Nonlinear State Feedback Controller for Nonlinear Systems: Stability Analysis and Design Based on Fuzzy Plant Model

H. K. Lam, F. H. F. Leung, and P. K. S. Tam

***Abstract*—This paper presents the stability analysis of a fuzzy-model-based control system consisting of a nonlinear plant and a nonlinear state feedback controller and the design of the nonlinear gains of the controller. The nonlinear plant is represented by a fuzzy model having p rules. A nonlinear state feedback controller is designed to close the feedback loop. Under this design, the stability condition is reduced to p linear matrix inequalities. An application example on stabilizing a mass-spring-damper system will be given.**

***Index Terms*—Nonlinear controller, nonlinear systems, stability analysis.**

I. INTRODUCTION

FUZZY control has been found capable of tackling ill-defined nonlinear plants [1], [2]. However, without carrying out an in-depth analysis, the design of the controller may come with no guarantee of system stability. One common approach of analysis is based on a fuzzy plant model [3], [7], which expresses a nonlinear system as a weighted sum of some linear subsystems. Under this structure, some linear control techniques and stability analysis methods can be applied. Some authors proposed a fuzzy controller to control this class of nonlinear systems. This fuzzy controller is a weighted sum of some linear state feedback controllers [4], [6], [8]–[10], [13]. A $p \times c$ linear matrix inequality (LMI) problem was derived in [4], [6], and [8]–[10], where p and c are the numbers of fuzzy rules of the fuzzy plant model and the fuzzy controller, respectively. The LMI problem can be solved by using some LMI tools, such as MATLAB, numerically [12]. In the case where $p = c$, and the premises of the fuzzy plant model and fuzzy controller are the same, $p(p+1)$ LMI conditions were derived in [6] and [8]–[10]. In this paper, we propose a design method that can further reduce the number of LMI conditions to p.

The contributions of this paper are threefold. First, we propose a nonlinear state feedback controller [11] to control the nonlinear plant. The only conceptual difference between the conventional fuzzy controller and the proposed nonlinear state feedback controller is that the grades of membership of the former controller are now regarded as nonlinear gains that can take positive or negative values. Second, by introducing the signed nonlinear gains, the number of LMI stability conditions is reduced to p, which is independent of the number of rules of the controller. As the number of LMI conditions is reduced, the chance of finding the solution will be increased. Third, we provide a methodology for designing the nonlinear gains of the proposed nonlinear state feedback controller. An application example on stabilizing a nonlinear mass-spring-damper system will be given to verify the results of this paper.

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II. FUZZY PLANT MODEL AND NONLINEAR STATE FEEDBACK CONTROLLER

We consider a multivariable nonlinear control system with all system states assumed to be accessible. The plant can be represented by a fuzzy plant model. A nonlinear state feedback controller is to be designed to close the feedback loop.

A. Fuzzy Plant Model

Let p be the number of fuzzy rules describing the nonlinear plant [3], [7]. The ith rule is of the following format:

$$\text{Rule } i\text{: IF } x_1(t) \text{ is } M_1^i \text{ and } \ldots \text{ and } x_n(t) \text{ is } M_n^i$$
$$\text{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t) \tag{1}$$

where M_k^i is a fuzzy term of rule i corresponding to the state $x_k(t), k = 1, 2, \ldots, n, i = 1, 2, \ldots, p; \mathbf{A}_i \in \Re^{n\times n}$ and $\mathbf{B}_i \in \Re^{n\times m}$ are the system matrix and input matrix, respectively; $\mathbf{x}(t) \in \Re^{n\times 1}$ is the system state vector; and $\mathbf{u}(t) \in \Re^{m\times 1}$ is the input vector. The system dynamics is described by

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p} w_i(\mathbf{x}(t))(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)) \tag{2}$$

where

$$\sum_{i=1}^{p} w_i(\mathbf{x}(t)) = 1, \quad w_i(\mathbf{x}(t)) \in [0 \quad 1] \quad \text{for all } i \tag{3}$$

is a known nonlinear function of $\mathbf{x}(t)$ and (4) shown at the bottom of the next page. $\mu_{M_k^i}(x_k(t))$ is the grade of membership of the fuzzy term M_k^i.

