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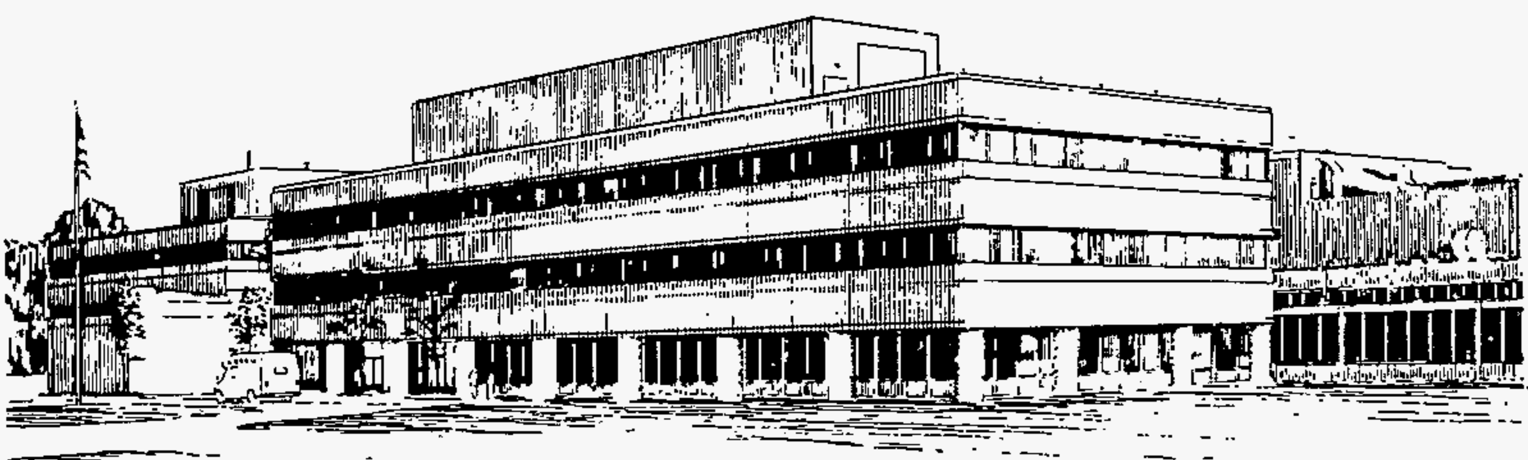

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Nonlinear Saturation of Toroidal Alfvén Eigenmodes via Ion Compton Scattering

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The nonlinear interactions of high mode number Toroidal Alfvén Eigenmodes (TAE), mediated via Compton scattering off the bulk ions, are investigated. It is shown that nonlinear $\boldsymbol{J}_\perp \times \boldsymbol{B}_\perp$ ponderomotive force produced by TAE's interaction drives sound wave like density fluctuation with low phase velocity which can resonantly interact with the bulk ion parallel motion. Consequently, fluctuation energy of TAE's is transferred to lower frequency and eventually absorbed by linearly stable TAE's near the lower shear-Alfvén continuum, leading to nonlinear saturation. Explicit expression for the saturated magnetic amplitude is derived.

