Nonlinear transmission and spatiotemporal solitons in metamaterials with negative refraction

Nina A. Zharova[1,2], **Ilya V. Shadrivov**[1], **Alexander A. Zharov**[1,3], **and Yuri S. Kivshar**[1]

[1]*Nonlinear Physics Centre and Center for Ultra-high bandwidth Devices for Optical Systems (CUDOS), Research School of Physical Sciences and Engineering, Australian National University, Canberra, Australia*

[2]*Institute of Applied Physics, Russian Academy of Sciences, Nizhny Novgorod 603600, Russia*

[3]*Institute for Physics of Microstructures, Russian Academy of Sciences, Nizhny Novgorod 603950, Russia*

http://wwwrsphysse.anu.edu.au/nonlinear/

Abstract: We study one- and two-dimensional transmission of electromagnetic waves through a finite slab of a dielectric material with negative refraction. In the case when the dielectric slab possesses an intensity-dependent nonlinear response, we observe the nonlinearity-induced wave transmission through an opaque slab accompanied by the generation of *spatiotemporal solitons*. We solve this problem numerically, by employing the finite-difference time-domain simulations, for the parameters of microstructured materials with the negative refractive index in the microwave region, but our results can be useful for a design of nonlinear metamaterials with the left-handed properties in other frequency range.

OCIS codes: (190.1450) Bistability; (190.5530) Pulse propagation and solitons; (999.9999) Metamaterials.

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1. Introduction

A number of theoretical studies [1, 2, 3, 4] and experimental results [5, 6, 7] demonstrated the existence of a novel type of microstructured materials which can be characterized, for some parameters, by a negative real part of the magnetic permeability and a negative real part of the dielectric permittivity in the microwave frequency range, being therefore described by the negative effective refractive index. These materials are often referred to as *left-handed metamaterials* (LHMs), double-negative materials, or materials with negative refraction. Properties of the left-handed materials were analyzed theoretically by Veselago long time ago [8], but only very recently such materials were demonstrated experimentally, as the composite structures created by arrays of metallic wires and split-ring resonators.

The microstructured design based on the lattices of the split-ring resonators and wires has already been demonstrated to give magnetism in the THz region of the spectrum [9]. It is believed that the concept can be extended into the infrared, bringing us close to the realization of magnetism at *optical frequencies*. Indeed, there has already been speculation that silver nanowires could be used to produce magnetic effects in the visible region [10]. Ultimately the design is limited by losses in the metallic components, losses which become severe as we attempt to extend the design to optical frequencies. Nevertheless, metamaterials open new doors for us in electromagnetism, giving access to material parameters not available in nature. Internal resonances are associated with these negative parameters, which couple strongly to the near field and can be used to control and manipulate the near field in ways not previously thought possible.

It has already been noticed that the left-handed metamaterials may possess quite complicated *nonlinear magnetic response* [11, 12], their properties can be altered by inserting *diodes* into the split-ring resonators [13], and *nonlinear metamaterials* can demonstrate interesting features of bistability [14] and the second-harmonic generation [15]. Importantly, the microscopic electric field in such composite structures can become much higher than the macroscopic electric field carried by the propagating electromagnetic wave. This provides a simple physical mechanism for enhancing nonlinear effects in the resonant structure with the left-handed properties. Moreover, any future effort in creating *tunable structures*, where the field intensity change the transmission of a composite structure, would require the study of nonlinear properties of such metamaterials, which are expected to be quite unusual.

In this paper, we study numerically the wave transmission through a slab of the left-handed

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metamaterial assuming that it possesses a hysteresis nonlinear response [11]. We make a step forward in comparison with the recent predictions in Ref. [11], and simulate numerically, with the help of the finite-difference time-domain simulations, a nonlinear microstructured material. When the slab possesses an intensity-dependent nonlinear response due to nonlinear dielectric inclusions in split-ring resonators, we observe the nonlinearity-induced transmission of the slab for larger input powers even the slab is opaque and totally reflecting for low-amplitude wave scattering. In addition, we observe that the spatiotemporal dynamics in the case of the over-critical transmission can be characterized by the generation and propagation of *spatiotemporal solitons*. We present the results for one- and two-dimensional geometries.

