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## Part I Nonlinear Effects in Reduced-Dimensional Structures: From Wave Guide Arrays to Slow Light

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