

Nonlinear propagation of ion-acoustic waves in a hot magnetized plasma with vortexlike electron distribution

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Effects of external magnetic field, ion temperature, and vortexlike electron distribution are incorporated in the study of nonlinear ion-acoustic waves in a hot magnetized plasma that consists of a positively charged, hot ion fluid and trapped, as well as, free electrons. It is found that, owing to the departure from the Boltzmann electron distribution to a vortexlike one, the dynamics of small but finite amplitude ion-acoustic waves is governed by a nonlinear equation of K-dV (Korteweg-de Vries) type. The latter admits a stationary ion-acoustic solitary wave solution, which has larger amplitude, smaller width, and higher propagation velocity, than that involving isothermal electrons. The effects of external magnetic field and ion temperature on the properties of these ion-acoustic solitary structures are also discussed. © 1998 American Institute of Physics. [S1070-664X(97)03012-7]

It is well known from computer simulations¹⁻³ and experiments^{4,5} that plasmas, which are strongly excited by means of the injection of particle beams, are often found to evolve toward a coherent trapped-particle state, instead of developing into a turbulent one. The nonlinear behavior of electrostatic waves in a plasma with this trapped particle state⁶⁻⁸ has received considerable attention and been studied by a number of authors in last few years.⁶⁻¹¹ To the best knowledge of the author, most of these studies⁶⁻¹¹ are restricted to the unmagnetized case and cold plasma limit. As the effects of external magnetic field and ion temperature, which have not been considered in these earlier investigations,⁶⁻¹¹ drastically modifies the properties of electrostatic solitary structures,¹²⁻¹⁵ we, in the present work, have studied the obliquely propagating ion-acoustic solitary structures in a hot magnetized plasma that consists of a positively charged, hot ion fluid and electrons having vortexlike distribution.

We consider a plasma, which consists of a positively charged, hot ion fluid and electrons with trapped particles, in the presence of an external static magnetic field ($\mathbf{B}_0 \parallel \hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is a unit vector along the z direction). The nonlinear behavior of ion-acoustic waves in this plasma system may be described by the following set of fluid equations:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0, \quad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla \varphi + \omega_c (\mathbf{u} \times \hat{\mathbf{z}}) - \frac{5}{3} \frac{\sigma}{n^{1/3}} \nabla n, \quad (2)$$

$$\nabla^2 \varphi = n_e - n, \quad (3)$$

where n is the ion number density normalized to equilibrium plasma density n_0 ; \mathbf{u} is the ion fluid velocity normalized to the ion-acoustic speed $C_s = (T_e/m)^{1/2}$ with T_e being the electron temperature (in energy units), and m being the mass of positively charged ions; φ is the electrostatic wave potential normalized to T_e/e with e being the magnitude of the electron charge. $\sigma = T_i/T_e$ with T_i being the ion temperature (in energy units). The time and space variables are in the units of

the ion plasma period $\omega_p^{-1} = (m/4\pi n_0 e^2)^{1/2}$ and the Debye length $\lambda_D = (T_e/4\pi n_0 e^2)^{1/2}$, respectively. $\omega_c = (eB_0/m)/\omega_p$ is the ion cyclotron frequency normalized to ω_p .

To model an electron distribution with trapped particles, we employ a vortexlike electron distribution function of Schamel,^{6,7} which solves the electron Vlasov equation. Thus we have

$$f_{ef} = \frac{1}{\sqrt{2\pi}} e^{-1/2(v^2 - 2\varphi)}, \quad |v| > \sqrt{2\varphi}, \quad (4)$$

$$f_{et} = \frac{1}{\sqrt{2\pi}} e^{-1/2\alpha(v^2 - 2\varphi)}, \quad |v| \leq \sqrt{2\varphi},$$

where the subscript $f(t)$ represents the free (trapped) electron contribution. It may be noted here that the distribution function, as presented above, is continuous in velocity space and satisfies the regularity requirements for an admissible BGK solution.⁸ Here, the velocity v is normalized to the electron thermal velocity v_{te} and α , which is the ratio of free-electron temperature (T_{ef}) to trapped electron temperature (T_{et}), is a parameter determining the number of trapped electrons. It has been assumed that the velocity of nonlinear ion-acoustic waves is small in comparison with the electron thermal velocity.