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Since the importance of the toroidicity–induced Alfvén eigenmode (TAE) [1] in present and future generation tokamak devices has been indicated, significant progress has been made in experimental identification [2–5] and detailed predictions on the linear stability [6,7]. High-N (toroidal mode number) TAE's have higher linear growth rates and it is important to know their amplitudes at nonlinear saturation and the implications on the ensuing alpha (α) particle loss. Previous studies [8–11] on the nonlinear interaction of high-N shear Alfvén fluctuations are based on a working paradigm of kinetic Alfvén wave (KAW). We note that TAE's are qualitatively different from KAW's in their mode structure and dispersion characteristics. Nonlinear behavior of TAE's, on the other hand, has been theoretically studied only for low-N modes in the context of single wave trapping, profile modification [12–14], and magnetohydrodynamic (MHD) mode coupling [15,16]. In this letter, we consider a different nonlinear mechanism which is relevant to high–N TAE's saturation. As well-known, the linear coupling of different poloidal harmonics due to the toroidal magnetic field variation on a given flux surface induces the formation of a TAE. A TAE typically contains a few dominant poloidal harmonics and its eigenfrequency is most sensitive to the equilibrium parameters at the amplitude peak. Therefore, there exist many ($O(Nq)$, here q is the safety factor) high-N TAE modes with the same toroidal mode number, and they differ by their radial locations and eigenfrequencies. We investigate, as a candidate for the saturation mechanism, the nonlinear interactions among them mediated via Compton scattering off thermal ions. Specifically, we show that nonlinear $\boldsymbol{J}_\perp \times \boldsymbol{B}_\perp$ ponderomotive force produced by TAE's interaction drives sound-wave-like density fluctuation which in turn induces the spectral transfer of fluctuation energy toward lower–frequency TAE's. The fluctuation energy is eventually absorbed by linearly stable TAE's near the lower shear Alfvén continuum. The fluctuation level at nonlinear saturation is then predicted to be

$$\left(\frac{\delta B_r}{B_0}\right)^2 \simeq \frac{1}{4\pi}(1+\frac{T_e}{T_i})^2\epsilon^4\left(\frac{\bar{\gamma}_L}{\omega_A}\right),$$

where $\epsilon \equiv 2(r_o/R_o+\Delta')$ is the effective local inverse aspect ratio which quantifies the strength of the toroidal coupling [17]. Δ' is the radial derivative of the Shafranov shift, $\omega_A = v_A/2qR_0$, $v_A^2 = B_0^2/4\pi\rho$, and $\bar{\gamma}_L$ is the spectrum average value of the linear growth rates (Eq. (16)).

We perform a third order nonlinear perturbation theory. To the first order, linear evolution of a test TAE (wave number $\boldsymbol{k}$) is described by the ideal MHD equation which consists of the following frozen-in-flux constraint and vorticity equation:

$$\frac{\partial}{\partial t}\psi_{\boldsymbol{k}}^{(1)} = -\mathbf{n}\cdot\nabla\phi_{\boldsymbol{k}}^{(1)}, \tag{1}$$

and

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi_{\boldsymbol{k}}^{(1)} = -v_A^2\mathbf{n}\cdot\nabla\nabla_\perp^2\psi_{\boldsymbol{k}}^{(1)} \tag{2}$$

where $\phi_{\boldsymbol{k}}$ and $\psi_{\boldsymbol{k}}$ are the perturbed electrostatic potential and parallel vector potential respectively, and $\mathbf{n} = \boldsymbol{B}_0/B_0$. In this work, we shall focus exclusively on the nonlinear interactions and refer the reader to the literature for details of the linear theory. With this approach in mind, no explicit expressions for the linear drive and damping will be given which, in any case, have little effects on the mode structure and the nonlinear interactions.

To the second order, the interaction of counter propagating components of two TAE's modes produces $\delta\boldsymbol{J}_\perp\times\delta\boldsymbol{B}_\perp\cdot\mathbf{n}$ ponderomotive force which drives sound wave like density perturbation with low phase velocity. Here, it is crucial to recall that each poloidal harmonics of a given TAE has a different radially varying $k_\|$, although they share the same eigenfrequency. Each poloidal harmonics can propagate either parallel or anti-parallel to the magnetic field depending on the radial location. A particular combination of two TAE components which produces low phase velocity density perturbation is $(\omega_{\boldsymbol{k}}, N, m)$ and $(\omega_{\boldsymbol{k}'}, N, m+1)$, because they have the opposite signs of $k_{\parallel}$ (i.e., a back scattering) at the gap position. Here, m is the poloidal mode number. The phase velocity of a beat wave is

$$\frac{\omega''}{k''_{\parallel}} = \frac{\omega - \omega'}{k_{\parallel} - k'_{\parallel}} \simeq qR(\omega - \omega')$$

at the gap position where the amplitude is significant. We also note that the radial overlap of the amplitude is most easily achieved for the interaction of TAE modes sharing the same toroidal mode number. The production of low frequency density fluctuation is described by the nonlinear ion drift kinetic equation [18]:

