In past decades, rigorous linear analysis of these structures has been presented (see, for example, papers [3, 4, 5]).

Nonlinear plasmonic waveguides offer additional possibilities for the mode control. Nonlinear plasmons can be excited at the metal-dielectric interfaces where the dielectric possesses the Kerr nonlinear response [6, 7, 8, 9]. Since only TM electromagnetic waves are supported by a metal-dielectric interface, the corresponding nonlinear Maxwell equations involve two components of the electric field; they cannot be solved analytically in a general case and several approximations and simplifications have been employed. The first, uniaxial approximation was proposed by Agranovich *et al.* [6] and it allows to solve Maxwell's equations when nonlinearity depends on the longitudinal component of the electric field. However, quite often the longitudinal component is weaker than the transverse component and, therefore, this assumption is valid only in specific cases. Stegeman *et al.* [7] assumed that nonlinearity is caused only by the transverse field component; this approximation has also some limitations. The problem has further been analyzed numerically [8, 9] and solved in quadratures [10, 11]. It was shown that the previous approximate methods do not give a complete picture of the mode structure and dynamics.

Based on the uniaxial approximation [7], the studies of nonlinear long-range plasmon-polaritons in metallic films embedded into nonlinear materials have been carried out in [12]. But, to the best of our knowledge, the modes of nonlinear plasmonic slot waveguides have not been studied yet. In this paper, we provide detailed numerical analysis of the problem of *nonlinear plasmonic slot waveguide* calculating all possible nonlinear modes in the metal-insulator-metal structure and show the mode splitting and bifurcations. We demonstrate that the symmetric nonlinear mode undergoes bifurcation via symmetry breaking, and a new asymmetric mode with the energy localized near one of the interfaces emerges. In addition, we demonstrate an interesting effect of the splitting of antisymmetric modes that is associated with the multivalued solutions for linear guided modes, which were not discussed in earlier literature [13]. Our results suggest interesting applications of nonlinear plasmonic guided modes for power enhancement and nonlinear switching.

## 2. Linear guided modes

We analyze the guided modes in a symmetric slot waveguide created by a three-layer structure, shown schematically in the inset of Fig. 1. For simplicity, we assume that metal is lossless. In the case of low losses, we expect that the mode dispersion will be qualitatively similar, But the losses will introduce non-zero imaginary part to the mode index, which is responsible for a finite propagation length of the plasmons.

In such structures, only TM modes exist, so that we can present the field components in the form,  $\mathbf{H} = \mathbf{e}_y H_y(x) \exp(-j\beta z)$  and  $\mathbf{E} = [\mathbf{e}_x E_x(x) + j\mathbf{e}_z E_z(x)] \exp(-j\beta z)$ ; also taking into

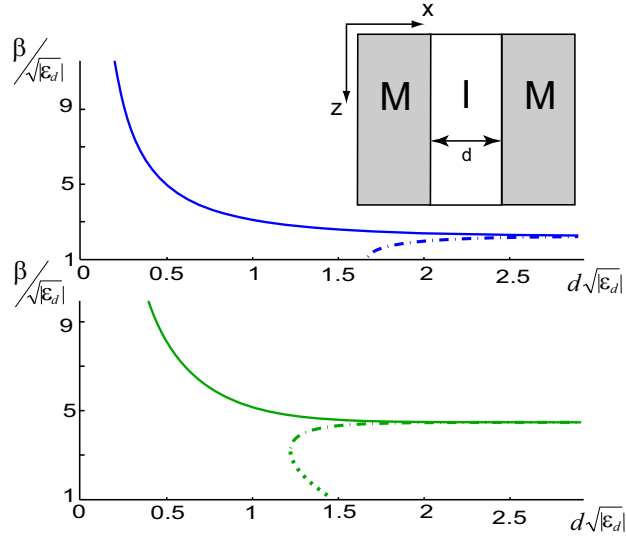


Fig. 1. Dispersion of linear guided modes shown as the dependence of the normalized guide index vs. normalized slot width, for different values of  $\rho = |\epsilon_m|/\epsilon_d$ :  $\rho = 1.3$  (top) and  $\rho = 1.05$  (bottom), for symmetric (solid) and two antisymmetric modes (dashed and dotted). Inset shows a schematic of the structure, shaded regions correspond to metal.

