Nonlinear reduced Braginskii equations with ion thermal dynamics in toroidal plasma

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Starting from the Braginskii fluid equations, a set of nonlinear reduced equations are derived which describe the low frequency dynamics of electron and ion energy and density in a toroidal plasma. The equations have an energy integral. The equations are appropriate for studying the relation between electron and ion thermal transport and particle transport in low temperature plasma near the edge of plasma confinement devices.
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I. INTRODUCTION

The understanding of anomalous transport in magnetically confined plasmas requires the development of models which can describe the dynamical evolution of potential, density, temperature, and possibly magnetic perturbations. In systems in which the ambient scale lengths are long compared to the characteristic Larmor radii of either ions or electrons $\rho_{e,i}$, instabilities driven by density or pressure gradients typically have characteristic time/space scales which are slower than the characteristic gyro-frequencies $\omega_{ce,i}$ and longer than the Larmor radii,

$$\frac{\partial}{\partial t} \sim \frac{\rho_i^2}{L_\perp^2} \omega_{ci} \ll \omega_{ci}, \tag{1}$$

with $L_\perp$ a characteristic transverse scale length of the plasma. In this case it is useful to derive a set of reduced equations in which the cyclotron motion of the particles is eliminated. The resulting equations describe only the low frequency motion of the system and therefore are suitable for studying the long time evolution of low frequency instabilities in magnetized plasmas.

We focus specifically on low β plasmas which are sufficiently collisional that the Braginskii equations[1] are valid. Reduced equations of various subsets of the Braginskii equations have been derived by many authors (see, for example, Ref. 2), neglecting either ion or electron temperature dynamics, assuming an adiabatic electron response or applying other additional approximations.[3–6] For a survey see Ref. 7. For the cold ion limit the complete set of reduced equations is derived in Ref. 5 including the corresponding energy integral. The earlier treatments of the ion pressure (see, for example, Ref. 4) suffer from the neglect of the compression due to the ion polarization drift. Recently Kim *et al.*[8] demonstrated how to treat this term properly. The electron part of the dynamics, however, is still neglected in this work. Since previous derivations remain incomplete it is the primary goal of this paper to extend Ref. 5 to fully include ion thermal effects. This work is complicated by the fact that finite Larmor radius effects must be systematically included as outlined in Ref. 8.

Although the electrostatic approximation is prevalent in numerical studies of turbulent edge transport, the models discussed here avoid that approximation and retain the magnetic perturbations. Estimating the importance of these perturbations from Ohm's law as shown later, one expects an electrostatic model to be sufficient for $\alpha_{mhd} = -q^2 R\beta' \ll 1$—a condition which is often violated in the edge of high performance tokamaks.

In Section II we present a brief derivation of the reduced Braginskii equations with full ion dynamics and then in Section III discuss the energy conservation properties of the equations. In Section IV we show that the general reduced Braginskii equations can be further truncated using a linearization procedure when the characteristic scale length of the turbulence is small compared with the equilibrium scale length. The final equations are suitable for studying both electron and ion thermal transport in collisional plasmas such as in the edge region of tokamaks and other confinement machines.

II. REDUCED BRAGINSKII EQUATIONS

We start the derivation of the reduced Braginskii equations from the momentum balance relations

$$m_i n\left(\frac{\partial}{\partial t} + \vec{v}_i \cdot \nabla\right)\vec{v}_i = -\nabla p_i - \nabla\cdot P_i + en\left[\vec{E} + \frac{1}{c}\vec{v}_i \times \vec{B}\right] + \vec{R}_{ie}, \tag{2}$$

$$0 = -\nabla p_e - en\left[\vec{E} + \frac{1}{c}\vec{v}_e \times \vec{B}\right] + \vec{R}_{ei}, \tag{3}$$

where the electron mass is neglected, electron and ion densities are both denoted by n, assuming quasi-neutrality, and $\vec{E} = -\nabla\phi - (1/c)\partial\vec{A}/\partial t$. Keeping collisional effects only in the direction parallel to the magnetic field, the momentum transfer term takes the form

$$R_{ei\|} = -R_{ie\|} = (ne\eta_\| j_\| - 0.71 n \nabla_\| T_e), \quad j_\| = en(v_{\|i} - v_{\|e}) \tag{4}$$

