

# Nonlinear Regular Wave Generation in Numerical and Physical Flumes

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## ABSTRACT

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The generation of nonlinear regular waves in a numerical wave flume using first-order wavemaker theory is discussed comparing numerical results with free surface data from large scale physical tests (CIEM wave flume) and Stokes wave theories. A general formulation for the analysis of fluid-structure interaction problems is employed to simulate the numerical wave flume using the Particle Finite Element Method (PFEM). This method uses a Lagrangian description to model the motion of particles in both the fluid and the structure domains. With this work we can conclude that PFEM formulations simulate the generation of naturally-occurring nonlinear waves with different paddle types, for varied wave conditions and at different scales. Like in physical flumes if we use first-order wavemaker theory in numerical flumes unwanted nonlinearities can be found for some wave conditions.

**ADDITIONAL INDEX WORDS:** *Particle Finite Element Method, first-order wavemaker theory, unwanted nonlinearities*

## INTRODUCTION

Physical wave flumes have been widely applied in laboratory studies on hydraulic and stability behavior of coastal structures, beach profile evolution and other related coastal phenomena involving waves. The wave generation is one of most important tasks in this kind of laboratory studies.

The most common way to generate waves in physical flumes is through the movement of a paddle that is generally located at one of the ends of the flume. Of the several types of paddles used, we can identify as the most frequent the flap, piston and wedge types. They differ among themselves by the kind of movement executed and consequently, the imposed water boundary condition and the necessary mechanisms to control their movements.

An analytical solution for waves generated by piston and flap wavemakers based on linear wave theory was derived by HAVELOCK (1929). The first order wavemaker theory for a piston was experimentally verified by URSELL *et al.* (1960) and FLICK and GUZA (1980). HUDSPETH *et al.* (1981) did an experimental verification for the flap first order wavemaker theory. Due to the difference in the type of movement, the velocity field in the area near the paddle changes depending on the type of paddle used. If we compare the form of the velocity profiles generated by the three most common paddles near the wavemaker with the profiles according to linear theory, we can conclude that each paddle reproduces different conditions. Thus, for a flap we have profiles that are similar to deep water wave conditions, for piston shallow water wave conditions (DEAN AND DALRYMPLE, 1992) and for wedge intermediate water wave conditions (GIRONELLA, 2004). The maximum wave heights generated by a paddle depend on the wave period, the water depth in front of the paddle, and the power of the actuator. The firsts two are related with wave breaking and

the third with the maximum paddle stroke and velocity allowed (GIRONELLA, 2004).

In the last years numerical waves flumes begun to be considered as a possible tool to support the design of vertical breakwaters (OUMERACI *et al.*, 2001), the design of low crested structures (HAWKINS *et al.*, 2007) and the overtopping calculation (Overtopping Manual, 2007).

The numerical wave flumes presented in the scientific literature can be grouped in several ways. One simple way is to divide them into two groups, one based on Non Linear Shallow Water (NLSW) equations and another one based on the Navier Stokes equations.

Examples of numerical waves flumes based on NLSW can be found in VAN GENT (1994), DODD (1998) and HU *et al.* (2000).

Numerical wave flumes based on the Navier-Stokes equations are in most cases controlled by two techniques: i) Volume of Fluid (VOF) described the first time by HIRT and NICHOLS (1981) and ii) Smooth Particle Hydrodynamics (SPH) technique developed at the end of the 70s in the astrophysics community by GINGOLD and MONAGHAN (1977).

LEMOS (1990) developed a VOF numerical model for the study of the movement of two-dimensional waves. VAN GENT *et al.* (1994) presented a model that can simulate plunging waves breaking into porous structures using the VOF technique for solving (2D-V) Navier-Stokes equations for incompressible fluids. LIN and LIU (1998) described the development of the COBRA numerical model to study the evolution of groups of waves, shoaling and breaking in Swash zone. LARA *et al.* (2006) show the ability of the COBRA model to simulate the interaction of irregular waves with permeable slope structures.