account that  $E_z$  is shifted in phase with respect to  $E_x$  and  $H_y$  [8]. Maxwell's equations for TM modes can be written in the form,

$$\frac{dH_y}{dx} = -\epsilon E_z, \quad \beta H_y = \epsilon E_x, \quad \frac{dE_z}{dx} + \beta E_x = H_y, \quad (1)$$

where  $\beta$  is the effective index (propagation constant), and the coordinates  $x$  and  $z$  are normalized to  $2\pi/\lambda$ , where  $\lambda$  is free space wavelength.

The variation of the refractive index across the structure is taken in the following form,

$$\epsilon = \begin{cases} \epsilon_d, & 0 < x < d, \\ \epsilon_m, & x < 0, x > d, \end{cases} \quad (2)$$

where  $d$  is the thickness of the dielectric layer.

Implying the boundary conditions of continuity of the field components  $H_y$  and  $E_z$  at the interface between metals and dielectric, we derive the dispersion relation [4] for  $\beta > \sqrt{\epsilon_d}$ ,

$$\tanh(\lambda_d d) [\epsilon_m^2 \lambda_d^2 + \epsilon_d^2 \lambda_m^2] + 2\epsilon_m \epsilon_d \lambda_m \lambda_d = 0 \quad (3)$$

where  $\lambda_m = \sqrt{\beta^2 - \epsilon_m}$  and  $\lambda_d = \sqrt{\beta^2 - \epsilon_d}$ .

Detailed analysis of this dispersion relation [3, 4] demonstrate the existence of two types of solutions for guided modes, with respect to the structure of the magnetic field, symmetric and antisymmetric, also refereed in literature as even and odd modes, respectively.

A systematic analysis [5] demonstrated that the mode existence and structure are defined by the values of the ratio  $\rho = |\epsilon_m|/\epsilon_d$ , so that for  $\rho \leq 1$  there exists only one (antisymmetric) mode with higher cutoff with respect to the slot width  $d$ . For  $\rho > 1$  two modes exist, symmetric, without cutoff, and antisymmetric, with lower cutoff. However, our analysis reveals that at  $\rho$  close

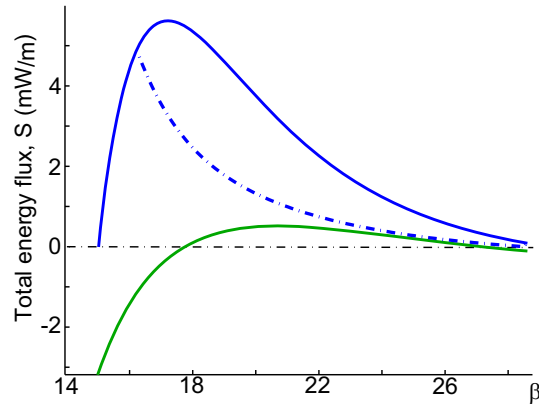


Fig. 2. Dispersion of nonlinear guided modes shown as the total power flow vs. guide index for  $\lambda = 480\text{nm}$ ,  $\varepsilon_m = -8.25$ ,  $\varepsilon_d = 7.84$ ,  $\rho = 1.05$ ,  $\alpha = 1.4 \times 10^{-18} (\text{m}^2/\text{V}^2)$ , and  $d = 25\text{nm}$ .

to 1 and for certain slot widths, the lower branch becomes multi-valued so that formally there exist three modes for a fixed value of the slot width. Figure 1 presents the modal dispersion as a function of the slot width  $d$ , at different values of  $\rho$ . As was predicted earlier [5], at  $\rho \gg 1$  here exist two modes: symmetric (solid curve), and antisymmetric (dashed curve). For  $\rho$  approaching 1 we observe three modes, see Fig. 1 (bottom), symmetric mode and two antisymmetric modes. One of the antisymmetric modes is more confined to the interfaces and has lower cutoff (dashed curve), the other mode is less confined (dotted curve), and it appears for certain values of the slot width [13]. Since metals are frequency dispersive, we can always find a frequency range for different insulators where those regimes can be realized.