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with $\eta_\parallel$ the Spitzer resistivity. In the ion stress-tensor we neglect terms depending on the collisional time (keeping only the finite Larmor radius contribution) and parallel gradients. The remaining components yield[3]

$$\nabla\cdot P_i = -m_i n \vec{v}_{di}\cdot\nabla\vec{v}_i + p_i\nabla\times\frac{\vec{b}}{\omega_{ci}}\cdot\nabla\vec{v}_i + \nabla_\perp\left(\frac{p_i}{2\omega_{ci}}\nabla\cdot\vec{b}\times\vec{v}_i\right) + \vec{b}\times\nabla\left(\frac{p_i}{2\omega_{ci}}\nabla_\perp\cdot\vec{v}_i\right), \quad (5)$$

with $\vec{b}=\vec{B}/B$, the ion cyclotron frequency $\omega_{ci}=eB/m_i c$ and the ion diamagnetic drift-velocity $\vec{v}_{di}=c/(enB^2)\vec{B}\times\nabla p_i$.

A fundamental assumption made in deriving a set of reduced equations for plasma dynamics is that the characteristic time scales are slower than the ion cyclotron frequency and space scales are longer than the ion Larmor radius. Under these conditions the perpendicular components of the first and third terms on the right hand side of Eq. (2) dominate and to lowest order the ion velocity perpendicular to $\vec{B}$ is given by

$$\vec{v}_{\perp i0} = \vec{v}_{di} + \vec{v}_E, \quad (6)$$

with $\vec{v}_E = c\vec{E}\times\vec{B}/B^2$. The remaining terms in Eq. (2) are of order $\rho_i^2/L_\perp^2 \ll 1$. We formally obtain the correction to the lowest order velocity $\vec{v}_{\mathrm{pol}} = \vec{v}_{\perp i} - \vec{v}_{\perp i0}$ by inserting Eq. (5) into Eq. (2), crossing with $\vec{b}$ and keeping corrections of order m_i,

$$\vec{v}_{\mathrm{pol}} = \frac{\vec{b}}{\omega_{ci}}\times\frac{d}{dt}\vec{v}_{\perp i0} + \frac{1}{nm_i\omega_{ci}}\left\{\vec{b}\times\left[p_i\nabla\times\frac{\vec{b}}{\omega_{ci}}\cdot\nabla\vec{v}_{\perp i0}\right] + \vec{b}\times\nabla_\perp\left[\frac{p_i}{2\omega_{ci}}\nabla\cdot\vec{b}\times\vec{v}_{\perp i0}\right] - \nabla_\perp\left[\frac{p_i}{2\omega_{ci}}\nabla_\perp\cdot\vec{v}_{\perp i0}\right]\right\} \quad (7)$$

with $d/dt = \partial/\partial t + (\vec{v}_E + \vec{v}_{\mathrm{pol}} + \vec{v}_{\parallel i})\cdot\nabla$. As discussed previously,[5] energy conservation requires the retention of the polarization drift in the convective derivative even though it is formally small in the ordering. Similarly we obtain from Eq. (3) the electron perpendicular velocity

$$\vec{v}_{\perp e} = \vec{v}_{de} + \vec{v}_E, \quad (8)$$

with the electron diamagnetic drift-velocity $\vec{v}_{de} = -c/(enB^2)\vec{B}\times\nabla p_e$.

Because the frequency of compressional magnetosonic waves, $\omega_m \sim V_A\nabla_\perp$ with $V_A \equiv B/\sqrt{4\pi\rho}$, typically satisfies $\omega \ll \omega_{ci}$, the description of such waves is still contained in the formalism at this point. Now, however, we extend the low-frequency assumption to the regime $\omega \ll \omega_m$, where the plasma motion is nearly incompressible. In this regime the electromagnetic contribution to $\vec{E}_\perp$, which is associated with perturbations in the toroidal component of $\vec{B}$, can be estimated from the condition of total pressure balance, and [for ∂_t given by Eq. (1)] leads to corrections that are $O(\beta) \ll 1$. As a result, in $\vec{v}_{\mathrm{pol}}$ as well as in the expressions for d/dt that follow, we can take the perpendicular electric field to be electrostatic so that $\vec{v}_E \simeq (c/B^2)\vec{B}\times\nabla\phi$.