The SPH application to Coastal Engineering began at the end of the 90's (MONAGHAN AND KOS, 1999). DALRYMPLE and ROGERS (2006) studied the plunging wave type breaking using a model

based on SPH method. SHAO *et al.* (2006) presented an incompressible SPH model to investigate overtopping in coastal structures.

The Moving Particle Semi-implicit (MPS) method proposed by KOSHIZUKA *et al.* (1995) and the Particle Finite Element Method (PFEM) revised in OÑATE *et al.* (2004) are other two less common methods based on Navier Stokes equations and used as numerical wave flumes (KOSHIZUKA *et al.* 1998 and OLIVEIRA *et al.* 2007).

In numerical wave flumes based on non fixed mesh methods as SPH, MPS and PFEM the generation of waves by means of different wavemaker types is possible. In these cases the wavemakers can be simulated by means of solid bodies located at one end of the flume and moving according to the transfer functions, the same ones used in physical flumes to determine the paddle movement. The selection of the paddle type could be based on the wave condition required and contrary to physical flumes the channel dimensions, the stroke and actuator velocity is not a limitation in the maximum generated wave height.

Within this context, the main aim of this paper is to study and compare the nonlinear regular wave generation in a numerical wave flume based on the PFEM formulation and using the same first order wavemaker theory that is used in physical flumes to generate waves.

## PARTICLE FINITE ELEMENT METHOD

The PFEM is a well known method in literature (OÑATE *et al.*, 2004). However, some important key features of the PFEM are presented in this paper.

The PFEM is a particular class of Lagrangian flow formulation developed at the International Center for Numerical Methods in Engineering (CIMNE) in Barcelona to solve free surface flow problems involving large motions of the free surface, as well as the interaction with rigid bodies.

The PFEM treats the mesh nodes in the fluid and solid domains as particles which can freely move and even separate from the main fluid domain representing, for instance, the effect of water drops or melted zones. The finite element method (FEM) is used to solve the continuum equations in both domains. Hence a mesh discretizing these domains must be generated in order to solve the governing equations for the fluid, in the standard FEM fashion. To do this, the nodes discretizing the fluid and solid domains are treated as material particles whose motion is tracked during the transient solution.

In the PFEM the motion of the individual particles are followed using a Lagrangian description and, consequently, nodes in a finite element mesh can be viewed as moving *particles*. Hence, the motion of the mesh discretizing the total domain (including both the fluid and solid parts) is followed during the transient solution.

An obvious advantage of the Lagrangian formulation used in the PFEM is that the convective terms disappear from the fluid and energy equations. The difficulty is however transferred to the problem of adequately (and efficiently) moving the mesh nodes. Indeed for large mesh motions remeshing may be a frequent necessity along the time solution. An innovative mesh regeneration procedure is used, based on the well known Delaunay Tessellation (IDELSOHN *et al.*, 2003).

It must be noted that the information in the PFEM is typically nodal-based, i.e. the element mesh is mainly used to obtain the values of the state variables (i.e. velocities, pressure, viscosity, etc.) at the nodes. A difficulty arises in the identification of the boundary of the domain from a given collection of nodes. Indeed the *boundary* can include the free surface in the fluid and the individual particles moving outside the fluid domain. The Alpha

Shape technique is used to identify the boundary nodes (IDELSOHN *et al.*, 2003).

In the PFEM both the fluid and the solid domains are modelled using an updated Lagrangian description of the motion. That is, all variables in the fluid and solid domains are assumed to be known in the current configuration at time  $t$ . The new set of variables in both domains is sought for in the next or updated configuration at the next time step.

## NONLINEAR WAVE GENERATION

In the following two points the validation of naturally-occurring nonlinearities in wave generation is made comparing free surface data from a numerical flume with experimental data and with theoretical free surface profiles given by Stokes Wave theories.

Then, scale effects in wave generation are analyzed comparing data from three different scale numerical flumes.

Finally, an example of unwanted nonlinearities generated in a numerical flume using first-order wavemaker theory is presented.