$$\begin{aligned}\left(\frac{\partial}{\partial t} + v_{\parallel}\boldsymbol{b}\cdot\boldsymbol{\nabla}\right)\delta f + \boldsymbol{v}_E\cdot\nabla f_0 + \left(\frac{e}{M_i}E_{\parallel} - \boldsymbol{b}\cdot\frac{d}{dt}\boldsymbol{v}_E\right)\frac{\partial f_0}{\partial v_{\parallel}} \\ = \left(\boldsymbol{b}\cdot\frac{d}{dt}\boldsymbol{v}_E - \frac{e}{M_i}E_{\parallel}\right)\frac{\partial \delta f}{\partial v_{\parallel}} - \boldsymbol{v}_E\cdot\nabla\delta f, \qquad (3)\end{aligned}$$

where $\boldsymbol{v}_E = (\mathbf{n}\times\nabla\phi)/B_0$, f_0 and δf are the equilibrium distribution function and the perturbed distribution function respectively, and $\boldsymbol{b} \equiv \mathbf{n} + \delta\boldsymbol{B}/B_0$ is the unit vector along the total magnetic field. Since a test TAE and the background TAE's (wave number $\boldsymbol{k}'$) are both high frequency ($\omega \gg \omega_{*e} \equiv (cT_e/eB_0)k_\theta/L_n$) fluctuations, we have $\delta f_{\boldsymbol{k}}/f_0 \ll e\phi_{\boldsymbol{k}}/T_e$, and $\delta f_{\boldsymbol{k}'}/f_0 \ll e\phi_{\boldsymbol{k}'}/T_e$, and the nonlinear terms on the right hand side (RHS) are negligible. Then, the evolution equation for $\delta f_{\boldsymbol{k}''}$ (sound wave like beat wave) simplifies to

$$\left(\frac{\partial}{\partial t} + v_{\parallel}\mathbf{n}\cdot\nabla\right)\delta f_{\boldsymbol{k}''} = \left\{\left(\frac{\delta\boldsymbol{B}}{B_0}\cdot\frac{\partial}{\partial t}\boldsymbol{v}_E\right)_{\boldsymbol{k}''} + \frac{|e|}{M_i}\mathbf{n}\cdot\nabla\phi_{\boldsymbol{k}''}\right\}\frac{\partial f_0}{\partial v_{\parallel}}, \qquad (4)$$

where $\boldsymbol{k}'' \equiv \boldsymbol{k} - \boldsymbol{k}'$. Since $\partial\boldsymbol{v}_E/\partial t = \boldsymbol{v}_{\text{pol}} \times \boldsymbol{B}_0/M_i$, the first term on the RHS is proportional to $(\delta\boldsymbol{B}_{\perp} \times \boldsymbol{v}_{\text{pol}} \cdot \mathbf{n})_{\boldsymbol{k}''}$. Noting that the perturbed perpendicular current is carried by the ion polarization drift, we can write this term as

$$\frac{1}{n_0|e|}(\delta \boldsymbol{B}_{\perp \boldsymbol{k}} \times \delta \boldsymbol{J}^*_{\perp \boldsymbol{k}'} + \delta \boldsymbol{B}^*_{\perp \boldsymbol{k}} \times \delta \boldsymbol{J}_{\perp \boldsymbol{k}}) \cdot \mathrm{n}$$

which clearly indicates that it is the ponderomotive force produced by nonlinear interaction of two high frequency TAE' ($\boldsymbol{k}$ and $\boldsymbol{k}'$). This nonlinearity has been studied within MHD fluid description in a straight ambient B_0 [19,20]. In Eq. (4), we have not kept the nonlinearity due to the $\boldsymbol{E} \times \boldsymbol{B}$ convection of the magnetic flux ($\boldsymbol{v}_E \cdot \nabla\psi = \delta \mathrm{B} \cdot \nabla\phi / B_0$) assuming $(k_\perp \rho_i)^2 \ll \omega/\Omega_{ci}$. In the opposite limit which corresponds to the short wavelength KAW's, that nonlinearity has been shown to be important [8]. Finally, $\omega'' \ll \omega, \omega'$ makes the induction field contribution $(\partial \psi_{\boldsymbol{k}''}/\partial t)$ to the RHS of Eq. (4) insignificant. A standard Fourier decomposition and straightforward vector algebra lead to