In this low-frequency limit, the motions of the plasma are governed by the vorticity equation, which can be obtained as follows. Since we assume the plasma to be quasineutral the ion and electron continuity equations

$$\frac{\partial n}{\partial t} + \nabla\cdot n(\vec{v}_E + \vec{v}_{di} + \vec{v}_{\mathrm{pol}} + \vec{v}_{\parallel i}) = 0,$$
$$\frac{\partial n}{\partial t} + \nabla\cdot n(\vec{v}_E + \vec{v}_{de} + \vec{v}_{\parallel e}) = 0 \quad (9)$$

must yield the same evolution of n. The implied constraint is obtained by subtracting the two equations and leads to the vorticity equation

$$\nabla\cdot n\vec{v}_{\mathrm{pol}} + \nabla_\parallel\frac{j_\parallel}{e} + \nabla\cdot n(\vec{v}_{di} - \vec{v}_{de}) = 0, \quad (10)$$

where we have used the relation for $j_\parallel$ given in Eq. (4) and $\nabla_\parallel = \vec{b}\cdot\nabla$.

In the low β limit the diamagnetic depression of the toroidal field can be neglected and the magnetic field can be written in terms of the vacuum toroidal component and a poloidal flux function $\psi = -A_\parallel$ as

$$\vec{B} = R_0 B_0 \nabla\zeta + R_0\nabla\zeta\times\nabla\psi, \quad (11)$$

where B_0 is the vacuum toroidal field at $R = R_0$, and ζ is the toroidal angle. An equation for ψ follows from the parallel component of the generalized Ohm's law in Eq. (3), with $R_{ei\parallel}$ given in Eq. (4),

$$\eta_\parallel j_\parallel = -\nabla_\parallel\phi + \frac{1}{c}\frac{\partial\psi}{\partial t} + \frac{1}{en}\nabla_\parallel p_e + \frac{0.71}{e}\nabla_\parallel T_e. \quad (12)$$

Neglecting the displacement current, $j_\parallel \simeq c\nabla_\perp^2\psi/4\pi$.

To obtain an equation for the evolution of $v_{\parallel i}$ we add the parallel components of Eqs. (2) and (3) using Eq. (5), which gives

$$m_i n\frac{d}{dt}v_{\parallel i} = -\nabla_\parallel(p_i + p_e) - p_i\nabla\times\frac{\vec{b}}{\omega_{ci}}\cdot\nabla v_{\parallel i}. \quad (13)$$

The ion temperature T_i is evolved according to the Braginskii equation

$$\frac{3}{2}n\frac{dT_i}{dt} + \frac{3}{2}n\vec{v}_{di}\cdot\nabla T_i + p_i\nabla\cdot\vec{v}_i + \frac{5}{2}\frac{c}{e}\nabla\cdot\left[p_i\left(\frac{\vec{b}}{B}\times\nabla T_i\right)\right] + P_i:\nabla\vec{v}_i = 0, \quad (14)$$

where we neglect the collisional perpendicular heat flux and the parallel heat conduction. Keeping only the finite Larmor radius parts of the stress tensor and neglecting parallel gradients, we obtain $P_i(\vec{v}):\nabla\vec{v} = 0$ (an explicit specification of $\vec{v}$ is not required to obtain this result). $\nabla\cdot\vec{v}_i$ is eliminated using Eq. (9),

$$p_i\nabla\cdot\vec{v}_i = -T_i\left(\frac{dn}{dt} + \vec{v}_{di}\cdot\nabla n\right) = -T_i\frac{dn}{dt} + n\vec{v}_{di}\cdot\nabla T_i, \quad (15)$$

where we have used the relations $\vec{v}_{di}\cdot\nabla p_i = 0$ and $p_i = nT_i$. The fourth term in Eq. (14) is transformed to

$$\frac{5}{2}\frac{c}{e}\nabla\cdot\left[p_i\left(\frac{\vec{b}}{B}\times\nabla T_i\right)\right]=-\frac{5}{2}n\vec{v}_{di}\cdot\nabla T_i + \frac{5}{2}\frac{c}{e}p_i\left(\nabla\times\frac{\vec{b}}{B}\right)\cdot\nabla T_i. \qquad (16)$$

Inserting Eqs. (15) and (16) into Eq. (14) we obtain the ion temperature equation

$$\frac{3}{2}n\frac{dT_i}{dt}-T_i\frac{dn}{dt}+\frac{5}{2}\frac{c}{e}p_i\left(\nabla\times\frac{\vec{b}}{B}\right)\cdot\nabla T_i=0. \qquad (17)$$