## Comparison with experimental data

The experiments were carried out at the Maritime, Experimental and Research Flume (CIEM, Canal d'Investigació I Experimentació Marítima) of the Maritime Engineering Laboratory (LIM, Laboratori d'Enginyeria Marítima) of the Technical University of Catalonia (UPC, Universitat Politècnica de Catalunya). The flume is 100m long, 3m wide and 5m deep. Due to its dimensions the CIEM flume is an excellent tool for scaled tests and analyses close to reality allowing a reduction of scale effects inherent to all scaled experiments. Controlled wave generation is achieved by a wedge-type wave paddle, particularly suited for intermediate depth waves.

The experimental set-up is presented in Figure 1. A rigid bottom was used across the flume and a dissipative rock beach was constructed at the right end side. A constant 2.62m water depth zone in front of the paddle has been separated from a second 1.50m constant water depth zone by a 1V:9.3H follow by a 1V:36.2H slopes. Six resistance wave gauges (WG) were used to measure the free surface elevation. Their positions are represented in Figure 1. A positional sensor was used to measure the paddle movement. Twenty two regular wave cases were tested for different wave heights ( $0.1\text{m} < H < 0.6\text{m}$ ) and wave periods ( $0.5\text{s} < T < 4.0\text{s}$ ).

The firsts 100s of experiments were simulated using PFEM for the three cases denoted in this work as case 1 ( $H=0.182\text{m}$ ,  $T=3.0\text{s}$ ), case 2 ( $H=0.351$ ,  $T=4.0\text{s}$ ) and case 3 ( $H=0.546\text{m}$ ,  $T=3.0\text{s}$ ). The maximum time step used during the simulations was 0.01s and the nodes distance 0.10m corresponding to 14045 initial nodes.

The free surface for case 3 at 58s of simulation is presented at Figure 1. The waves were generated reproducing the paddle movement recorded by the positional sensor used during the physical experiments.

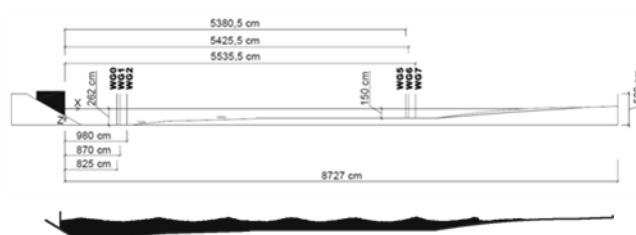


Figure 1. Experimental set-up and numerical flume for  $H=0.546\text{m}$ ,  $T=3.0\text{s}$  (case 3) after 58s of simulation

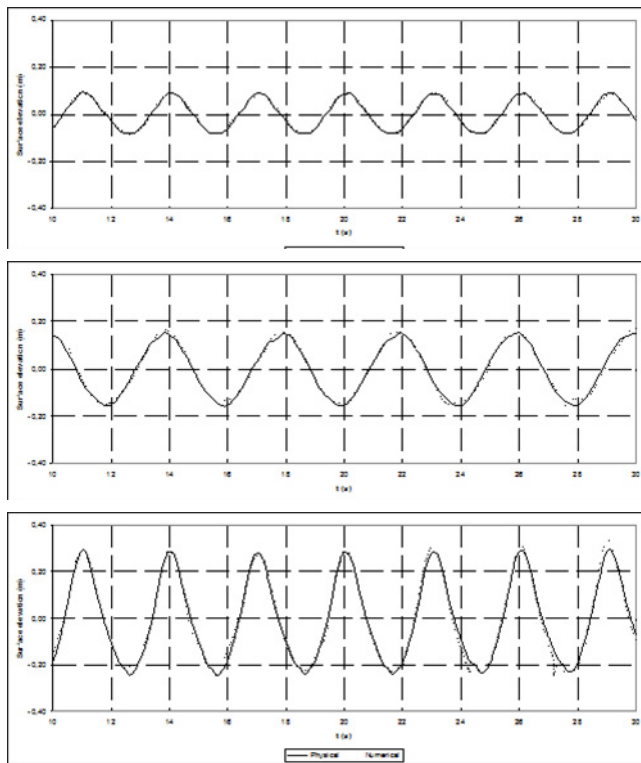


Figure 2. Comparison between physical and numerical free surface at WG0; case 1 ( $H=0.182\text{m}$ ,  $T=3.0\text{s}$ ), case 2 ( $H=0.351$ ,  $T=4.0\text{s}$ ) and case 3 ( $H=0.546\text{m}$ ,  $T=3.0\text{s}$ )

The paddle was simulated as a vertical solid wall moving through a 30 degrees inclined plane according with CIEM wavemaker system layout.