$$-i(\omega'' - k''_\parallel v_\parallel)\delta f^{(2)}_{\boldsymbol{k}''} = -\frac{iv_\parallel}{v_{Ti}^2}\left\{\frac{|e|}{M_i}k''_\parallel \phi^{(2)}_{\boldsymbol{k}''} + \frac{1}{B_0^2}\left(k_\parallel \frac{\omega'}{\omega} - k'_\parallel \frac{\omega}{\omega'}\right)\nabla\phi^{(1)*}_{\boldsymbol{k}'} \times \mathrm{n} \cdot \nabla\phi^{(1)}_{\boldsymbol{k}} \times \mathrm{n}\right\} f_0. \quad (5)$$

The corresponding density perturbation is

$$\frac{\delta n^{(2)}_{\boldsymbol{k}''}}{n_0} = \chi_i(\boldsymbol{k}'')\left\{|e|\phi^{(2)}_{\boldsymbol{k}''} + \frac{M_i}{B_0^2}\frac{(k_\parallel \omega'/\omega - k'_\parallel \omega/\omega')}{k_\parallel}\nabla\phi^{(1)*}_{\boldsymbol{k}'} \times \mathrm{n} \cdot \nabla\phi^{(1)}_{\boldsymbol{k}} \times \mathrm{n}\right\}, \quad (6)$$

where

$$\chi_i(\boldsymbol{k}'') \equiv \frac{1}{T_i n_o}\int d^3v \frac{k''_\parallel v_\parallel}{\omega'' - k''_\parallel v_\parallel} f_{oi}$$

is the ion linear susceptibility at $\boldsymbol{k}''$. Since the electron nonlinearity is negligible due to smaller mass, the shielding potential $\phi^{(2)}_{\boldsymbol{k}''}$ can be obtained via the following quasi-neutrality condition

$$\frac{\delta n^{(2)}_{\boldsymbol{k}''}}{n_0} = \frac{\delta n^{(2)}_{e \cdot \boldsymbol{k}''}}{n_0} = -\chi_e(\boldsymbol{k}'')|e|\phi^{(2)}_{\boldsymbol{k}''}.$$

Here,

$$\chi_e(\boldsymbol{k}'') \equiv \frac{1}{T_e n_0}\int d^3v \frac{k''_\parallel v_\parallel}{\omega'' - k''_\parallel v_\parallel} f_{oe}$$

is the electron linear susceptibility. The partial shielding contribution reduces the nonlinear density perturbation to, from Eq. (6),

$$\frac{\delta n^{(2)}_{\boldsymbol{k}''}}{n_0} = \frac{\chi_i\chi_e}{\chi_i+\chi_e}\frac{M_i}{B_0^2}\frac{1}{k''_\parallel}\Big(\frac{k_\parallel\omega'}{\omega} - \frac{k'_\parallel\omega}{\omega'}\Big)\nabla\phi^{(1)*}_{\boldsymbol{k}'}\times\mathrm{n}\cdot\nabla\phi^{(1)}_{\boldsymbol{k}}\times\mathrm{n}. \tag{7}$$

In this work, we shall concentrate on the resonant (imaginary) part of Eq. (7) which is responsible for the ion Compton scattering leading to the spectral transfer of fluctuation energy to lower frequency region. This restricts the applicability regime of our theory to low β_i ($\beta_i < \epsilon^2$). For higher value of β_i, the nonlinear shift of real frequency caused by the real part of Eq. (7) must be considered. Noting

$$Im\chi_e \ll Im\chi_i \equiv -\sqrt{\frac{\pi}{2}}\frac{sgn(k''_\parallel)}{T_i}\frac{\omega''/k''_\parallel}{v_{T_i}}e^{-(\omega''/k''_\parallel)^2/2v^2_{T_i}},$$