B. Nonlinear State Feedback Controller

A nonlinear state feedback controller similar to a fuzzy controller with c rules is to be designed for the plant. The output of the nonlinear state feedback controller is given by

$$\mathbf{u}(t) = \sum_{j=1}^{c} m_j(\mathbf{x}(t))\mathbf{G}_j\mathbf{x}(t) \tag{5}$$

where $\mathbf{G}_j \in \Re^{m\times n}, j = 1, 2, \ldots, c$, is a feedback gain vector to be designed

$$\sum_{j=1}^{c} m_j(\mathbf{x}(t)) = 1. \tag{6}$$

$m_j(\mathbf{x}(t))$ is a nonlinear gain function of $\mathbf{x}(t)$ defined as

$$m_j(\mathbf{x}(t)) = \frac{\mu_{N^j}(\mathbf{x}(t))}{\sum_{k=1}^{c}(\mu_{N^k}(\mathbf{x}(t)))} \tag{7}$$

and $\mu_{N^j}(\mathbf{x}(t))$ is to be designed.

III. Stability Analysis and Design

In this section, stability of the nonlinear control system formed by the fuzzy plant model and nonlinear state feedback controller connected in closed-loop will be investigated. A design methodology of $m_j(\mathbf{x}(t)), j = 1, 2, \ldots, c$, will be provided under the consideration of the closed-loop system stability. For simplicity, we write $w_i(\mathbf{x}(t))$ as w_i and $m_j(\mathbf{x}(t))$ as m_j. From (2) and (5) and the property that $\sum_{i=1}^{p} w_i = \sum_{j=1}^{c} m_j = \sum_{i=1}^{p}\sum_{j=1}^{c} w_i m_j = 1$, the nonlinear control system becomes

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \sum_{i=1}^{p} w_i \left(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i \sum_{j=1}^{c} m_j \mathbf{G}_j \mathbf{x}(t)\right)\\ &= \sum_{i=1}^{p}\sum_{j=1}^{c} w_i m_j (\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j)\mathbf{x}(t)\\ &= \sum_{i=1}^{p}\sum_{j=1}^{c} w_i m_j \mathbf{H}_{ij}\mathbf{x}(t)\end{aligned} \tag{8}$$

where

$$\mathbf{H}_{ij} = \mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j. \tag{9}$$

To investigate the stability of (8), we employ the following Lyapunov function in quadratic form:

$$V = \frac{1}{2}\mathbf{x}(t)^T\mathbf{P}\mathbf{x}(t) \tag{10}$$

where $(\cdot)^T$ denotes the transpose of a vector or matrix and $\mathbf{P} \in \Re^{n\times n}$ is a symmetric positive definite matrix. Differentiating (10), we have

$$\dot{V} = \frac{1}{2}(\dot{\mathbf{x}}(t)^T\mathbf{P}\mathbf{x}(t) + \mathbf{x}(t)^T\mathbf{P}\dot{\mathbf{x}}(t)). \tag{11}$$

From (8) and (11), we have

$$\begin{aligned}\dot{V} &= \frac{1}{2}\left[\left(\sum_{i=1}^{p}\sum_{j=1}^{c} w_i m_j \mathbf{H}_{ij}\mathbf{x}(t)\right)^T \mathbf{P}\mathbf{x}(t) + \mathbf{x}(t)^T\mathbf{P}\sum_{i=1}^{p}\sum_{j=1}^{c} w_i m_j \mathbf{H}_{ij}\mathbf{x}(t)\right]\\ &= \frac{1}{2}\sum_{i=1}^{p}\sum_{j=1}^{c} w_i m_j \mathbf{x}(t)^T\left(\mathbf{H}_{ij}^T\mathbf{P} + \mathbf{P}\mathbf{H}_{ij}\right)\mathbf{x}(t).\end{aligned} \tag{12}$$

Let

$$\mathbf{Q}_{ij} = -\left(\mathbf{H}_{ij}^T\mathbf{P} + \mathbf{P}\mathbf{H}_{ij}\right). \tag{13}$$

From (12)

$$\begin{aligned}\dot{V} &= -\frac{1}{2}\sum_{i=1}^{p}\sum_{j=1}^{c} w_i m_j \mathbf{x}(t)^T\mathbf{Q}_{ij}\mathbf{x}(t)\\ &= -\frac{1}{2}\sum_{j=1}^{c} m_j\left(\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{ij}\mathbf{x}(t)\right).\end{aligned} \tag{14}$$