The ion pressure equation results if dn/dt is evaluated using the ion continuity equation Eq. (9),

$$\frac{3}{2}\frac{dp_i}{dt}+\frac{5}{2}p_i\nabla\cdot(\vec{v}_E+\vec{v}_{\|i}+\vec{v}_{\mathrm{pol}})+\frac{5}{2}\frac{c}{e}\left(\nabla\times\frac{\vec{b}}{B}\right)\cdot\nabla(p_iT_i)=0. \qquad (18)$$

The electron temperature T_e is determined by the Braginskii equation

$$\frac{3}{2}n\frac{DT_e}{Dt}+\frac{3}{2}n\vec{v}_{de}\cdot\nabla T_e+p_e\nabla\cdot\vec{v}_e-\nabla_\|\kappa_\|\nabla_\|T_e -\frac{5}{2}\frac{c}{e}\nabla\cdot p_e\left(\frac{\vec{b}}{B}\times\nabla T_e\right)-\frac{0.71}{e}T_e\nabla_\|j_\|=0, \qquad (19)$$

with $D/Dt=\partial/\partial t+(\vec{v}_E+\vec{v}_{\|e})\cdot\nabla$ and $\vec{v}_e=\vec{v}_{\perp e}+\vec{v}_{\|e}$. The parallel electron velocity $v_{\|e}$ can be written in terms of $v_{\|i}$ and $j_\|$ as $v_{\|e}=v_{\|i}-j_\|/ne$. Similar manipulations as before yield the electron temperature equation

$$\frac{3}{2}n\frac{DT_e}{Dt}-T_e\frac{Dn}{Dt}-\nabla_\|\kappa_\|\nabla_\|T_e-\frac{5}{2}\frac{c}{e}p_e\left(\nabla\times\frac{\vec{b}}{B}\right)\cdot\nabla T_e -\frac{0.71}{e}T_e\nabla_\|j_\|=0, \qquad (20)$$

and the electron pressure equation

$$\frac{3}{2}\frac{Dp_e}{Dt}+\frac{5}{2}p_e\nabla\cdot(\vec{v}_E+\vec{v}_{\|e})-\nabla_\|\kappa_\|\nabla_\|T_e -\frac{5}{2}\frac{c}{e}\left(\nabla\times\frac{\vec{b}}{B}\right)\cdot\nabla(p_eT_e)-\frac{0.71}{e}T_e\nabla_\|j_\|=0. \qquad (21)$$

Equations (9), (10), (12), (13), (18), and (21) completely describe the dynamics of plasma with finite electron and ion pressure.

At this point we clarify the accuracy with which the magnetic field must be specified in the various locations in the equations. The magnetic field $\vec{B}$ which appears in the $\vec{E}\times\vec{B}$, diamagnetic drifts and the ion and electron pressure equations can be approximated by the vacuum toroidal field [first term on the right side of Eq. (11)]. The $\nabla_\|$ operator must be written as

$$\nabla_\|=R_0\nabla\zeta\cdot\nabla+(R_0/B_0)\nabla\zeta\times\nabla\psi\cdot\nabla. \qquad (22)$$

Aside from Ohm's law [Eq. (12)], this is the only location where the poloidal magnetic field and poloidal magnetic perturbations enter the equations.

As noted in the introduction, the importance of the magnetic perturbations can be estimated from Ohm's law [Eq. (12)] by comparing the time derivative of the poloidal flux $\partial\psi/\partial t$ to the resistive term $c\,\eta_\|j_\|\simeq(c^2/4\pi)\,\eta_\|\nabla_\perp^2\psi$. When the resistive term dominates, magnetic perturbations can be neglected. Assuming $\partial/\partial t\sim\gamma_0$ and $\nabla_\perp^2\sim 1/L_0^2$ for γ_0, L_0 characteristic of drift-resistive ballooning modes:[9] $\gamma_0=|2p'/Rm_in|^{1/2}$, $L_0^2=(qRc)^2\pi\eta_\|\gamma_0/V_A^2$, one finds $\partial\psi/\partial t$ is negligible under the condition given earlier, $\alpha_{mhd}\ll 1$.

