Nonlinear semigroups and differential equations in Banach spaces

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1.1 The equation
$$\frac{d^2u}{dt^2} + Au + M\left(\frac{du}{dt}\right) \ni f$$
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