The free experimental water surface obtained experimentally at WG0 (see Figure 1) is compared with numerical results for case 1, 2 and 3 in Figure 2 where the dashed line represents physical data and the dot line numerical results.

### Comparison with Stokes Waves Theory

A numerical prototype scale flume with a 15m constant water depth with 250m length and with a dissipative beach of 250m length was simulated with PFEM. Four regular wave cases, called here case A ( $H=0.20\text{m}$ ,  $T=6.93\text{s}$ ), case B ( $H=1.25\text{m}$ ,  $T=6.93\text{s}$ ), case C ( $H=5.00\text{m}$ ,  $T=6.93\text{s}$ ) and case D ( $H=7.00\text{m}$ ,  $T=6.93\text{s}$ ) were generated using a numerical piston paddle. The paddle movement was determined using the corresponding first order wavemaker theory.

The maximum time step used during the simulations was 0.02s and the nodal distance 0.50m corresponding to 23078 initial nodes. A snapshot of the four cases is presented in Figure 3 after



Figure 3. Snapshot for case A, B, C and D (order from top to bottom), 58s after the beginning of simulation

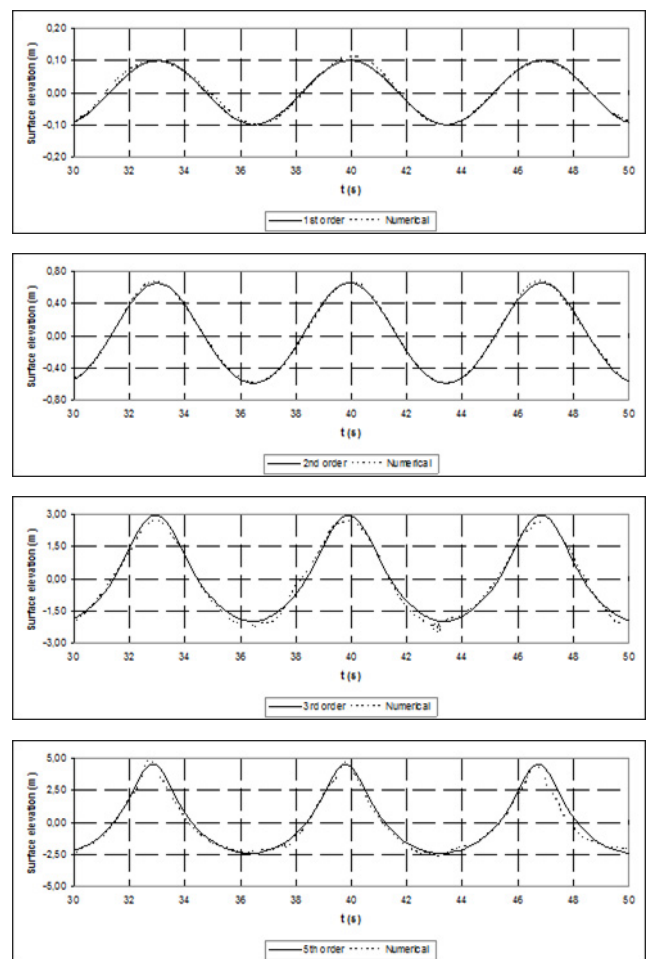


Figure 4. Comparison between numerical and Stokes wave theory for case A ( $H=0.20\text{m}$ ,  $T=6.93\text{s}$ ), case B ( $H=1.25\text{m}$ ,  $T=6.93\text{s}$ ), case C ( $H=5.00\text{m}$ ,  $T=6.93\text{s}$ ) and case D ( $H=7.00\text{m}$ ,  $T=6.93\text{s}$ )

58s of simulation.