III. ENERGY CONSERVATION

To derive the energy conservation we multiply Eq. (10) by $e\phi$ and integrate over all space. Integrating by parts and dropping the surface integrals yields

$$\int dV[e\phi\nabla\cdot n\vec{v}_{\mathrm{pol}}-j_\|\nabla_\|\phi-(p_i+p_e)\nabla\cdot\vec{v}_E]=0. \qquad (23)$$

Integration of the ion pressure equation Eq. (18) yields

$$\int dV\left[\frac{3}{2}\frac{\partial p_i}{\partial t}+p_i\nabla\cdot(\vec{v}_E+\vec{v}_{\|i}+\vec{v}_{\mathrm{pol}})\right]=0. \qquad (24)$$

Adding the terms with $\vec{v}_{\mathrm{pol}}$ in Eqs. (23) and (24) and inserting $\vec{v}_{\mathrm{pol}}$ [Eq. (7)] leads to

$$\int dV[e\phi\nabla\cdot n\vec{v}_{\mathrm{pol}}+p_i\nabla\cdot\vec{v}_{\mathrm{pol}}]$$

$$=\int dV[nm_i\omega_{ci}\vec{b}\times(\vec{v}_E+\vec{v}_{di})\cdot\vec{v}_{\mathrm{pol}}]$$

$$=\int dV\left[(\vec{v}_E+\vec{v}_{di})\cdot\left(nm_i\frac{d}{dt}+p_i\nabla\times\frac{\vec{b}}{\omega_{ci}}\cdot\nabla\right)(\vec{v}_E+\vec{v}_{di})\right]$$

$$=\int dV\left[\frac{nm_i}{2}\left(\frac{d}{dt}+\vec{v}_{di}\cdot\nabla\right)(\vec{v}_E+\vec{v}_{di})^2\right]$$

where the last two terms in Eq. (7) for $\vec{v}_{\mathrm{pol}}$ arising from the ion stress tensor exactly cancel and where we have used the relation

$$\int dVp_i\nabla\times\frac{\vec{b}}{\omega_{ci}}\cdot\nabla f=\int dVm_in\vec{v}_{di}\cdot\nabla f.$$

Terms $\int dV[n\ df/dt]$ are treated using the continuity equation

$$\int dV\left[n\left(\frac{d}{dt}+\vec{v}_{di}\cdot\nabla\right)f\right]=\int dV\left[n\frac{\partial f}{\partial t}-f\nabla\cdot n\vec{v}\right] =\int dV\left[n\frac{\partial f}{\partial t}+f\frac{\partial n}{\partial t}\right].$$

Therefore the sum of Eqs. (23) and (24) takes the form

$$\frac{d}{dt}\int dV\left[\frac{nm_i}{2}(\vec{v}_E+\vec{v}_{di})^2+\frac{3}{2}p_i\right]$$

$$=\int dV[j_\parallel\nabla_\parallel\phi+p_e\nabla\cdot\vec{v}_E-p_i\nabla_\parallel v_{\parallel i}]. \tag{25}$$

Treating the electron pressure equation Eq. (21) analogous to the ion pressure equation yields

$$\frac{d}{dt}\frac{3}{2}\int dVp_e=-\int dV\left[p_e\nabla\cdot(\vec{v}_E+\vec{v}_{\parallel i})+\frac{j_\parallel}{en}\nabla_\parallel p_e+\frac{0.71}{e}j_\parallel\nabla_\parallel T_e\right]. \tag{26}$$

The equation for the parallel ion velocity is multiplied by $v_{\parallel i}$, and we obtain

$$\frac{d}{dt}\int dV\frac{nm_i}{2}v_{\parallel i}^2=\int dV[(p_i+p_e)\nabla_\parallel v_{\parallel i}]. \tag{27}$$

Adding Eqs. (25), (26), and (27) and using Eq. (12) leads to the total energy theorem

$$\frac{d}{dt}\int dV\left[\frac{nm_i}{2}|\vec{v}_E+\vec{v}_{di}+v_{\parallel i}|^2+\frac{3}{2}(p_e+p_i)+\frac{1}{8\pi}|\nabla_\perp\psi|^2\right]=-\int dV\eta_\parallel j_\parallel^2, \tag{28}$$

where the sink term on the right side reflects the Ohmic heating, which we did not include in the temperature equations.