Integrating the electron distribution functions over the velocity space, we readily obtain the electron number density n_e as

$$n_e = I(\varphi) + \frac{e^{a\varphi}}{\sqrt{|\alpha|}} \operatorname{erf}(\sqrt{\alpha\varphi}), \quad \alpha \geq 0,$$

$$n_e = I(\varphi) + \frac{2}{\sqrt{\pi|\alpha|}} W(\sqrt{-\alpha\varphi}), \quad \alpha < 0, \quad (5)$$

where

$$I(\varphi) = [1 - \operatorname{erf}(\sqrt{\alpha\varphi})]e^\varphi,$$

$$\operatorname{erf}(\sqrt{\alpha\varphi}) = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\alpha\varphi}} e^{-y^2} dy, \quad (6)$$

$$W(\sqrt{-\alpha\varphi}) = e^{\alpha\varphi} \int_0^{\sqrt{-\alpha\varphi}} e^{y^2} dy.$$

If we expand this n_e for the small amplitude limit and keep the terms up to φ^2 , it is found that n_e is the same for both $\alpha \geq 0$ and $\alpha < 0$ and is finally given by

$$n_e = 1 + \varphi - \frac{4}{3} \frac{1 - \sigma}{\sqrt{\pi}} (\varphi)^{3/2} + \frac{1}{2} \varphi^2. \quad (7)$$

In order to derive a nonlinear dynamical equation for the ion-acoustic waves from (1)–(3) and (7), one must find an appropriate coordinate frame where the wave can be described smoothly. To find this frame, we need to know the width Δ and nonlinear velocity v_0 of the wave, which can be taken from an equilibrium theory using vortexlike electron distributions.^{6,7} Thus we find $\Delta \propto \epsilon^{-1/4}$ and $(v_0 - 1) \propto \epsilon^{1/2}$, where ϵ is a smallness parameter measuring the weakness of the dispersion (amplitude of the perturbation). These lead immediately to the following stretched coordinates:

$$\begin{aligned} \xi &= \epsilon^{1/4}(l_x x + l_y y + l_z z - v_0 t), \\ \tau &= \epsilon^{3/4} t, \end{aligned} \quad (8)$$

where v_0 is normalized to C_s ; l_x , l_y , and l_z are the directional cosines of the wave vector \mathbf{k} along the x , y , and z axes, respectively, so that $l_x^2 + l_y^2 + l_z^2 = 1$. It should be mentioned here that in the case of isothermal electrons¹⁶ the quantities Δ and v_0 behave as $\Delta \propto \epsilon^{-1/2}$ and $(v_0 - 1) \propto \epsilon$, respectively, and the ordering (8) is no longer applicable, and we must return to the ordering of Washimi and Taniuti¹⁶ yielding the K-dV (Korteweg-de Vries) equation of standard form in a cold unmagnetized plasma. We can expand the perturbed quantities n , φ , and $u_{x,y,z}$ (where terms of $\epsilon^{5/4}$, $\epsilon^{3/2}$, \dots are taken into account) about their equilibrium values in powers of ϵ , including terms of $\epsilon^{3/2}$:

$$\begin{aligned} n &= 1 + \epsilon n^{(1)} + \epsilon^{3/2} n^{(2)} + \dots, \\ \varphi &= \epsilon \varphi^{(1)} + \epsilon^{3/2} \varphi^{(2)} + \dots, \\ u_z &= \epsilon u_z^{(1)} + \epsilon^{3/2} u_z^{(2)} + \dots, \\ u_{x,y} &= \epsilon^{5/4} u_{x,y}^{(1)} + \epsilon^{3/2} u_{x,y}^{(2)} + \dots. \end{aligned} \quad (9)$$

Next, substituting (7)–(9) into (1)–(3) one can obtain the lowest order continuity equation, momentum equation, and Poisson's equation which in turn can be solved as $n^{(1)} = l_z u_z^{(1)}/v_0 = \varphi^{(1)}$ and $v_0 = l_z \sqrt{1 + 5/3\sigma}$. We can write the first order x and y components of the momentum equation as

$$\begin{aligned} u_y^{(1)} &= \frac{l_x}{\omega_c} \left(1 + \frac{5}{3}\sigma\right) \frac{\partial \varphi^{(1)}}{\partial \xi}, \\ u_x^{(1)} &= -\frac{l_y}{\omega_c} \left(1 + \frac{5}{3}\sigma\right) \frac{\partial \varphi^{(1)}}{\partial \xi}. \end{aligned} \quad (10)$$

These, respectively, represent the y and x components of the sum of electric and diamagnetic drifts. These equations are also satisfied by the second order continuity equation.

Again, using (7)–(9) in (2) and (3), and eliminating $u_{x,y}^{(1)}$, we obtain the next higher order x and y components of the momentum equation and Poisson's equation as

$$\begin{aligned} u_y^{(2)} &= \frac{l_y v_0}{\omega_c^2} \left(1 + \frac{5}{3}\sigma\right) \frac{\partial^2 \varphi^{(1)}}{\partial \xi^2}, \\ u_x^{(2)} &= \frac{l_x v_0}{\omega_c^2} \left(1 + \frac{5}{3}\sigma\right) \frac{\partial^2 \varphi^{(1)}}{\partial \xi^2}, \\ \frac{\partial^2 \varphi^{(1)}}{\partial \xi^2} + n^{(2)} - \varphi^{(2)} + \frac{4(1 - \alpha)}{3\sqrt{\pi}} \varphi^{3/2} &= 0. \end{aligned} \quad (11)$$

