NONLINEAR SPATIO-TEMPORAL NOISE SUPPRESSION TECHNIQUES WITH APPLICATIONS IN IMAGE SEQUENCE PROCESSING

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ABSTRACT

In this paper, statistical properties of the spatiotemporal center weighted median (CWM) filter for image sequences are investigated. It is statistically shown that the CWM filter preserves image structures under motion at the expense of noise suppression. To improve the CWM filter, an adaptive CWM (ACWM) filter having a variable central weight is presented. We show that the ACWM filter can preserve image structures under motion while suppressing noise, and thus can be effectively used in image sequence filtering.

1. INTRODUCTION

The processing of image sequences involving motion has become increasingly important in a variety of areas including video signal coding, medical imaging, and robot vision [1],[2]. Recently so-called *high definition television* (HDTV) systems which are an application of image sequence processing have received a great deal of attention.

Some noise reduction techniques for image sequences use 3-D linear filters based on the assumption of the spatiotemporal stationarity [3],[4]. Such linear filters smear abruptly sustained changes (edges), and distort moving objects where the assumption of the stationarity is not justified.

A variety of motion-compensated temporal filtering techniques have been proposed to overcome this problem [5]-[7]. Some of these techniques utilize a combined segmentation and motion detection algorithm to segment the images into moving and non-moving regions, applying a temporal filter only in the non-moving areas. These methods can preserve image structures under motion, but cannot reduce noise in moving regions. Although noise in moving areas is perceptually masked to some extent by the motion, it will be visible in slowly moving areas. Others estimate the motion path of a pixel and the temporal filtering is performed over its path so that the moving objects are not distorted. However, the amount of noise suppression which can be attained with the 1-D temporal filter is quite limited. In order to increase noise suppression, larger temporal windows may be used. This leads to a computationally expensive estimation algorithm for the motion path since a large number of possible motion trajectories need to be processed.

The center weighted median (CWM) filter is an extension of the median filter which gives weight to the center sample in the window [8]-[10]. This filter allows a degree of control of smoothing behavior of the filter via the central weight, and thus is a promising image enhancement technique. Recently a spatiotemporal version of the CWM filter has been effectively applied for image sequence filtering [11]. In this paper, statistical properties of the spatiotemporal CWM filter are investigated for image sequence processing. It is statistically shown that the spatiotemporal CWM filter can preserve image structures under motion at the expense of the noise suppression. To improve the performance of the CWM filter, a spatiotemporal adaptive CWM (ACWM) filter having a variable central weight is presented by extending the 2-D ACWM filter [8] to three dimensions. We will show that the ACWM filter can suppress noise while preserving moving objects without the use of the motion estimation.

2. STATISTICAL PROPERTIES OF CWM FILTERS

In this section, we first review the spatiotemporal CWM filter and study its statistical properties.

Let $\{X(.,.,.)\}$ and $\{Y(.,.,.)\}$ be the input and output, respectively, of a spatiotemporal filter. Consider a spatiotemporal window W defined in terms of the 3-D coordinates in the neighborhood of the origin $(0,0,0)$. For example, a $(2N+1)$ x $(2N+1)$ x $(2N+1)$ cubic window is given by $W = \{(l,m,n) \mid -N \leq l,m,n \leq N\}$. The total number of points in a window is called the window size $2L+1$. Let (i,j) and (k), respectively, represent the spatial coodinates and the time coodinate for a spatiotemporal signal.

A CWM filter with window size $2L+1$ and central weight $2D+1$ is denoted by CWM(2L+1,2D+1) and defined by:

Definition 1: The output $Y(i,j,k)$ of the CWM filter is given by

$$Y(i,j,k) = median\{X(i-l,j-m,k-n),\ 2D \text{ copies of } X(i,j,k) \mid (l,m,n) \in W\}. \quad (1)$$

where D is a nonnegative integer.

The CWM filter is very simple to implement. It has been shown in [8] that the output value of the CWM filter is the median of two (lower and upper) order statistics and the central sample in the window.