We consider a finite slab of a composite structure consisting of a cubic lattice of the periodic arrays of conducting wires and split-ring resonators (SRRs). We assume that the unit-cell size d of the structure is much smaller then the wavelength of the propagating electromagnetic field and, for simplicity, we choose the single-ring geometry of a lattice of cylindrical SRRs. The results obtained for this case are qualitatively similar to those obtained in the more involved cases of double SRRs. This type of microstructured left-handed materials has recently been demonstrated experimentally [5]. Somewhat similar composite nanostructures were suggested for realization of left-handed metamaterials for optical frequencies (see, e.g., Ref. [10]).

2. Nonlinear metamaterials

We study the scattering of the electromagnetic waves by a slab of the metamaterial assuming that the metamaterial possesses a nonlinear response. We use the finite-difference time-domain (FDTD) numerical simulations which allow the most complete analysis of spatiotemporal effects in the wave scattering. To describe the nonlinearity response of the metamaterial, we employ the effective averaged Maxwell equations in the standard form

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{1}$$

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\langle \mathbf{j} \rangle + 4\pi \nabla \times \mathbf{M}, \tag{2}$$

where $\langle \mathbf{j} \rangle$ is the current density averaged over the period of the unit cell of the cubic lattice, and $\mathbf{M}$ is the magnetization of the metamaterial. We base our numerical simulations on the microscopic model of a nonlinear metamaterial that generalizes the linear model recently introduced in Ref. [16]. First, we write the averaged constitutive relations in the following form [16],

$$\begin{aligned} \sigma L_w S \frac{d\langle \mathbf{j} \rangle}{dt} + \langle \mathbf{j} \rangle &= \frac{\sigma S}{d_{\text{cell}}^2}\mathbf{E}, \\ \mathbf{M} &= \frac{n_m}{2c}\pi a^2 I_R \frac{\mathbf{B}}{|\mathbf{B}|}, \end{aligned} \tag{3}$$

where L_w is the inductance of a metallic wire per unit length, σ is the conductivity of metal used in the composite, a is the SRR radius, d_{cell} is the size of the unit cell of the composite structure, S is the effective cross-section of a wire, $S \approx \pi r_w^2$, for $\delta > r_w$, and $S \approx \pi\delta(2r-\delta)$, for $\delta < r_w$, where r_w is the radius of the wires, $\delta = c/\sqrt{2\pi\sigma\omega}$ is the skin-layer thickness, I_R is the current in SRR, n_m is the concentration of SRRs. The current in SRRs is governed by the equation

$$L\frac{dI_R}{dt} = -\frac{\pi a^2}{c}\frac{dH'}{dt} - U - RI_R, \tag{4}$$

where L is the inductance of SRR, R is the resistance of the SRR wire, U is the voltage on the SRR slit, and H' is the acting (microscopic) magnetic field, which differs from the average

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(macroscopic) magnetic field. Voltage U at the slit of SRR is coupled to the current I through the relation

$$C(U)\frac{dU}{dt} = I_R, \tag{5}$$

with the nonlinear capacitance introduced as

$$C(U) = \frac{r^2\varepsilon}{4d_g}\left(1+\alpha\frac{|U|^2}{U_c^2}\right), \tag{6}$$

where ε is the linear part of the permittivity of a dielectric material inside the SRR slit of the size d_g, U_c is the characteristic nonlinear voltage, and $\alpha = \pm 1$ corresponds to the case of the focusing and defocusing nonlinear response, i.e. permittivity of the nonlinear dielectric in the SRR slit is taken in the form $\varepsilon_D(|E|) = \varepsilon + \alpha|E|^2$.

The microscopic magnetic field $\mathbf{H}'$ can be expressed in terms of $\mathbf{M}$ and $\mathbf{B}$ using the Lorenz-Lorentz relation [17]:

$$\mathbf{H}' = \mathbf{B} - \frac{8\pi}{3}\mathbf{M}. \tag{7}$$

As a result, Eqs. (1) to (7) form a closed set of equations, and they are solved here numerically using the FDTD method. We also notice that, by substituting the harmonic fields into these equations, we recover the expression for the magnetic permeability of a nonlinear left-handed metamaterial similar to that derived earlier in Ref. [11],

$$\mu_{\text{eff}}(\mathbf{H}) = 1 + \frac{F\,\omega^2}{\omega_{0NL}^2(\mathbf{H}) - \omega^2(1+F/3) + i\Gamma\omega}, \tag{8}$$

where

$$\omega_{0NL}^2(\mathbf{H}) = \left(\frac{c}{a}\right)^2 \frac{d_g}{[\pi h \varepsilon_D(|\mathbf{E}_g(\mathbf{H})|^2)]}$$

is the eigenfrequency of nonlinear oscillations, $\Gamma = c^2/4\pi\sigma a r_w$ is the dumping coefficient, and $F = \pi^2 a^3/2d_{\text{cell}}^3[\ln(8a/r_w) - 7/4]$ is the filling factor.