IV. PARTIALLY LINEARIZED EQUATIONS

For numerical simulations the set of equations derived above is still very complicated. The complications arise because we have as yet made no assumptions with respect to the magnitude of the fluctuations. In principle, for example, the equations can describe density fluctuations of order unity. A simpler set of equations can be derived in the limit in which the perturbations have scale lengths $L_\perp$ much smaller than the characteristic equilibrium scale length L_p of the pressure or density profiles. In this limit the ratio of fluctuating to the average of a quantity scales like $L_\perp/L_p\ll 1$ and many of the nonlinearities which appear in the earlier equations can be discarded. We separate the turbulent variables into average n_0, T_{e0}, T_{i0} and fluctuating components (denoted by a tilde), and linearize the equations, but keep the $\vec{E}\times\vec{B}$-convection terms, which provide the dominant nonlinear interaction. The curvature drift of either the electrons or ions $v_{ce,i}\sim T_{e,i}/m_{e,i}R\omega_{ce,i}$ can be neglected in the convection terms since in a drift-wave ordering in which $e\phi/T_{e0}\sim\tilde{n}/n_0$, the characteristic time variation is given by $\partial/\partial t\sim v_E/L_\perp\sim v_{de,i}/L_\perp\sim(R/L_p)v_{ce,i}/L_\perp\gg v_{ce,i}/L_\perp$. Finally, the curvature terms can be evaluated explicitly as $\nabla\times(\vec{B}/B^2)=(2/B)\vec{b}\times\vec{\kappa}\simeq(2/B_0)\nabla R\times\nabla\zeta$ with $\vec{\kappa}=\vec{b}\cdot\nabla\vec{b}$. In the resulting equations, we use the abbreviations

$$T_e=\frac{\tilde{T}_e}{T_{e0}},\quad p_i=\frac{\tilde{p}_i}{p_{i0}},\quad n=\frac{\tilde{n}}{n_0},\quad \phi=\frac{e\tilde{\phi}}{T_{e0}},\quad v_\parallel=\frac{\tilde{v}_{\parallel i}}{c_s},$$
$$\psi=\frac{\omega_{ci}V_A\tilde{\psi}}{c_s^2B_0},\quad J=V_A\frac{c_s^2}{\omega_{ci}^2}\nabla_\perp^2\psi, \tag{29}$$

with $c_s^2=T_{e0}/m_i$, $V_A^2=B_0^2/(4\pi nm_i)$, and

$$h=\phi-p_e-0.71T_e,\quad \tau=\frac{T_{i0}}{T_{e0}},\quad \sigma=\frac{T_{e0}}{\eta_\parallel n_0e^2}. \tag{30}$$

Note the relation $p_e=n+T_e$ which holds for the normalized fluctuating components. After some manipulations we obtain

$$\frac{c_s^2}{\omega_{ci}^2}\nabla_\perp\cdot\frac{d}{dt}\nabla_\perp(\phi+\tau p_i)-\nabla_\parallel J-2\frac{c_s^2}{\omega_{ci}}\vec{b}\times\vec{\kappa}\cdot\nabla(p_e+\tau p_i)=0, \tag{31}$$

$$\frac{dn}{dt}+\frac{c_s^2}{\omega_{ci}}\frac{1}{L_n}\frac{\partial\phi}{\partial y}+2\frac{c_s^2}{\omega_{ci}}\vec{b}\times\vec{\kappa}\cdot\nabla(\phi-p_e)+c_s\nabla_\parallel v_\parallel-\nabla_\parallel J=0, \tag{32}$$

$$\frac{3}{2}\frac{dT_e}{dt}-\frac{dn}{dt}+\frac{c_s^2}{\omega_{ci}}\left(\frac{3}{2}\frac{1}{L_{T_e}}-\frac{1}{L_n}\right)\frac{\partial\phi}{\partial y}-\frac{\kappa_\parallel}{n_0}\nabla_\parallel\left(\nabla_\parallel T_e+\frac{c_s^2}{\omega_{ci}V_AL_{T_e}}\frac{\partial\psi}{\partial y}\right)-0.71\nabla_\parallel J=0, \tag{33}$$

$$\frac{3}{2}\frac{dp_i}{dt}+\frac{3}{2}\frac{c_s^2}{\omega_{ci}}\frac{1}{L_{p_i}}\frac{\partial\phi}{\partial y}+5\frac{c_s^2}{\omega_{ci}}\vec{b}\times\vec{\kappa}\cdot\nabla(\phi+\tau p_i)+\frac{5}{2}c_s\nabla_\parallel v_\parallel-\frac{5}{2}\frac{c_s^2}{\omega_{ci}^2}\nabla_\perp\cdot\frac{d}{dt}\nabla_\perp(\phi+\tau p_i)=0, \tag{34}$$