Property 1: The output $Y(i,j,k)$ of a CWM(2L+1, 2D+1) filter is equivalent to

$$Y(i,j,k) = median\{X_{ijk}(L+1-D;2L+1), X_{ijk}(L+1+D;2L+1), X(i,j,k)\} \quad (2)$$

where $X_{ijk}(r;2L+1)$ is the r^{th} smallest one (order statistic) among $2L+1$ samples within the window centered at (i,j,k), and $X(i,j,k)$ is the central sample of the window.

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By varying the central weight of the CWM filter, various filter responses are obtained. For example, when $D = 0$, the CWM filter becomes the median filter, and when $D = L$, it becomes the identity filter (no filtering).

The motion preservation properties can be examined by considering 3-D simple inputs with moving step edges which are corrupted by additive white noise.

The input sequence representing a noisy step edge at $k \le k_1$ is expressed by

$$X(i,j,k) = \begin{cases} V(i,j,k), & j \le 0 \\ h + V(i,j,k), & j \ge 1 \end{cases} \tag{3}$$

where h is a constant representing edge height, $V(i,j,k)$ is i.i.d. noise with distribution $F_1(x)$. Let the distribution function of $h + V(i,j,k)$ be $F_2(x)$. Then, obviously, $F_2(x) = F_1(x - h)$.

Assume that at $k = k_1 + 1$, the edge is shifted horizontally by j_1 pixels, i.e.,

$$X(i,j,k_1+1) = \begin{cases} V(i,j,k_1+1), & j \le j_1 \\ h + V(i,j,k_1+1), & j \ge j_1 + 1 \end{cases} \tag{4}$$

We will examine the filter behavior near the noisy edge by using the expected value of the output and the root mean squared error (rmse). Here the rmse at (i,j,k) denoted by rmse(i,j,k), is defined as rmse$(i,j,k) = \sqrt{E[Y(i,j,k) - S(i,j,k)]^2}$ with $Y(i,j,k)$ as the filtered output, $S(i,j,k)$ equal to 0 if $j \le 0$, and equal to h if $j \ge 1$. In order to compute these quantities, we derive the distribution function $F_{\mathbf{Y}_{\mathbf{ijk}}}(y)$ of the CWM filtered output $Y(i,j,k)$ which is taken from m_{ijk} samples with distribution $F_1(x)$ and $2L+1-m_{ijk}$ samples with distribution $F_2(x)$ among $2L+1$ samples within the window centered at (i,j,k). Note that the number of samples having $F_1(x)$ in the window, m_{ijk}, depends on the location of the window.

Property 2: For the noisy step edge input in (3), the output distribution function $F_{\mathbf{Y}_{\mathbf{ijk}}}(y)$ of the CWM(2L+1, 2D+1) filter is given by

$$\begin{aligned} F_{\mathbf{Y}_{\mathbf{ijk}}}(y) = & \sum_{d=d_1-1}^{2L} \sum_{l=max(0,d-(2L-m))}^{min(d,m)} \binom{m}{l} \binom{2L-m}{d-l} F_1^l(y)(1-F_1(y))^{m-l} \\ & F_2^{d-l}(y)(1-F_2(y))^{2L-m-d+l} F_{ijk}(y) \\ & + \sum_{d=d_2}^{2L} \sum_{l=max(0,d-(2L-m))}^{min(d,m)} \binom{m}{l} \binom{2L-m}{d-l} F_1^l(y)(1-F_1(y))^{m-l} \\ & F_2^{d-l}(y)(1-F_2(y))^{2L-m-d+l}(1-F_{ijk}(y)), \end{aligned} \tag{5}$$

where $d_1 = L + 1 - D, d_2 = L + 1 + D$,

$$F_{ijk}(y) = \begin{cases} F_1(y), & j \le 0 \\ F_2(y), & j \ge 1, \end{cases} \quad \text{and} \quad m = \begin{cases} m_{ijk} - 1, & j \le 0 \\ m_{ijk}, & j \ge 1 \end{cases}$$