3. One-dimensional scattering

To study the spatiotemporal dynamics of the wave scattering by a slab of a nonlinear metamaterial in the framework of the model introduced above, first we consider a simpler one-dimensional problem that describes the interaction of the plane wave incident at the normal angle from air on a finite slab of the metamaterial. We consider *two types of nonlinear effects*: (i) nonlinearity-induced suppression of the wave transmission when initially transparent left-handed material becomes opaque with the growth of the input wave amplitude, and (ii) nonlinearity-induced transparency of the slab when an initially opaque composite material becomes left-handed (and, therefore, transparent) with the growth of the input wave amplitude.

In our simulations, we assume that the amplitude of the incident wave grows linearly for the first 50 periods, and then it becomes constant. The slab thickness is $1.3\lambda_0$, where λ_0 is a free-space wavelength. For the selected parameters, the metamaterial is left-handed in the linear regime for the frequency range from $f_1 = 5.787$ GHz to $f_2 = 6.05$ GHz.

Our FDTD numerical simulations show that for the incident wave with the frequency $f_0 = 5.9$ GHz (i.e. inside the left-handed transmission band), the electromagnetic field reaches a steady state independently of the sign of nonlinearity. Both reflection and transmission coefficients in *the stationary regime* are shown in Figs. 1 as functions of the incident field amplitude, for defocusing and focusing types of nonlinearity. Here and in the rest of the paper, the incident field intensity is normalized to the squared characteristic field, $E_c^2 = U_c^2/d_g^2$. In the linear regime,

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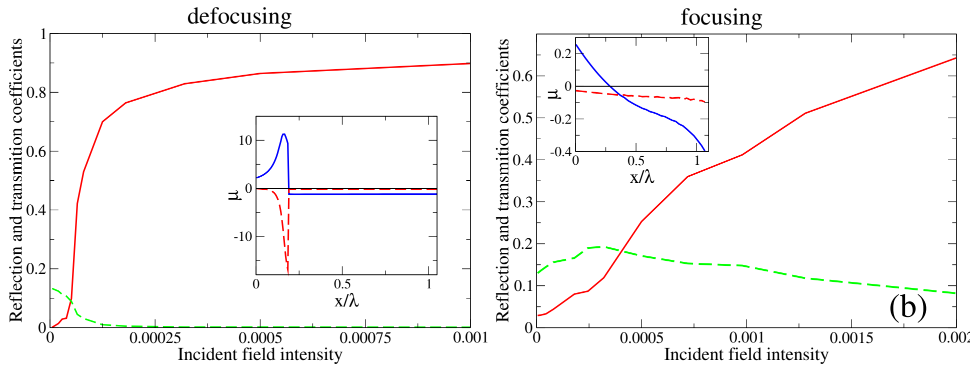

Fig. 1. Reflection (solid red) and transmission (dashed green) coefficients for a slab of nonlinear metamaterial *vs.* the normalized incident field intensity in a stationary regime, for the case of (a) defocusing nonlinearity ($\alpha = -1$), and (b) focusing nonlinearity ($\alpha = +1$). Insets show real (solid blue) and imaginary (dashed red) parts of the magnetic permeability inside the slab of a composite material.

the effective parameters of the metamaterial at the frequency f_0 are: $\varepsilon = -1.33 - 0.01i$ and $\mu = -1.27 - 0.3i$; this allows the impedance matching with surrounding air, so that the reflection coefficient vanishes for small intensities, as shown in Fig. 1(a).