The first two of these equations, respectively denote the y and x components of the ion polarization drift. Similarly, following the same procedure one can obtain the next higher order continuity equation and z component of the momentum equation as

$$\frac{\partial n^{(1)}}{\partial \tau} - v_0 \frac{\partial n^{(2)}}{\partial \xi} + l_x \frac{\partial u_x^{(2)}}{\partial \xi} + l_y \frac{\partial u_y^{(2)}}{\partial \xi} + l_z \frac{\partial u_z^{(2)}}{\partial \xi} = 0, \quad (12)$$

$$\frac{\partial u_z^{(1)}}{\partial \tau} - v_0 \frac{\partial u_z^{(2)}}{\partial \xi} + l_z \frac{\partial \varphi^{(2)}}{\partial \xi} + \frac{5}{3} \sigma l_z \frac{\partial n^{(2)}}{\partial \xi} = 0. \quad (13)$$

Now, using (10)–(13), one can eliminate $n^{(2)}$, $u_z^{(2)}$, and $\varphi^{(2)}$, and can obtain

$$\frac{\partial \varphi^{(1)}}{\partial \tau} + A \sqrt{\varphi^{(1)}} \frac{\partial \varphi^{(1)}}{\partial \xi} + B \frac{\partial^3 \varphi^{(1)}}{\partial \xi^3} = 0, \quad (14)$$

which is the K-dV (Korteweg-de Vries) type nonlinear equation with the coefficients A and B given by

$$\begin{aligned} A &= \frac{1 - \alpha}{\sqrt{\pi}} \frac{l_z}{\sqrt{1 + \frac{5}{3}\sigma}}, \\ B &= \frac{1}{2} \frac{l_z}{\sqrt{1 + \frac{5}{3}\sigma}} \left[1 + \left(\frac{1 - l_z^2}{\omega_c^2} \right) \left(1 + \frac{5}{3}\sigma \right)^2 \right]. \end{aligned} \quad (15)$$

The steady state solution of this K-dV type equation is obtained by transforming the independent variables ξ and τ to $\eta = \xi - u_0 \tau$ and $\tau = \tau$, where u_0 is a constant velocity normalized to C_s , and imposing the appropriate boundary conditions, viz., $\varphi \rightarrow 0$, $d\varphi^{(1)}/d\eta \rightarrow 0$, $d^2\varphi^{(1)}/d\eta^2 \rightarrow 0$ at $\eta \rightarrow \pm \infty$. Thus one can express the steady state solution of this K-dV type equation as

$$\varphi^{(1)} = \varphi_m^{(1)} \operatorname{sech}^2[(\xi - u_0 \tau)/\delta], \quad (16)$$

where the amplitude $\varphi_m^{(1)}$ and the width δ (normalized to λ_D) are given by $\varphi_m^{(1)} = (15u_0/8A)^2$ and $\delta = \sqrt{16B/u_0}$, respectively. This solution also stands for $n^{(1)}$. It should be noted here that the perturbation method, which is only valid for small but finite amplitude limit, is not valid for large propagation angle θ , which makes the wave amplitude large

enough to break the condition $1 > \epsilon n^{(1)}$. As $u_0 > 0$, there exist solitary waves with positive potential only, i.e., solitary structures with enhanced density only. It is seen that as u_0 increases, the amplitude increases while the width decreases and, that as $|\alpha|$ increases, the amplitude decreases for $\alpha < 0$ (a vortexlike excavated trapped electron distribution) and increases for $\alpha > 0$. It is clear that due to the trapped electrons, we have found solitonlike structures of larger amplitude, smaller width, and higher propagation velocity than that involving isothermal electrons.¹⁶

It is also obvious that as we decrease l_z , i.e., increase the angle between magnetic field (\mathbf{B}_0) and propagation vector (\mathbf{k}), the amplitude of these solitonlike structures increases, whereas their width decreases for $\omega_c \gg (1 - l_z)$ and increases for $\omega_c \ll (1 - l_z)$. It has been found here that as we increase ion temperature, the amplitude increases, whereas the width decreases for $\omega_c \gg (1 - l_z)$ and increases for $\omega_c \ll (1 - l_z)$.

It is seen that the magnitude of the external magnetic field has no effect on the amplitude of the solitary waves. However, it does have an effect on the width of these solitary waves. It is shown that as we increase the magnitude of the magnetic field, the width of these solitary waves decreases, i.e., the external magnetic field makes the solitary structures more spiky.

It may be stressed here that the results of this investigation should be useful in understanding the nonlinear features of localized electrostatic disturbances in laboratory and space plasmas where positively charged ions and free and trapped

electrons are the plasma species. To conclude it may be added that the time evolution and stability analysis of these solitary structures are also problems of great importance, but beyond the scope of the present work.

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