### 3. Nonlinear guided modes

We now assume that the insulator is a nonlinear dielectric with the Kerr nonlinear response. To be more specific, we consider that the nonlinear dielectric is chalcogenite glass  $\text{As}_2\text{Se}_3$  with self-focusing nonlinearity confined between two silver slabs. In this case, dielectric permittivity of the nonlinear slab can be presented as follows:

$$\varepsilon_{\text{nl}} = \varepsilon_d + \alpha(E_x^2 + E_z^2) \quad (4)$$

where  $\varepsilon_d = 7.84$ , and  $\alpha = 1.4 \times 10^{-18} (\text{m}^2/\text{V}^2)$  is the nonlinear coefficient [14].

We choose  $\rho = 1.05$ , so that three mode regime is realized, see Fig. 1 (bottom), thus  $\varepsilon_m = -8.25$ . For silver this is realized at  $\nu = 0.63 \times 10^{15} \text{Hz}$  (free space wavelength  $\lambda = 480\text{nm}$ ) [15].

We solve Eqs. (1) numerically by using shooting method and find the mode profiles for different values of the guide index and nonlinear parameter. To analyze the nonlinear modes we calculate the total energy flux per unit length in the direction of propagation,

$$S = \int ([\mathbf{E} \times \mathbf{H}] \cdot \mathbf{z}) dx. \quad (5)$$

and plot this value for three different values of the slot width, corresponding to different types of linear guided waves shown in Fig. 1.

First, we analyze the guided modes when there exists only one symmetric mode in the linear regime. For this case, we observe that for all values of the guide index  $\beta > \sqrt{\varepsilon_d}$  there appears

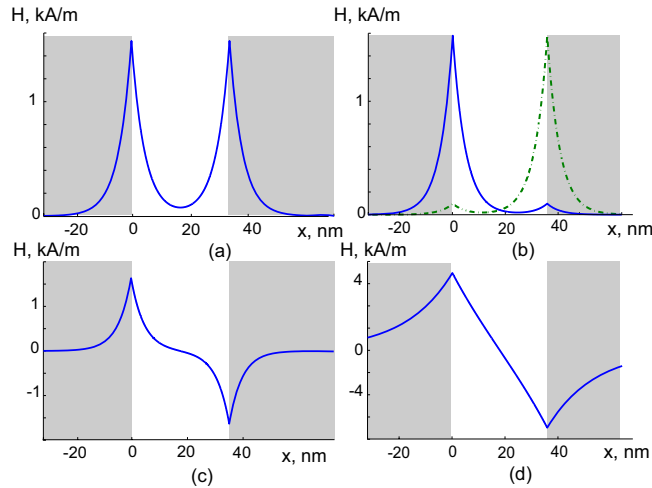


Fig. 3. (a-d) Characteristic profiles of nonlinear plasmonic modes shown as the magnetic field distribution for different branches of the dispersion curves marked by points in Fig. 4.

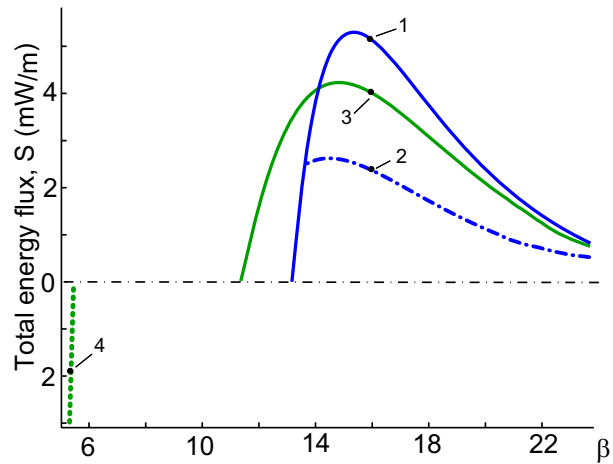


Fig. 4. Dispersion of nonlinear guided modes shown as the total power flow vs. effective guide index for  $d = 35\text{nm}$ . In the linear limit of vanishing power, the structure supports one symmetric and two antisymmetric plasmonic modes.