Using (5), we computed $E[Y(i,j,k)]$ and rmse(i,j,k) for $k = k_1$ through numerical intergration. Fig. 1 and 2 show plots of $E[Y(i,j,k)]$ and rmse(i,j,k), respectively, for the CWM filters with the 3x3x3 cubic window, when the step edge ($h = 4$) degraded by a Gaussian $N(0,1)$ noise is shifted horizontally by $j_1 = 1$ and 2 at $k = k_1 + 1$. As expected, the edge preservation characteristic of the CWM filter improves as the central weight decreases. It is seen that the CWM filter $2D + 1 \ge 15$ effects a greater degree of edge preservation under motion than median (CWM(27,1)) filtering. Similar results can be obtained for vertically and diagonally shifted edges.

In Fig. 1(b), the results associated with rmse(i,j,k) for $j \le -1$, where the 3x3x3 window of the CWM filter contains only i.i.d Gaussian inputs with N(0,1), show the noise suppression characteristics of the CWM filter in flat regions. It is seen that the CWM filter provides a wide range of smoothing performance depending on the selection of the central weight. The noise suppression of the CWM filter decreases with increasing the central weight.

In summary, the CWM filter can preserve edges under motion at the expense of noise suppression. The central weight should be carefully selected depending on the characteristics of both the input image sequence and its noise.

3. SPATIOTEMPORAL ACWM FILTERS

In this section, a spatiotemporal ACWM filter having a variable central weight $2D_{ijk} + 1$ is introduced. It will be shown that the ACWM filter can offer a desirable combination of motion preservation and noise suppression properties by allowing a variable central weight adjusted by local characteristics in the spatiotemporal neighborhood of each pixel.

Definition 2: The output $Y(i,j,k)$ of an ACWM filter with variable central weight $2D_{ijk} + 1$ is defined by

$$Y(i,j,k) = median\{X_{ijk}(L+1-D_{ijk}; 2L+1), X_{ijk}(L+1+D_{ijk}; 2L+1), X(i,j,k)\} \tag{6}$$

We will derive the adaptive parameter D_{ijk} by designing the ACWM filter according to the approach proposed in [12], which may be stated as follows: First choose a linear filter with desirable characterstics, and then search for a nonlinear filter whose linear component is close to the linear filter. Here the linear component of a 3-D nonlinear filter for i.i.d. inputs is a linear FIR filter with the impulse response given by

$$h(r,s,t) = Pr\{Y(i,j,k) = X(i-r, j-s, k-t),\ (r,s,t) \in W\} \tag{7}$$

which is the probability that $X(i-r, j-s, k-t)$ equals the output at (i,j,k).

Using (2) and (7), it is readily seen that the impulse response of the linear component of the CWM filter for i.i.d inputs is given by

$$h(r,s,t) = \begin{cases} \frac{2D+1}{2L+1}, & \text{if } (r,s,t) = (0,0,0) \\ \frac{1-D/L}{2L+1}, & \text{if } (r,s,t) \ne (0,0,0), \end{cases} \tag{8}$$

for each $(r,s,t) \in W$. Note that $\sum_{(r,s,t)\in W} h(r,s,t) = 1$. Comparison of (2) and (6) shows that the impulse response of the linear component of the ACWM filter with variable central weight $2D_{ijk} + 1$ can be expressed as

$$h_{ijk}(r,s,t) = \begin{cases} \frac{2D_{ijk}+1}{2L+1}, & \text{if } (r,s,t) = (0,0,0) \\ \frac{1-D_{ijk}/L}{2L+1}, & \text{if } (r,s,t) \ne (0,0,0), \end{cases} \tag{9}$$

by replacing D in (8) by D_{ijk}.