$$\frac{d}{dt}v_\parallel=-c_s\left(\nabla_\parallel(p_e+\tau p_i)+\frac{c_s^2}{\omega_{ci}V_A}\left(\frac{1}{L_{p_e}}+\frac{\tau}{L_{p_i}}\right)\frac{\partial\psi}{\partial y}\right), \tag{35}$$

$$\frac{1}{V_A}\frac{\partial\psi}{\partial t}+\frac{c_s^2}{\omega_{ci}V_A}\left(\frac{1}{L_{p_e}}+\frac{0.71}{L_{T_e}}\right)\frac{\partial\psi}{\partial y}-\nabla_\parallel h=\frac{J}{\sigma}, \tag{36}$$

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\frac{c_s^2R_0}{\omega_{ci}}\nabla\zeta\times\nabla\phi\cdot\nabla, \tag{37}$$

$$\nabla_\parallel=\nabla_{\parallel 0}+\frac{c_s^2R_0}{\omega_{ci}V_A}\nabla\zeta\times\nabla\psi\cdot\nabla, \tag{38}$$

$$\vec{b}\times\vec{\kappa}=\nabla R\times\nabla\zeta, \tag{39}$$

where $\nabla_{\parallel 0}$ is the equilibrium $\nabla_\parallel$, $L_n=|n/n'|$, $L_{T_e}=|T_e/T_e'|$, etc. In the linearized equations the ion cyclotron frequency ω_{ci} should be evaluated with the magnetic field B_0. These equations are a generalization of those presented in Ref. 5 to include finite T_i and a generalization of those of Ref. 8 to include the non-adiabatic dynamics of electrons and electromagnetic perturbations.

A straightforward calculation, in which for simplicity the ambient gradients are discarded, reveals that Eqs. (31)–(35) satisfy the conservation law,

$$\frac{d}{dt}\frac{1}{2}\int dV\left[\frac{c_s^2}{\omega_{ci}^2}[|\nabla_\perp(\phi+\tau p_i)|^2+|\nabla_\perp\psi|^2]+v_\parallel^2+n^2+\frac{3}{2}T_e^2 + \frac{3}{5}\tau p_i^2\right]=-\int dV\left[\frac{J^2}{\sigma}+\frac{\kappa_\parallel}{n_0}(\nabla_\parallel T_e)^2\right], \tag{40}$$

which plays the role of the energy in this partially linearized system.

V. CONCLUSION

A set of nonlinear reduced fluid equations have been derived to describe the dynamics of electron and ion thermal fluctuations, and potential, density, and magnetic fluctuations in magnetically confined plasma. Two limits have been considered. When the scale lengths of the fluctuations and equilibrium are comparable, the fluctuations can be comparable in amplitude to equilibrium quantities and no distinction between equilibrium and fluctuation has been made. The energy conservation law is of the classical form: kinetic energy associated with the motion of the ion fluid parallel and perpendicular to the magnetic field, thermal energy associated with both the electrons and ions, and magnetic energy. When the scale length of the fluctuations is small compared with equilibrium scale lengths, the fluctuation amplitudes are small and many of the nonlinearities in the system of equations can be discarded. The resulting equations are less complicated and can be readily utilized to study the relationship between particle and electron and ion thermal flux in low temperature plasma.

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[1] S. I. Braginskii, in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1965), Vol. I, p. 205.
[2] S.-T. Tsai, F. W. Perkins, and T. H. Stix, Phys. Fluids **13**, 2108 (1970).
[3] F. L. Hinton and C. W. Horton, Phys. Fluids **14**, 116 (1971).
[4] W. Horton, R. D. Estes, and D. Biskamp, Plasma Phys. **22**, 663 (1980).
[5] J. F. Drake and T. M. Antonsen, Phys. Fluids **27**, 898 (1984).
[6] R. D. Hazeltine, C. T. Hsu, and P. J. Morrison, Phys. Fluids **30**, 3204 (1987).
[7] W. Horton, Phys. Rep. **192**, 177 (1990).
[8] C. B. Kim, W. Horton, and S. Hamaguchi, Phys. Fluids B **5**, 1516 (1993).
[9] D. R. McCarthy, P. N. Guzdar, J. F. Drake, T. M. Antonsen, Jr., and A. B. Hassam, Phys. Fluids B **4**, 1846 (1992).