an antisymmetric nonlinear mode, see Fig. 2. Even the total flux for this mode may vanish, this nonlinear mode has no linear analog, and it requires a finite power to be excited in the structure. Our analysis reveals that for smaller slot widths (not shown here) the total energy flux of this antisymmetric mode becomes negative for all effective guide indices. When the slot width grows, the total energy flux of this mode becomes positive for some values of  $\beta$ , see Fig. 2. The typical mode profile is shown in Fig. 3(c).

For  $\beta \simeq 15$ , a symmetric mode appears, and for larger  $\beta$  the flux increases up to its maximum ( $S \simeq 6\text{mW/m}$ ) at  $\beta \simeq 17$  but then decreases again. Also, at  $\beta \simeq 16$  we observe a symmetry breaking bifurcation (at  $S \simeq 5\text{mW/m}$ ) which leads to the appearance of an *asymmetric mode* and

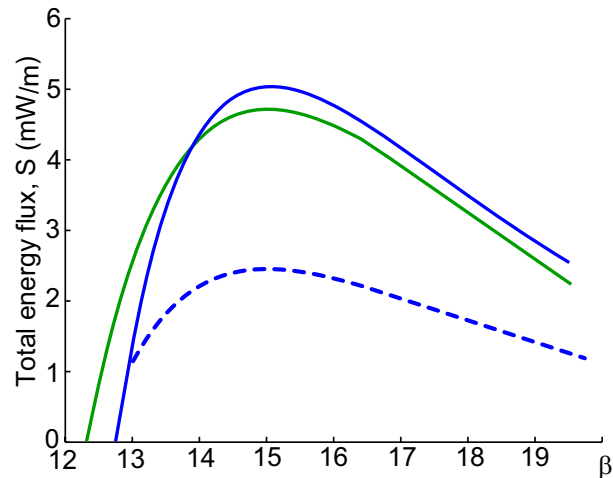


Fig. 5. Dispersion of nonlinear guided modes shown as the total power flow vs. guide index for  $d = 50\text{nm}$ . In the linear limit of vanishing power, the structure supports one symmetric and one antisymmetric plasmonic modes.

its power decreases monotonously with  $\beta$ . For larger  $\beta$  the field becomes strongly confined to the interfaces, thus the interaction between plasmons localized at different interfaces becomes weaker, so that the dispersion resembles that of a single interface plasmon [9]. Typical mode profiles of symmetric and asymmetric modes are presented in Figs. 3(a,b), respectively.

For larger values of the slot width, three modes appear, and the antisymmetric mode splits into two branches. With larger  $d$  (see Fig. 4), the character of the symmetric mode and bifurcated asymmetric branch does not change much. However, the bifurcation point is observed at smaller values of power. Also, the power of the asymmetric mode reaches its maximum. The mode profiles are shown in Figs. 3(a-d) for the marked points (1-4). Long-wave antisymmetric mode (see Fig. 4, dotted) has a negative flux because the field resides mainly in metal, see Fig. 3(d), and the mode is less confined to the interfaces in comparison to the short-wave antisymmetric mode, [see Fig. 4, dashed; and Fig. 3(c)]. We note that for all slot widths there is a range of effective guide indices where several modes at different powers can coexist.

Further increase of the slot width leads to broadening of the gap between the antisymmetric branches bringing to a degeneracy of the long-wave antisymmetric branch since it becomes radiative. The bifurcation power decreases, and it is observed at  $S \simeq 1\text{mW}/m$ , see Fig. 5.

#### 4. Conclusion and acknowledgements

We have studied the families of nonlinear plasmonic modes in metal-dielectric slot waveguide and predicted the symmetry-breaking bifurcation of the symmetric modes with the critical power depending on the slot width. We have also discussed a complex structure of the asymmetric plasmonic modes that originate from the multi-valued linear plasmonic modes.

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