We then have, in Eq. (7),

$$Im\Big(\frac{\chi_e\chi_i}{\chi_e+\chi_i}\Big) \simeq \frac{\chi_e^2}{|\chi_e+\chi_i|^2}Im\chi_i.$$

To the third order, the nonlinear evolution equation of a test TAE in the presence of the turbulent bath of other TAE's ($\boldsymbol{k}'$) and the low frequency density perturbartion ($\boldsymbol{k}''$) is derived. Since the frozen-in-flux constraint is not affected by $\delta n^{(2)}_{\boldsymbol{k}''}$, it suffices to study vorticity equation. Noting that the polarization current, $\delta J_\perp = (1/B_0)\mathrm{n}\times\rho(d\boldsymbol{v}_E/dt)$ depends on the ion number density, we obtain the following modified vorticity equation:

$$v_A^2\mathrm{n}\cdot\nabla\nabla_\perp^2\psi_{\boldsymbol{k}} + \frac{\partial}{\partial t}\nabla_\perp^2\psi_{\boldsymbol{k}} + \sum_{\boldsymbol{k}'}\nabla\cdot\Big(\frac{\delta n^{(2)}_{\boldsymbol{k}''}}{n_0}\Big)\frac{\partial}{\partial t}\nabla_\perp\phi^{(1)}_{\boldsymbol{k}'} = 0. \tag{8}$$

Here, we note that the nonlinearities due to field-line-bending $\nabla\psi_{\boldsymbol{k}'}\times\mathrm{n}\cdot\nabla\nabla_\perp^2\psi_{\boldsymbol{k}}$ and $\mathrm{E}\times\mathrm{B}$ convection of vorticity $(\nabla\phi_{\boldsymbol{k}'}\times\mathrm{n}\cdot\nabla\nabla_\perp^2\phi_{\boldsymbol{k}})$ are subdominant to the last term [19]. Using the linear dispersion relation and the fact that $k_r^{(')} \gg k_\theta^{(')} \gg k_\parallel^{(')}$, we can greatly simplify the nonlinear term of Eq. (8). By multiplying $\phi^*_{\boldsymbol{k}}$ to $\partial(Eq.\ (8))\partial t$ and taking the imaginary part of the spatial average, we finally arrive at the following wave-kinetic equation for $I_{\boldsymbol{k}} \equiv < |\nabla_\perp \phi_{\boldsymbol{k}}|^2 >$,

$$\frac{\partial}{\partial t} I_{\boldsymbol{k}} = \gamma_L(\boldsymbol{k}) I_{\boldsymbol{k}} - \sum_{\boldsymbol{k}'} M_{\boldsymbol{k},\boldsymbol{k}'} I_{\boldsymbol{k}'} I_{\boldsymbol{k}}, \tag{9}$$

where

$$M_{\boldsymbol{k},\boldsymbol{k}'} \equiv \frac{\omega'}{2} \frac{\chi_e^2 Im\chi_i}{|\chi_i + \chi_e|^2} \frac{M_i}{B_0^2}.$$

At nonlinear saturation, the RHS must vanish. The mode summation over various nonlinear interaction channels can be approximated by an integral (the continuum approximation), if there exist many TAE's within the strong nonlinear interaction range $v_{Ti}/qR \sim \beta_i^{1/2}\omega_A$. Since eigenfrequencies of TAE's separated by Δr differ by $\omega_A/L_A \Delta r$ ($L_A^{-1} = |\partial \ln \omega_A/\partial r|$), the adjacent TAE's frequency difference is given by $\Delta\omega \simeq r_0\omega_A/(Nq\hat{s}L_A)$, ($\hat{s}$ is the magnetic shear). Therefore, the continuum approximation is justified for $\beta_i^{1/2} \gg 1/Nq\hat{s}$, taking $L_A \sim r_0$. Now, denoting TAE's with their eigenfrequencies, the nonlinear saturation condition becomes

$$\gamma_L(\omega) = \int d\omega' M_{\omega,\omega'} I(\omega'), \tag{10}$$

where $\sum_{\boldsymbol{k}'} I_{\boldsymbol{k}'}\delta(\omega' - \omega_{\boldsymbol{k}'})$ has been replaced by its continuum version $I(\omega')$.