Reflection and transmission coefficients differ qualitatively for two types of nonlinearity. For *defocusing nonlinearity*, the reflection coefficient varies from low to high values when the incident field exceeds some threshold value, see Fig. 1(a). Such a sharp transition can be explained in terms of the hysteresis behavior of the magnetic permeability described by Eq. (8) and discussed in Ref. [11]. When the field amplitude in the metamaterial becomes higher than a certain critical amplitude, magnetic permeability changes its sign, and the metamaterial becomes opaque. Our FDTD simulations show that for the overcritical amplitudes of the incident field, the opaque region of positive magnetic permeability appears inside the slab [see the inset in Fig. 1(a)], and the magnetic permeability experiences an abrupt change at the boundary between the transparent and opaque regions. The dependencies shown in Fig. 1(a) are obtained for the case when the incident field grows from zero to a steady-state value. However, taking different temporal behavior of the incident wave, e.g. increasing the amplitude above the threshold value and then decreasing it to a steady state, one can get different values of the stationary reflection and transmission coefficients, and different distributions of the magnetic permeability inside the metamaterial slab. Such properties of the nonlinear metamaterial slab are consistent with the multi-valued dependence of the magnetic permeability 8.

In the case of *focusing nonlinearity* [see Fig. 1(b)], the dependence of the reflection and transmission coefficients on the amplitude of the incident field is smooth. First of all, this effect originates from a gradual detuning from the impedance matching condition, and, second, from the appearance of an opaque layer with a positive value of the magnetic permeability for higher powers that is a continuous function of the coordinate inside the slab, as shown in the inset of Fig. 1(b).

Now we consider the other type of nonlinear effects mentioned above when an initially opaque composite metamaterial becomes transparent with the growth of the incident field amplitude. We take the frequency of the incident field to be $f_0 = 5.67$ GHz, so that the magnetic permeability is positive in the linear regime and, correspondingly, the metamaterial is opaque for the incoming waves. In the case of the self-focusing nonlinear response ($\alpha = +1$), the material properties can be “switched” to the regime with the negative magnetic permeability when

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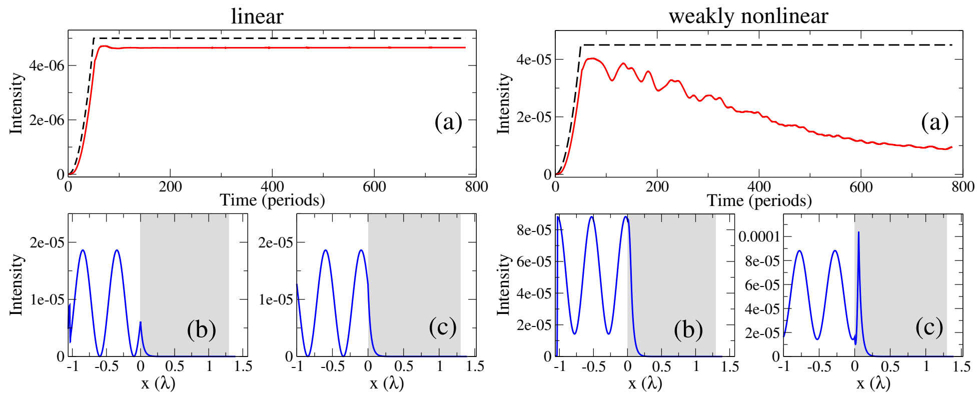

Fig. 2. (a) Temporal evolution of the reflected (solid) and incident (dashed) wave intensity. (b,c) Spatial distribution of the magnetic and electric fields, respectively, at the end of the simulation domain; the metamaterial region is shaded. Left: low-amplitude (linear) regime, right: weakly nonlinear regime.

the slab becomes left-handed and, therefore, transparent. Figure 2(a, right) shows the temporal evolution of the incident and reflected wave intensities for small input amplitudes, this case corresponds to the linear regime. The reflection coefficient reaches a steady state after approximately 100 periods. The final spatial distribution of the electric and magnetic fields is shown in Figs. 2(b,c), respectively.

In a weakly nonlinear regime [see Fig. 2(right)], the intensity of the reflected beam decreases approaching a steady state. In this case we observe the formation of a localized state inside the metamaterial slab and near the interface, as can be seen more distinctly in Fig. 2(c, right). This effect gives an additional contribution to the absorption of the electromagnetic energy, and it leads to a decrease of the reflection coefficient.