The comparison between the free surface given by the smallest order wave theory applicable to wave conditions tested according LE MÉHAUTÉ (1976) and the numerical results obtained at 50m far from the paddle is presented in Figure 4 for all cases. For case A the 1<sup>st</sup> order Stokes theory is applicable, for case B the 2<sup>nd</sup> order, case C with the 3<sup>rd</sup> order, and case D with 5<sup>th</sup> order.

### Scale effects analysis

A regular wave of  $H=0.75\text{m}$  and  $T=6.93\text{s}$  was generated in the prototype scale numerical wave flume presented above. This situation was repeated in a 1:5 large scale numerical flume and in a 1:30 small scale numerical flume, with corresponding regular waves of  $H=0.15\text{m}$ ,  $T=3.10\text{s}$  and  $H=0.025\text{m}$ ,  $T=1.27\text{s}$ .

Table 1 summarizes the three flume dimensions and the numerical features used in each scale.

Table 1: Numerical features and flume dimensions.

Scale	Mesh size (m)	Time step (s)	Depth (m)	Flume length (m)
1:1	0.50	0.020	15.0	500.0
1:5	0.10	0.009	3.0	100.0
1:30	0.02	0.004	0.5	16.6

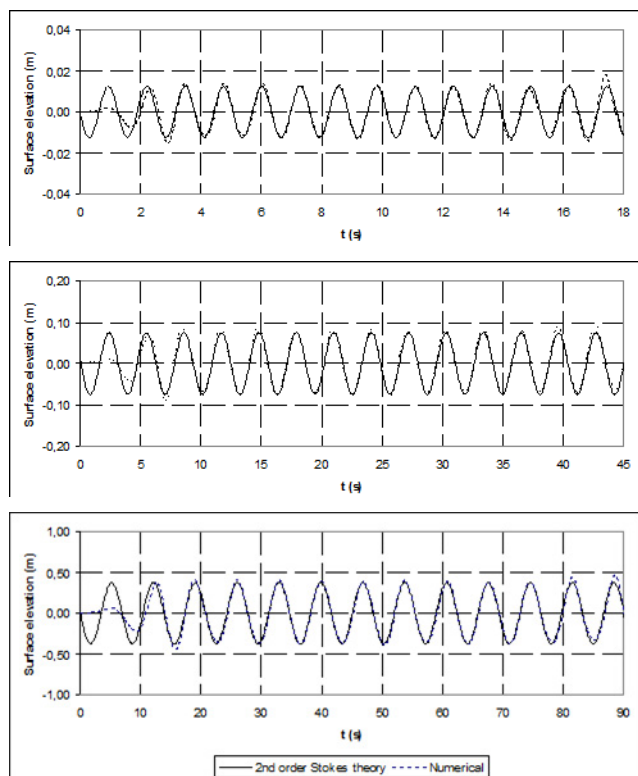


Figure 5. Comparison between numerical results and 2<sup>nd</sup> order Stokes theory for small scale ( $H=0.025\text{m}$ ,  $T=1.27\text{s}$ ), large scale ( $H=0.15\text{m}$ ,  $T=3.10\text{s}$ ) and prototype scale ( $H=0.75\text{m}$ ,  $T=6.93\text{s}$ )

The paddle movement in the three scales was generated using the first order wavemaker theory for a piston paddle type. In Figure 5 the free surface obtained at small, large and prototype scale at 1.67m, 10m and 50m respectively in front of paddle is presented and compared with the free surface given by second order Stokes wave theory.

### Unwanted nonlinearities generation

A regular wave of  $H=5.00\text{m}$  and  $T=13.0\text{s}$  was generated in the prototype scale numerical wave flume presented above. A piston paddle type was used to generate the wave and its movement was calculated using the first order wavemaker theory. The maximum time step used during the simulations was 0.02s and the nodes distance 0.50m corresponding to 23078 initial nodes.

The free surface obtained at 50m, 100m, and 150m far from the paddle is presented in Figure 6.