We design the nonlinear gains $\mu_{N^j}(\mathbf{x}(t))$ in (7) as shown in (15) and (16) at the bottom of the next page. From (7), (15), and (16), it can be seen that $\sum_{j=1}^{c} m_j = \sum_{j=1}^{c}\mu_{N^j}(\mathbf{x}(t)) = 1$. From (14)–(16) and considering the case that $\sum_{j=1}^{c}|\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{ij}\mathbf{x}(t)| \neq 0$, we have

$$\begin{aligned}\dot{V} &= -\frac{1}{2}\left(\frac{\sum_{j=2}^{c}\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{i1}\mathbf{x}(t)}{\sum_{j=1}^{c}|\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{ij}\mathbf{x}(t)|}\right) \times \mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{i1}\mathbf{x}(t)\\ &\quad \times -\frac{1}{2}\sum_{j=2}^{c}\frac{\left(\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{ij}\mathbf{x}(t)\right)^2}{\sum_{j=1}^{c}|\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{ij}\mathbf{x}(t)|}\\ &\leq -\frac{1}{2}\left(1 - \frac{\sum_{j=2}^{c}\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{i1}\mathbf{x}(t)}{\sum_{j=1}^{c}|\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{ij}\mathbf{x}(t)|}\right) \times \mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{i1}\mathbf{x}(t).\end{aligned} \tag{17}$$

We choose

$$\mathbf{Q}_{i1} > \mathbf{0} \quad \text{for } i = 1, 2, \ldots, p. \tag{18}$$

Then $\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{i1}\mathbf{x}(t) > 0$ when $\mathbf{x}(t) \neq \mathbf{0}$ and $\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{i1}\mathbf{x}(t) = 0$ when $\mathbf{x}(t) = \mathbf{0}$. From (17) and (18) and as $1 - (\sum_{j=2}^{c}\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{i1}\mathbf{x}(t))/(\sum_{j=1}^{c}|\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{ij}\mathbf{x}(t)|) > 0$, we can conclude that

$$\dot{V} \leq -\frac{1}{2}\left(1 - \frac{\sum_{j=2}^{c}\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{i1}\mathbf{x}(t)}{\sum_{j=1}^{c}|\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{ij}\mathbf{x}(t)|}\right) \times \sum_{i=1}^{p} w_i\mathbf{x}(t)^T\mathbf{Q}_{i1}\mathbf{x}(t) \leq 0. \tag{19}$$

Equality holds when $\mathbf{x} = \mathbf{0}$. Next, we consider the case that $\sum_{j=1}^{c}|\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{ij}\mathbf{x}(t)| = 0$. From (18), as $\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{i1}\mathbf{x}(t) = 0$ only when $\mathbf{x}(t) = \mathbf{0}$, it implies that $\sum_{j=1}^{c}|\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{ij}\mathbf{x}(t)| = 0$ occurs only when $\mathbf{x}(t) = \mathbf{0}$. Hence, we can conclude that the system is asymptotically stable. The analysis can be summarized by the following lemma.

$$w_i(\mathbf{x}(t)) = \frac{\mu_{M_1^i}(x_1(t)) \times \mu_{M_2^i}(x_2) \times \cdots \times \mu_{M_n^i}(x_n)}{\sum_{j=1}^{p}\left(\mu_{M_1^j}(x_1(t)) \times \mu_{M_2^j}(x_2(t)) \times \cdots \times \mu_{M_n^j}(x_n(t))\right)}. \tag{4}$$

Lemma 1: The nonlinear control system of (8) is guaranteed to be asymptotically stable if the following p LMI conditions are satisfied:

$$\mathbf{Q}_{i1} = -\left(\mathbf{H}_{i1}^T\mathbf{P} + \mathbf{P}\mathbf{H}_{i1}\right) < \mathbf{0} \quad \text{for all } i = 1, 2, \ldots, p$$

and the nonlinear gains of the controller are designed as shown in (15) and (16).

From Lemma 1, we can see that the number of LMI conditions is p. It should be noted that the LMI conditions in Lemma 1 is actually a subset of the LMI conditions corresponding to (12). Hence, if the LMIs derived from (12) are satisfied, so are the LMIs in Lemma 1. Our approach does not result in more conservative IMI conditions. The procedure for finding the nonlinear state feedback controller can be summarized as follows.