In a strongly nonlinear, overcritical regime, we observe the effect of the dynamical self-modulation of the reflected electromagnetic wave that results from the periodic generation of the self-localized states inside the metamaterial, as oscillating localized states near the interface [see Fig. 3(left)], or as propagating pulses [see Fig. 3(right)]. Somewhat related effect of the formation of self-focused localized states inside a nonlinear material was previously discussed for the problem of interaction of the intense electromagnetic waves with over-dense plasma [18, 19]. Such localized states can be termed as *spatiotemporal electromagnetic solitons* [20], and they can transfer the energy away from the interface. Figure 3(c, right) shows an example when two localized states enter the metamaterial. These localized states appear at the jumps of the magnetic permeability and, as a result, we observe a change of the sign of the derivative of the electric field at the maximum of the soliton intensity, and subsequent appearance of transparent regions in the metamaterial. Unlike all previous cases, the field structure in this regime does not reach any steady state for high intensities of the incident field.

4. Two-dimensional scattering

Now we consider the two-dimensional beam scattering by a slab of the nonlinear metamaterial, and present the FDTD results for the nonlinearity-induced transparency of the metamaterial. We launch a TM-polarized beam of the width $2\lambda_0$ at the angle 45^o from the left towards the surface of the metamaterial slab of the thickness $0.9\lambda_0$. Figures 4 and 5 (top) show the snapshots of the magnetic field distribution at the linear stage (simulation time $t = 67$T, T is the field oscillation period) and nonlinear stage (simulation time $t = 381T$) of simulations. Modi-

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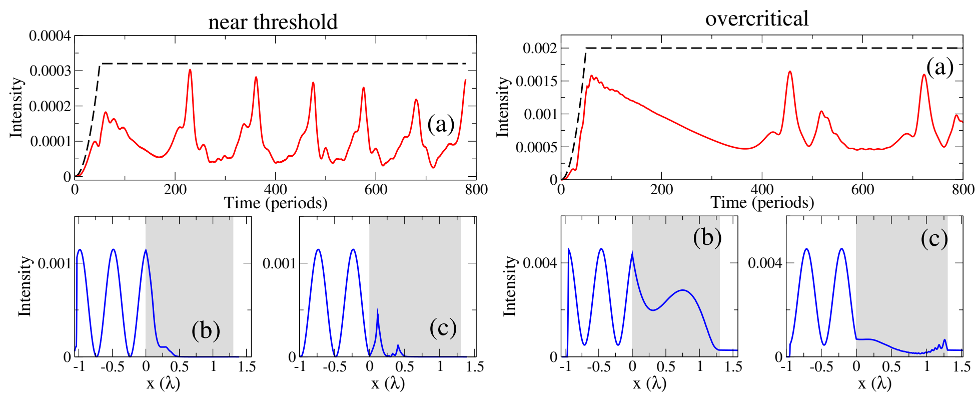

Fig. 3. (a) Temporal evolution of the reflected (solid) and incident (dashed) wave intensity in the strongly nonlinear regime (i.e., for the overcritical amplitude of the incident wave). (b,c) Spatial distribution of the magnetic and electric fields, respectively, at the end of simulation domain; the metamaterial is shaded. Left: generation of an oscillating localized state at the surface, right: soliton generation in the overcritical regime.