### DISCUSSION

As it is seen in Figure 2 the numerical free surface results are in a good agreement with physical data. The wave height, wave period and wave shape are well reproduced by the numerical model in the three cases. However, in maximum steepness wave cases (case 3) differences up to 0.05m at the wave crest are found. Differences between the three cases in wave shape due to nonlinear effects are reproduced with numerical and physical models.

Analyzing Figure 4 it is possible to see that for case A and B the wave shapes obtained with PFEM agree well with the wave shapes proposed by the Stokes wave theories. Increasing wave height and

consequently wave steepness some differences can be found (case C and D).

For case C the wave height generated is 0.20m less than the theoretical wave height being the largest differences at the wave crests. For case D at the wave crest the free surface is steeper for numerical results than in theory. As we can expect is possible to see also differences induced by nonlinear effects in wave shapes for all four cases.

Observing Figure 5 we can see that the numerical results for the three scales studied are in good agreement with 2<sup>nd</sup> Stokes wave theory. The free surface in the three scaled numerical flumes is qualitative equivalent. At the three cases the first arriving wave is smaller than expected and the second one hasn't the imposed period. This characteristic can be considered as transient waves due to wavemaker movement start effect.

As is possible to see in numerical free surface presented in Figure 6 a second crest is generated also by the paddle. The second crest position in the wave profile is not the same one at different distances from the paddle. This means that the second crest travels at a velocity different from that of the principal wave.

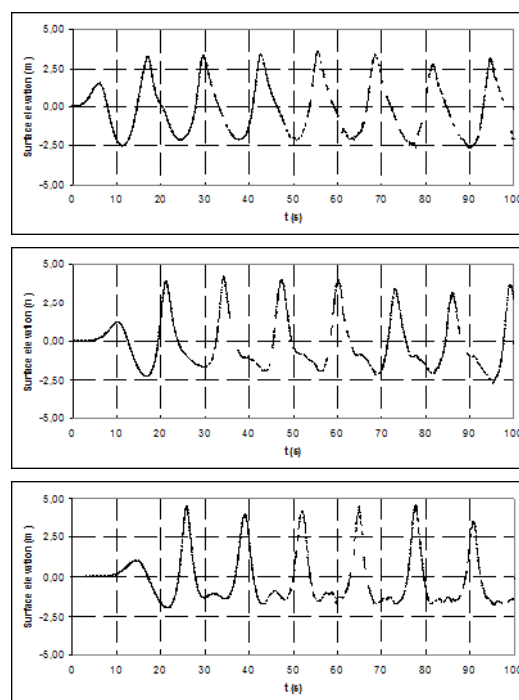


Figure 6. Numerical free surface at 50m, 100m and 150m far from the paddle ( $H=5\text{m}$ ;  $T=13\text{s}$ )

### CONCLUSIONS

With the capacity of PFEM to simulate solid-fluid interactions the generation of linear and nonlinear regular waves by means of different paddles types is possible.

Based on results here presented, with the PFEM formulation it is possible to simulate and to obtain the same waves generated in physical flumes reproducing the recorded physical paddle movement in the numerical paddle. By this way the real paddle performance can be imposed in the numerical paddle. This is an advantaged in front of others numerical flumes that can't add the "imperfections" in the real wavemaker systems.

The capacity of PFEM to simulate a large motion of the free surface allows the reproduction of very steep waves.

The first-order wavemaker theory used in physical flumes for different paddle types can be used in PFEM to generate a bigger range of wave conditions.

Generation of waves at different scales, including prototype dimensions can be reproduced with this numerical facility.

Like in physical flumes transient waves appear in PFEM numerical flume and should be taken into account in numerical wave studies.

The unwanted nonlinearities presents in laboratories with first-order wavemaker performance also occur in PFEM numerical flume.

PFEM numerical wave flume is a possible tool to find the range of applicability of first order wavemaker theory without the generation of unwanted nonlinearities.

Higher order wavemaker theories can be a possible solution to avoid the generation of unwanted wave nonlinearities in numerical flumes. The second-order wavemaker theories being used successfully at some physical flumes can be tested in numerical flumes based on the PFEM formulation.

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