Step I) Obtain the mathematical model of the nonlinear plant to be controlled.

Step II) Obtain the fuzzy plant model for the system stated in Step I) by means of a fuzzy modeling method as that proposed in [3] and [7].

Step III) Choose the gains $(\mathbf{G}_j)$ of the nonlinear state feedback controller.

Step IV) Find $\mathbf{P}$ by solving the p linear matrix inequalities of (18). If $\mathbf{P}$ cannot be found, go back to Step III) and choose other gains $(\mathbf{G}_j)$ for the nonlinear state feedback controller.

Step V) Design the nonlinear gains of the nonlinear state feedback controller based on Lemma 1.

IV. Application Example

An application example based on a nonlinear mass-spring-damper system [5] is given in this section to illustrate the design procedure of the nonlinear state feedback controller.

Step I) Fig. 1 shows the diagram of a mass-spring-damper system. Its dynamic equation is given by

$$M\ddot{x}(t) + g(x(t), \dot{x}(t)) + f(x(t)) = \phi(\dot{x}(t))u(t) \qquad (20)$$

where M is mass; u is force; $f(x(t))$ is spring nonlinearity; $g(x(t), \dot{x}(t))$ is damper nonlinearity; and $\phi(\dot{x}(t))$ is input nonlinearity. Let

$$\begin{aligned} g(x(t), \dot{x}(t)) &= c_1 x(t) + c_2\dot{x}(t) \\ f(x(t)) &= c_3 x(t) + c_4 x(t)^3 \\ \phi(\dot{x}(t)) &= 1 + c_5\dot{x}(t)^2. \end{aligned} \qquad (21)$$

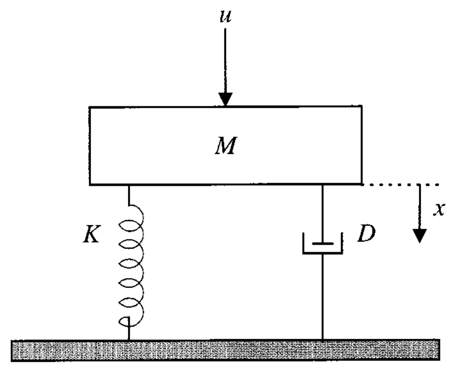

Fig. 1. A mass-spring-damper system.

The operating range of the states is assumed to be within the interval [−1.5, 1.5]. (The fuzzy plant model derived later will not be valid when the system states are outside this interval.) The parameters are chosen as follows: $M = 1.0, c_1 = 0, c_2 = 1, c_3 = 0.01, c_4 = 0.1$, and $c_5 = 0.13$. The system then becomes

$$\begin{aligned} \ddot{x}(t) = &-\dot{x}(t) - 0.01x(t) - 0.1x(t)^3 \\ &+ (1.4387 - 0.13\dot{x}(t)^2)u(t). \end{aligned} \qquad (22)$$

Step II) The nonlinear plant can be represented by a fuzzy model. The ith rules are given by

$$\begin{aligned} &\text{Rule } i\text{: IF } x(t) \text{ is } M_1^i \text{ AND } \dot{x}(t) \text{ is } M_2^i \\ &\quad \text{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i u(t), \\ &\qquad i = 1, 2, 3, 4 \end{aligned} \qquad (23)$$

where the membership functions of $M_k^i, k = 1, 2, i = 1, 2, 3, 4$, as shown in Figs. 2 and 3, respectively, are given by

$$\begin{aligned} \mu_{M_1^1}(x(t)) &= \mu_{M_1^2}(x(t)) = 1 - \frac{x(t)^2}{2.25} \\ \mu_{M_1^3}(x(t)) &= \mu_{M_1^4}(x(t)) = \frac{x(t)^2}{2.25} \\ \mu_{M_2^1}(\dot{x}(t)) &= \mu_{M_2^3}(\dot{x}(t)) = 1 - \frac{\dot{x}(t)^2}{6.75} \\ \mu_{M_2^2}(\dot{x}(t)) &= \mu_{M_2^4}(x(t)) = \frac{\dot{x}(t)^2}{6.75} \end{aligned} \qquad (24)$$