Noting that $(k_\parallel \omega'/\omega) - (k'_\parallel \omega/\omega') \simeq k''_\parallel$, at the gap position ($k_\parallel = -k'_\parallel \simeq 1/2qR$, $|\omega - \omega'| \lesssim (r_0/R)\omega$), we can write $M_{\omega,\omega'} = \omega' V(\omega'')$, where $V(\omega'') \equiv (M_i/2B_0^2)\chi_e^2 Im\chi_i/|\chi_i + \chi_e|$. Then, Eq. (10) can be converted to a differential equation if $\epsilon > \beta_i^{1/2}$, because the kernel $V(\omega'')$ varies faster than $\omega' I(\omega')$ as a function of ω'. Using $\omega' I(\omega') \simeq \omega I(\omega) - \omega'' \partial(\omega I)/\partial\omega$, we obtain

$$\gamma_L(\omega) = \int d\omega' V(\omega'')\Big\{\omega I - \omega''\partial(\omega I)/\partial\omega\Big\}. \tag{11}$$

The higher frequency modes tend to be TAE's excited near the plasma center (recall that $\omega_A = v_A/2qR_0$, $q(r)$ is typically an increasing function of r) where the hot particle pressure gradient peaks, they are more unstable. The lower frequency TAE's tend to be in a region where the hot particle drive is weaker. Furthermore, if a TAE frequency is close enough to the lower continuum, TAE is linearly stable due to strong continuum damping [21,22]. With this in mind, we can specify the integral limits of Eq. (11). The upper limit is given by ω_M, the highest frequency of linearly unstable TAE's, and the lower limit is given by ω_1, the lowest frequency with $I(\omega) \geq 0$. We note that ω_1 corresponds to a linearly stable TAE because the downward spectral transfer due to ion Compton scattering can nonlinearly excite a linearly stable mode. Since both ω_m and ω_1 are within a gap, $\omega_M - \omega_1 \lesssim \epsilon\omega_A$. Changing the integration variable to ω'', we obtain

$$\gamma_L(\omega) = U_0(\omega)\omega I(\omega) - U_1(\omega)\frac{\partial}{\partial\omega}\{\omega I(\omega)\}, \tag{12}$$

where $U_0(\omega) \equiv \int_{\omega-\omega_M}^{\omega-\omega_1} d\omega'' V(\omega'')$ and $U_1(\omega) \equiv \int_{\omega-\omega_M}^{\omega-\omega_1} d\omega'' \omega'' V(\omega'')$.

Although it is possible to write the formal solution of Eq. (12) in terms of integrals of the error function, we simply present a more illuminating approximate solution. For $\omega - \omega_1$, $\omega_M - \omega \gtrsim v_{Ti}/qR$, $U_0(\omega)$ becomes exponentially small because $V(\omega'')$ is an odd function. Then, approximating $U_1(\omega)$ by

$$\bar{U}_1 \equiv \int_{-\infty}^{\infty} d\omega'' \omega'' V(\omega'') = \frac{\pi}{2}\frac{1}{\{1+(T_e/T_i)\}^2}\frac{1}{B_0^2}\frac{1}{(qR)^2},$$

we can integrate Eq. (12) to obtain

$$I(\omega) = \frac{1}{\omega}\Big\{\omega_M I(\omega_M) + \frac{1}{\bar{U}_1}\int_{\omega}^{\omega_M} d\omega \gamma_L(\omega)\Big\}. \tag{13}$$

Now let us estimate $I(\omega_M)$. For ω very close to $\omega_M(\omega_M - \omega \ll v_{Ti}/qR)$,

$$U_1(\omega) \simeq \int_{\omega-\omega_M}^{\infty} d\omega'' V(\omega'')\omega'' = O\Big(\frac{(\omega_M - \omega)^2}{v_{Ti}^2 (qR)^2}\Big)\bar{U}_1.$$