Next we make an interesting observation indicating that the adaptive filter in [13] gives more weight only to the central value of each window. The output of a spatiotemporal version of this adaptive filter for additive white noise suppression can be represented by

$$Y(i,j,k) = A(i,j,k) + R_{ijk}[X(i,j,k) - A(i,j,k)] \qquad (10)$$

where $A(i,j,k)$ is the average of the values within a window centered at (i,j,k),

$$R_{ijk} = \begin{cases} 1 - \frac{\sigma_n^2}{\hat{\sigma}_X^2(i,j,k)}, & \text{if } \hat{\sigma}_X^2(i,j,k) \geq \sigma_n^2 \\ 0, & \text{o.w.}, \end{cases} \qquad (11)$$

where $\hat{\sigma}_{\mathbf{X}}^2(i,j,k)$ is the sample variance of the data inside the window, and σ_n^2 is the variance of additive white noise, which is assumed to be known. Note that this filter varies between the average filter and the identity filter depending on the local statistics.

We rewrite the output $Y(i,j,k)$ in (10) as

$$Y(i,j,k) = \sum_{(r,s,t)\in W} h_{ijk}(r,s,t) X(i-s, j-t) \qquad (12)$$

where

$$h_{ijk}(r,s,t) = \begin{cases} \frac{1}{2L+1}[2LR_{ijk} + 1], & \text{if } (r,s,t) = (0,0,0) \\ \frac{1}{2L+1}[1 - R_{ijk}], & \text{if } (r,s,t) \neq (0,0,0). \end{cases} \qquad (13)$$

Note that $\sum h_{ijk}(r,s,t) = 1$. It is straightforward to see that this filter imposes more weight only on the central value of each window.

Comparison of (9) and (13) shows that the linear component of the ACWM filter becomes the same as the filter of (12) if

$$D_{ijk} = LR_{ijk}. \qquad (14)$$

Since D_{ijk} must be a non-negative integer, LR_{ijk} may be rounded.

It may be noticed that the ACWM filter associated with (14) can become an identity filter if $R_{ijk} \simeq 1$ in some regions. One simple remedy for this is to limit the central weight by precluding any value which would cause the ACWM filter to become an identity filter. The parameter D_{ijk} of the proposed ACWM filter is determined by

$$D_{ijk} = \lceil (L-T)R_{ijk} \rfloor \qquad (15)$$

where T is an integer, $0 \leq T \leq L$, and $\lceil x \rfloor$ represents rounding of x. Since $0 \leq R_{ijk} \leq 1$, we get $0 \leq D_{ijk} \leq L - T$. Therefore, the ACWM filter varies between the median filter and the CWM filter with $2D+1 = 2(L-T)+1$. Generally speaking, it preserves the motion like the CWM filter with central weight $2(L-T)+1$ in the neighborhood of moving areas, and remove noise like the median filter in non-moving flat regions of an image. (A rule for the selection of parameter T is discussed in [10].)

4. EXPERIMENTAL RESULTS

Three successive image frames (k = 7, 8, and 9) of the original noise-free image sequence are shown in Fig. 3. Noisy images were generated by adding zero mean i.i.d. Gaussian noise of variance 100 to the original image frames, and then were passed through the spatiotemporal CWM, ACWM (T = 2), median filters with the 3x3x3 cubic window as well as the motion-compensated temporal median filter of size 3 in [6]. Fig. 4 shows the noisy image frame (k = 8) and its difference from the orignal image frame.

Table 1 summarizes the normalized mean square error (NMSE) of the filters. The minimal NMSE of the CWM filters is smaller than the NMSE of the median filter. The ACWM filter yields the smallest NMSE, while the motion-compensated temporal median filter gives the largest NMSE. Fig. 5 shows the filtered images and the difference between the original and the filtered images. As expected, the spatiotemporal median filter introduces the blur in the image, but performs well in suppressing noise in non-moving regions. It is seen that the CWM filter with a proper central weight ($2D+1 = 7$) preserves more image structure at the expense of less noise suppression. The motion-compensated temporal median filter introduces artifacts in the area of eyes where motion is not tracked. The motion preserving characteristic of the ACWM filter is clearly seen. The results in this section indicate that the ACWM filter is an effective motion-preserving filter that can suppress noise in image sequences.