$$\mu_{N^1}(\mathbf{x}(t)) = \begin{cases} \left(1 - \dfrac{\sum_{j=2}^{c}\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{ij}\mathbf{x}(t)}{\sum_{j=1}^{c}\left|\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{ij}\mathbf{x}(t)\right|}\right), & \text{if } \displaystyle\sum_{j=1}^{c}\left|\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{ij}\mathbf{x}(t)\right| \neq 0 \\ \dfrac{1}{c}, & \text{otherwise} \end{cases} \qquad (15)$$

$$\mu_{N^j}(\mathbf{x}(t)) = \begin{cases} \dfrac{\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{ij}\mathbf{x}(t)}{\sum_{j=1}^{c}\left|\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{ij}\mathbf{x}(t)\right|}, & \text{if } \displaystyle\sum_{j=1}^{c}\left|\mathbf{x}(t)^T\sum_{i=1}^{p} w_i\mathbf{Q}_{ij}\mathbf{x}(t)\right| \neq 0 \\ \dfrac{1}{c}, & \text{otherwise} \end{cases} \quad \text{for } j = 2, 3, \ldots, c. \qquad (16)$$

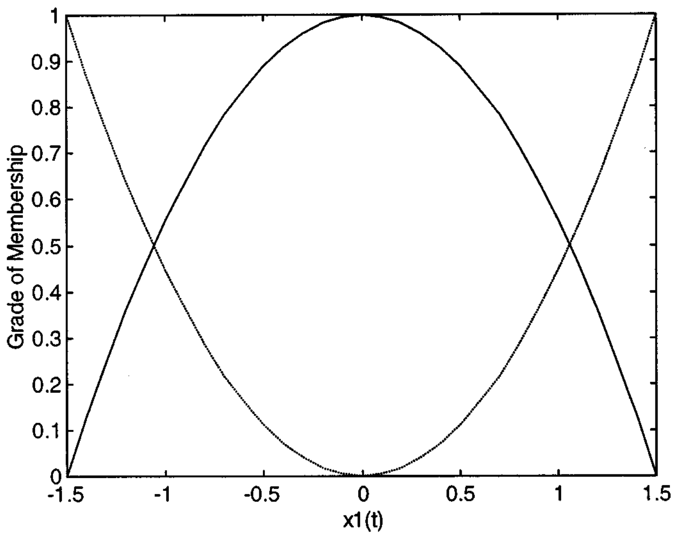

Fig. 2. Membership functions of the fuzzy plant model of the nonlinear mass-spring-damper system: $\mu_{M_1^1}(x) = \mu_{M_1^2}(x) = 1 - (x^2/2.25)$ (solid line), $\mu_{M_1^3}(x) = \mu_{M_1^4}(x) = (x^2/2.25)$ (dotted line).

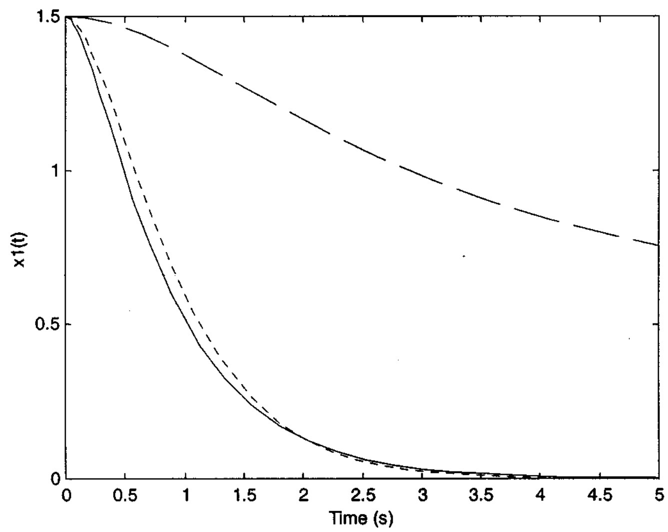

Fig. 4. Responses of $x_1(t)$ of the mass-spring-damper system controlled by a linear state feedback controller (dotted line), the proposed nonlinear state feedback controller (solid line), and $u(t) = 0$ (dashed line).