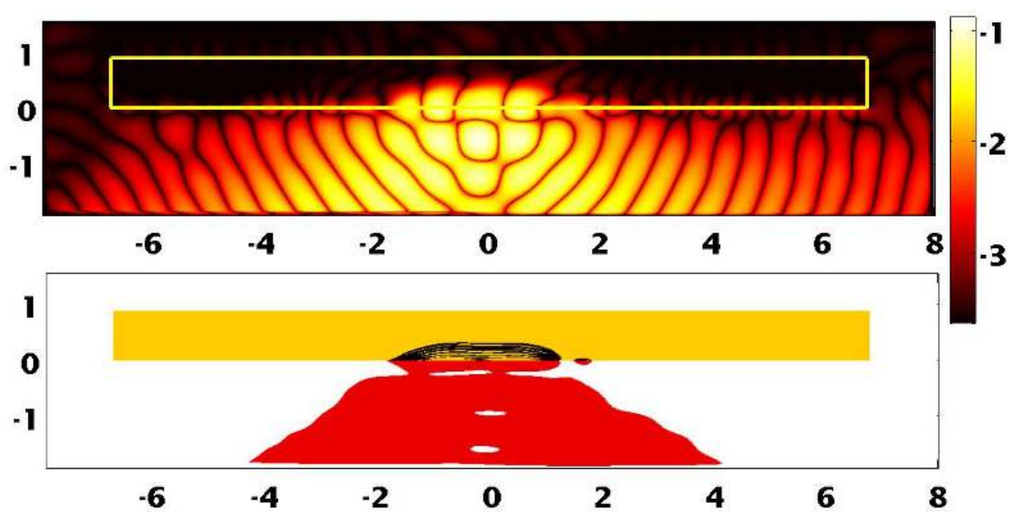

Fig. 4. Magnetic field distribution (in logarithmic scale) for the beam scattering by a metamaterial slab in low-intensity regime (top). Bottom – plot shows that the metamaterial is opaque (yellow) for the beam incident at 45 degrees from the left. Red color indicates the high field areas outside the slab. Coordinates are normalized on the free-space wavelength.

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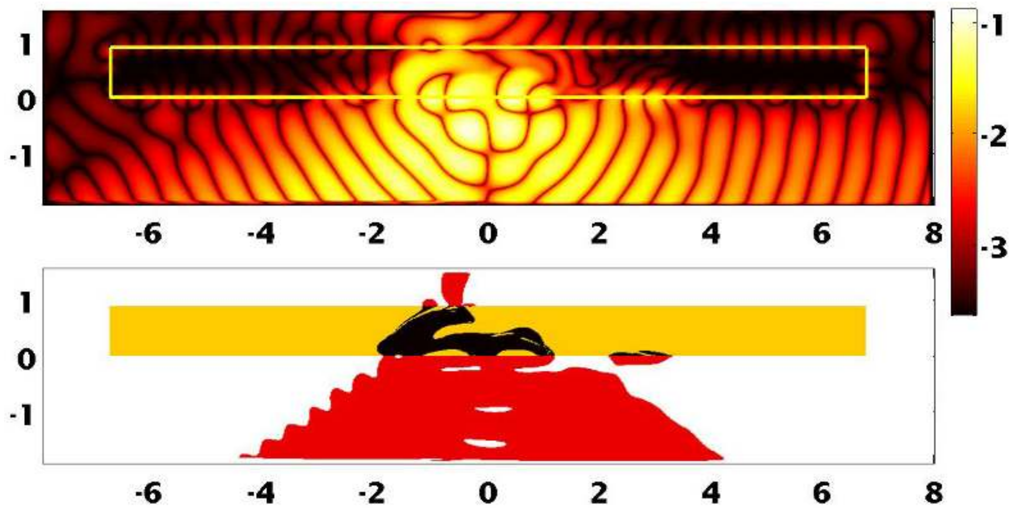

Fig. 5. Magnetic field distribution (in logarithmic scale) for the beam scattering by a metamaterial slab in high-intensity regime (top). Bottom – plot shows the transparent left-handed domain (black) formed in initially opaque metamaterial (yellow) by the beam incident at 45 degrees. Red color indicates the high field areas outside the slab. Coordinates are normalized on the free-space wavelength.

fication of the metamaterial parameters in the high intensity area results in the formation of the non-stationary spatiotemporal soliton inside the slab [see Fig. 5(top)], which makes possible for the electromagnetic energy to penetrate through the slab. The dynamics of the soliton formation is qualitatively similar to that in the overcritical one-dimensional case discussed above. Figures 4 and 5 (bottom) show the formation of the transparent left-handed domain (shown by black) inside the metamaterial slab, induced by the electromagnetic field. One can see that an initially opaque slab [see Fig. 4(bottom)] becomes transparent [see black area inside the slab in Fig. 5(bottom)] for the high enough field intensities. The shift of the transparent domain to the left indicates the negative refraction of the beam in the left-handed slab.

5. Conclusions

We have demonstrated novel effects associated with the nonlinear response of metamaterials. Using the FDTD numerical simulations, we have studied the spatiotemporal dynamics of the wave scattering by a slab of a nonlinear metamaterial and observed two types of nonlinear effects associated with a change of the metamaterial properties: (i) nonlinearity-induced suppression of the wave transmission, and (ii) the nonlinearity-induced transmission and the generation of spatiotemporal electromagnetic solitons.

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