Thus, the first term on the right hand side of Eq. (12) is larger than the second term. Using

$$U_0 \simeq \bar{U}_0 \equiv \int_0^{\infty} d\omega'' V(\omega'') = \frac{1}{2}\sqrt{\frac{\pi}{2}}\frac{1}{(1+T_e/T_i)^2 B_0^2 v_{Ti} qR},$$

We obtain $I(\omega_M) \simeq \gamma_L(\omega_M)/\omega_M \bar{U}_0$ from Eq. (13). Then Eq. (13) can be written as

$$I(\omega) = \frac{1}{\omega}\Big[\frac{\gamma_L(\omega_M)}{\bar{U}_0} + \frac{1}{\bar{U}_1}\int_{\omega}^{\omega_M} d\omega \gamma_L(\omega)\Big]. \tag{14}$$

As a consequence of the ion Compton scattering induced downward spectral transfer, the spectral intensity peaks at a frequency lower than that with maximum linear growth rate. Integrating the intensity over the fluctuation population zone, we obtain

$$\int_{\omega_1}^{\omega_M} d\omega I(\omega) = \frac{\gamma_L(\omega_M)}{\bar{U}_0}\ln\Big(\frac{\omega_M}{\omega_1}\Big) + \frac{1}{\bar{U}_1}\int_{\omega_1}^{\omega_M} d\omega \gamma_L(\omega)\ln\Big(\frac{\omega}{\omega_1}\Big). \tag{15}$$

Since $\bar{U}_0(\omega_M - \omega_1)/\bar{U}_1 = O(\epsilon/\beta_i^{1/2})$, we keep only the second term of Eq. (15). The corresponding magnetic fluctuation level at nonlinear saturation, which can be obtained by using the frozen-in-flux constraint, then becomes

$$\Big(\frac{\delta B_r}{B_0}\Big)^2 \simeq \Big(\frac{k_\theta}{k_r}\Big)^2\Big(1+\frac{T_e}{T_i}\Big)^2\frac{1}{2\pi\omega_A^2}\int_{\omega_1}^{\omega_M} d\omega \gamma_L(\omega)\ln\Big(\frac{\omega}{\omega_1}\Big). \tag{16}$$

For a rough estimation, we take $\gamma_L(\omega) \simeq \bar{\gamma}_L$, and expand the final expression in $\epsilon_{\text{eff}} \equiv 1 - \omega_1/\omega_M$, to get

$$\Big(\frac{\delta B_r}{B_0}\Big)^2 \simeq \frac{1}{4\pi}\Big(1+\frac{T_e}{T_i}\Big)^2\Big(\frac{\bar{\gamma}_L}{\omega_A}\Big)\epsilon^2\epsilon_{\text{eff}}^2, \tag{17}$$

where $(k_\theta/k_r)^2 = \epsilon^2$ has been used. For $\bar{\gamma}_L/\omega_A \lesssim 10^{-2}$, $\epsilon_{\text{eff}} \simeq \epsilon \sim 10^{-1}$, and $T_e/T_i \lesssim 1$, Eq. (17) yields $\delta B_r/B_0 \lesssim 10^{-3}$.

Since $\epsilon_{eff} \lesssim \epsilon$, the saturation level is a strong ($\sim \epsilon^4$) function of the inverse aspect ratio, and independent of β_i . The strong ϵ dependence originates from the radial localization of each poloidal component of TAE ($k_r \simeq k_\theta/\epsilon$), and the proportionality of the fluctuation population range ($\omega_M - \omega_1$) to ϵ. The β_i–scaling which appears in Compton scattering cross–section ($\sim Im\chi_i$) for each pair ($\boldsymbol{k}, \boldsymbol{k}'$) of back scattering goes away after the mode summation because the nonlinear interaction range is given by $|\omega - \omega'| \lesssim v_{Ti}/qR$. As a result, the effective "convective velocity" $\bar{U}_1$ in ω-space (see Eq. (13), and the saturation level is independent of β_i in the low β_i regime ($\beta_i < \epsilon^2$) considered in this work. Finally, the temperature ratio appears due to the partial shielding by $\phi^{(2)}_{\boldsymbol{k}''}$.

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