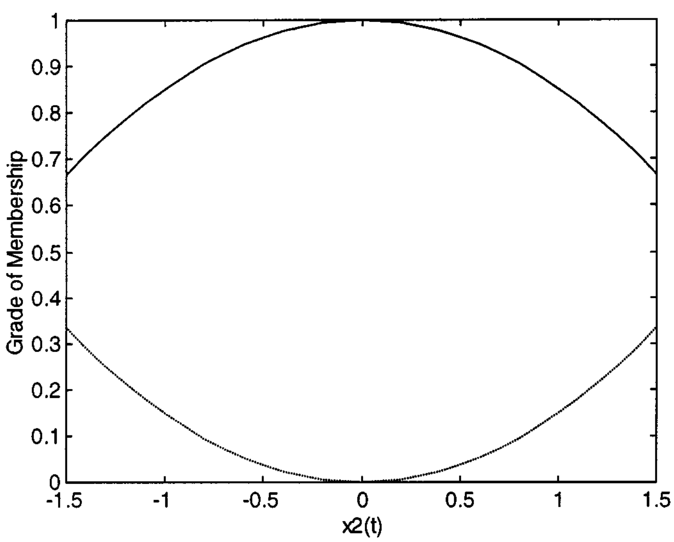

Fig. 3. Membership functions of the fuzzy plant model of the nonlinear mass-spring-damper system: $\mu_{M_2^1}(\dot{x}) = \mu_{M_2^3}(\dot{x}) = 1 - (\dot{x}^2/6.75)$ (solid line), $\mu_{M_2^2}(\dot{x}) = \mu_{M_2^4}(\dot{x}) = (\dot{x}^2/6.75)$ (dotted line).

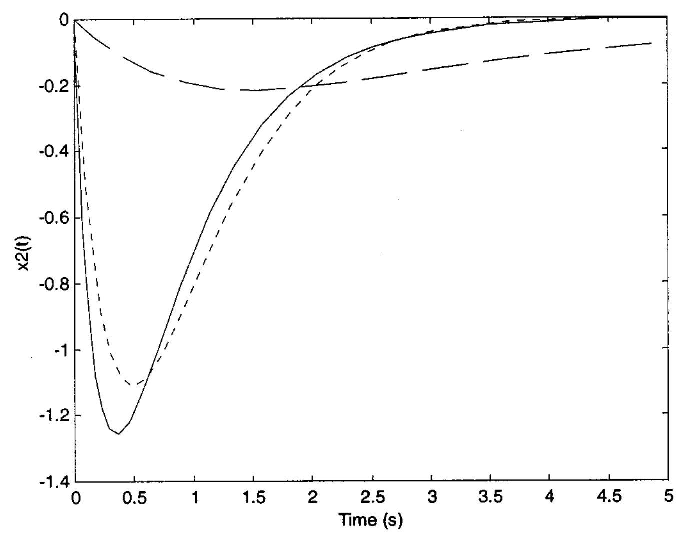

Fig. 5. Responses of $x_2(t)$ of the mass-spring-damper system controlled by a linear state feedback controller (dotted line), the proposed nonlinear state feedback controller (solid line), and $u(t) = 0$ (dashed line).

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}$$

$$\mathbf{A}_1 = \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ -0.01 & -1 \end{bmatrix}$$

$$\mathbf{A}_3 = \mathbf{A}_4 = \begin{bmatrix} 0 & 1 \\ -0.235 & -1 \end{bmatrix}$$

$$\mathbf{B}_1 = \mathbf{B}_3 = \begin{bmatrix} 0 \\ 1.4387 \end{bmatrix}$$

$$\mathbf{B}_2 = \mathbf{B}_4 = \begin{bmatrix} 0 \\ 0.5613 \end{bmatrix}.$$

(Details about the derivation of the fuzzy plant model for the mass-spring-damper system can be found in [5].)

Step III) A nonlinear state feedback controller similar to a two-rule fuzzy controller is designed for the plant of (23). The control law is given by

$$u(t) = \sum_{j=1}^{2} m_j(\mathbf{x}(t))\mathbf{G}_j\mathbf{x}(t). \tag{25}$$

The feedback gains are arbitrarily chosen as $\mathbf{G}_1 = [-2.7732 \quad -2.0852]$ and $\mathbf{G}_2 = [-6.7076 \quad -5.3447]$. As a result, we have $\mathbf{H}_{11} = \mathbf{H}_{24}$, and their eigenvalues are $-2, -2$.