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Table 1 NMSE (Gaussian noise $\sigma^2 = 100$)

Median	CWM(27,7)	CWM(27,13)	CWM(27,19)	*MC-median	ACWM(T=2)
0.39	0.34	0.48	0.68	0.88	0.27

* MC-median represents the motion-compensated temporal median filter.

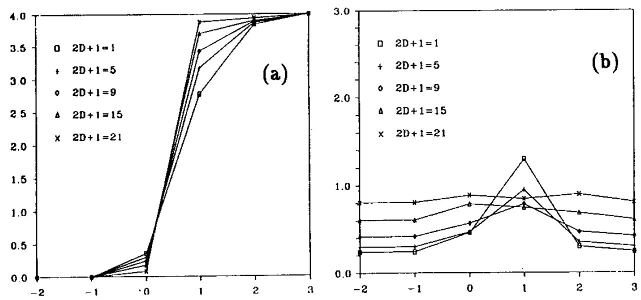


Fig. 1. Results of median and CWM filters, with the 3x3x3 cubic window, for the horizontally shifted noisy edge with $j_1 = 1$: (a) The output expected values, (b) The root mean square errors.

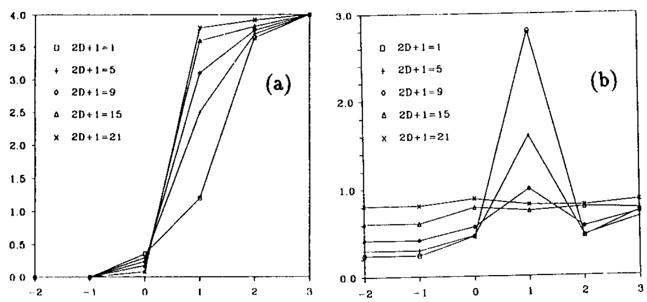
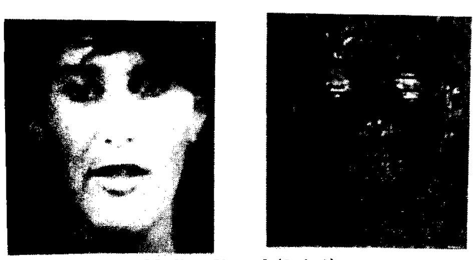
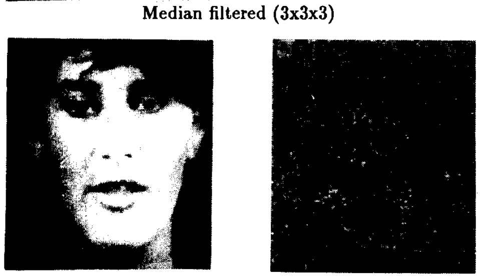


Fig. 2. Results of median and CWM filters, with the 3x3x3 cubic window, for the horizontally shifted noisy edge with $j_1 = 2$: (a) The output expected values, (b) The root mean square errors.



Median filtered (3x3x3)



CWM(27,7) filtered

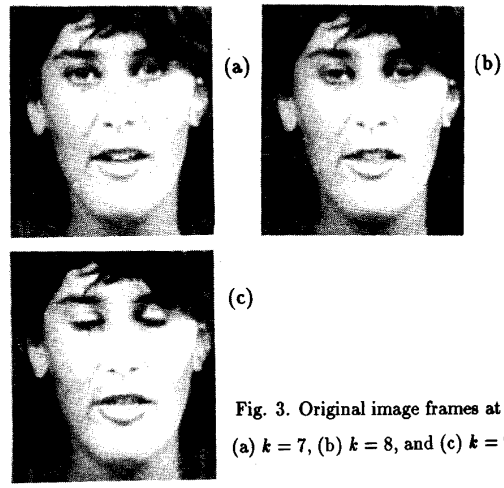


Fig. 3. Original image frames at (a) $k = 7$, (b) $k = 8$, and (c) $k = 9$.

Fig. 4. Noisy image frame 8 (Gaussian noise $\sigma^2 = 100$)

Motion-compensated temporal median filtered (size 3)

ACWM(T=2) filtered

Fig. 5. Filtered images and difference images between original and filtered images.