Step IV) We choose $\mathbf{P} = \begin{bmatrix} 1.1486 & 0,1580 \\ 0.1586 & 0.2225 \end{bmatrix}$ such that $\mathbf{Q}_{i1} = -(\mathbf{H}_{i1}^T\mathbf{P} + \mathbf{P}\mathbf{H}_{i1}) < \mathbf{0}$ for $i = 1, 2, 3, 4$.

$$\mu_{N^1}(\mathbf{x}(t)) = \begin{cases} \left(1 - \dfrac{\sum_{j=2}^{2} \mathbf{x}(t)^T \sum_{i=1}^{4} w_i \mathbf{Q}_{ij}\mathbf{x}(t)}{\sum_{j=1}^{2} \left|\mathbf{x}(t)^T \sum_{i=1}^{4} w_i \mathbf{Q}_{ij}\mathbf{x}(t)\right|}\right), & \text{if } \sum_{j=1}^{2} \left|\mathbf{x}(t)^T \sum_{i=1}^{4} w_i \mathbf{Q}_{ij}\mathbf{x}(t)\right| \neq 0 \\ \dfrac{1}{2}, & \text{otherwise} \end{cases}$$

$$\mu_{N^2}(\mathbf{x}(t)) = \begin{cases} \dfrac{\mathbf{x}(t)^T \sum_{i=1}^{4} w_i \mathbf{Q}_{i2}\mathbf{x}(t)}{\sum_{j=1}^{2} \left|\mathbf{x}(t)^T \sum_{i=1}^{4} w_i \mathbf{Q}_{ij}\mathbf{x}(t)\right|}, & \text{if } \sum_{j=1}^{2} \left|\mathbf{x}(t)^T \sum_{i=1}^{4} w_i \mathbf{Q}_{ij}\mathbf{x}(t)\right| \neq 0 \\ \dfrac{1}{2}, & \text{otherwise} \end{cases}$$

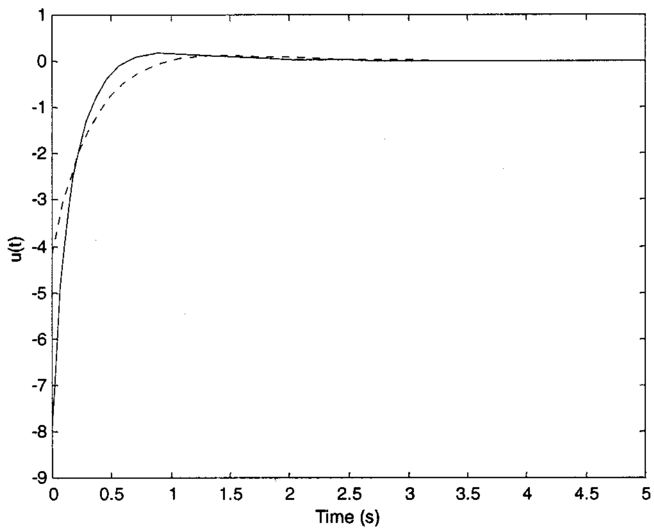

Fig. 6. Control signals of $u(t)$ given by the linear state feedback controller (dotted line) and the proposed nonlinear state feedback controller (solid line).

Step V) The nonlinear gains of the nonlinear state feedback controller are chosen according to Lemma 1, as shown in the equation at the top of the page.

Figs. 4 and 5 show the system responses of $x_1(t)$ and $x_2(t)$ of the mass-spring-damper system under the control of the nonlinear state feedback controller (solid lines) with the initial condition $\mathbf{x}(0) = [1.5\ 0]^T$. The responses are compared with those from a linear state feedback controller with $\mathbf{u}(t) = \mathbf{G}_1 x(t)$ (dotted lines). The open-loop system responses (dashed lines) are also displayed in Figs. 4 and 5, respectively. We can see that the responses given by the nonlinear state feedback controller are better. Furthermore, the closed-loop responses are much better than the open-loop responses. Fig. 6 shows the control signals given by the nonlinear state feedback controller and the linear state feedback controller.

V. Conclusion

A nonlinear state feedback controller has been proposed for nonlinear systems represented by a p-rule fuzzy plant model. The system stability of this fuzzy control system has been proven, which requires the satisfaction of p LMI conditions. A design methodology of the nonlinear gains of the nonlinear state feedback controller has been given under the consideration of the system stability. An application example has been presented to show the merits and the design procedures of the proposed nonlinear state feedback controller.

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