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NONLINEARITY AND THE PERMANENT EFFECTS OF RECESSIONS

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SUMMARY

This paper presents a new nonlinear time series model that captures a post-recession 'bounce-back' in the level of aggregate output. While a number of studies have examined this type of business cycle asymmetry using recession-based dummy variables and threshold models, we relate the 'bounce-back' effect to an endogenously estimated unobservable Markov-switching state variable. When the model is applied to US real GDP, we find that the Markov-switching regimes are closely related to NBER-dated recessions and expansions. Also, the Markov-switching form of nonlinearity is statistically significant and the 'bounce-back' effect is large, implying that the permanent effects of recessions are small. Meanwhile, having accounted for the 'bounce-back' effect, we find little or no remaining serial correlation in the data, suggesting that our model is sufficient to capture the defining features of US business cycle dynamics. When the model is applied to other countries, we find larger permanent effects of recessions. Copyright © 2005 John Wiley & Sons, Ltd.

1. INTRODUCTION

In his seminal paper, Hamilton (1989) captures asymmetry in US business cycles using a regime-switching model of real output. His model portrays the short, violent nature of recessions relative to expansions. However, other studies emphasize another distinctive feature of US business cycles not captured by Hamilton's model: output growth tends to be relatively strong following recessions. A simple way to capture this feature is to exogenously allow growth dynamics to change in the quarters immediately after a decline in output below its historical maximum (see, for example, Beaudry and Koop, 1993).

In this paper we show that Hamilton's model can be extended to allow for a post-recession 'bounce-back' in the level of output, while maintaining endogenously estimated business cycle regimes. When we extend Hamilton's model, we find that the Markov-switching form of nonlinearity is statistically significant and that the 'bounce-back' effect is large. An attractive feature of the model is that it provides a straightforward estimate of the permanent effects of recessions on the level of output. We find that these effects are substantially less than suggested by Hamilton or by most linear models (e.g. Nelson and Plosser, 1982; Campbell and Mankiw, 1987; Stock and Watson, 1988). In addition, once the 'bounce-back' effect is taken into account, there is little or no remaining serial correlation, suggesting that expansionary shocks are permanent and the nonlinearity in the model is sufficient to capture the defining features of US business cycle

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dynamics. Using a model comparison approach similar to that in Hess and Iwata (1997a), we find that our model is better able to reproduce business cycle features than other standard models. We also find that the 'bounce-back' effect is robust to allowing for a one-time structural break in business cycle volatility in the mid-1980s and to relating the size of the post-recession 'bounce-back' to the depth of the preceding recession. Finally, when we apply the model to output data for other countries including Australia, Canada and the United Kingdom, we find larger permanent effects of recessions.

2. BACKGROUND

The idea of inherently different dynamics in expansions and recessions has a long history in business cycle analysis, dating back at least to Mitchell (1927) and Keynes (1936). Recent advances in econometrics have allowed this idea to be formally modelled and tested. Hamilton (1989) captures asymmetries using a Markov-switching model that estimates two regimes in the mean growth rate of US GNP. Notably, even though the timing of the regimes is endogenously estimated, he finds that the regimes correspond closely to NBER-dated recessions and expansions. Despite this success, statistical tests of Hamilton's model have often failed to reject a linear null hypothesis (see Hansen, 1992; Garcia, 1998).[1]

The parameter estimates of Hamilton's model yield a striking implication: recessions have large permanent effects on the level of output. By one measure discussed in his paper and employed here, the expected level of output is permanently lowered by as much as 4.5% as a result of a transition into recession. However, one reason this estimate may be so large is that Hamilton's original model is unable to capture the high growth recovery phase typical of post-recession dynamics in the United States. This apparent 'bounce-back' in economic activity is evident in Table I, which reports the average growth rates for US real GDP in the quarters immediately following the troughs of NBER-dated post-war recessions.

One approach to modelling the high growth recovery phase is to add a distinct third regime to Hamilton's model, as in Sichel (1994). However, there is some evidence that recoveries are not independent of the preceding recession, as would be implied by a standard three-regime model, but rather the strength of the post-recession recovery is related to the length and severity of the recession (see Friedman, 1964, 1993; Wynne and Balke, 1992, 1996). Kim and Nelson (1999a) allow for this type of business cycle asymmetry by modelling regime switching in the cyclical component of output only. While this relates the strength of the recovery to the preceding recession, it constrains the effects of recessionary shocks to be completely transitory, *a priori*. Thus, we cannot use this approach to examine the permanent effects of recessions. Kim and Murray (2002) combine the Hamilton (1989) and Kim and Nelson (1999a) approaches in a multivariate model with regime switching in both the trend and cycle components of output. While this approach is capable of providing a measure of the permanent effects of recessions, it comes at the price of considerable added complexity and the need for strong identification assumptions.

A related literature models a post-recession 'bounce-back' using nonlinear ARMA processes in which dynamics change when an observed indicator variable exceeds a given threshold. In an important paper, Beaudry and Koop (1993) augment a standard ARMA model of output growth

[1] However, Hansen (1992) shows that allowing for regime shifts in the autoregressive coefficients of Hamilton's (1989) model can yield a statistical rejection of the linear null hypothesis.

Table I. US real GDP growth rates

Quarters after recession	Average growth	Observations
1	7.01	10
2	6.34	10
3	6.12	10
4	6.03	9
5	4.32	9
6	4.36	8
7	4.00	8
8	3.32	7
Full sample	3.33	217

Note: Average growth rates are measured as annualized percentages. The sample period is 1949:Q1 to 2003:Q1. For four quarters and longer, one observation is lost due to the termination of the expansion following the 1980 recession. For six quarters and longer, another observation is lost due to the end of the sample following the 2001 recession. For eight quarters, yet another observation is lost due to the termination of the expansion following the 1957–1958 recession.

with a 'current-depth-of-recession' dummy variable that measures the distance output has fallen below its previous historical maximum. They find that this additional variable is highly significant using a standard t-test, and that recessions have small permanent effects on the level of US real GDP. However, Hess and Iwata (1997b) argue that the dummy variable is nonstationary, implying that standard critical values overstate the significance of the t-statistic.[2] The Beaudry and Koop model has been extended and modified by several authors, most notably Pesaran and Potter (1997), who endogenize the threshold. Similarly, Tiao and Tsay (1994), van Dijk and Franses (1999) and Öcal and Osborn (2000) have estimated multiple regime threshold models in which one regime is a high growth phase following economic contractions.

Our approach in this paper is to augment Hamilton's original model with a 'bounce-back' term that is scaled by the length of each recession and can generate faster growth in the quarters immediately following a recession. In some sense, our model is closely related to Sichel's (1994) three-regime model in that it implies an expansion, a recession and a recovery phase. However, given the link between each recession and the strength of the subsequent recovery, our model is much like Beaudry and Koop's (1993). Of course, unlike the 'current-depth-of-recession' variable used in their paper, the 'bounce-back' term is directly related to the underlying recessionary regimes and is, therefore, endogenously estimated. The 'bounce-back' term is also stationary by construction and so does not suffer from the Hess and Iwata (1997b) critique. Meanwhile, our model places no constraints on the permanent effects of recessions and, like Hamilton's original model, yields a straightforward measure of the expected long-run effect.

[2] On the other hand, given the positive drift in real GDP, it is not clear that the 'current-depth-of-recession' variable is, in fact, nonstationary.

3. MODEL

Our model is given as follows:

$$\phi(L)\left(\Delta y_t - \mu_0 - \mu_1 S_t - \lambda \sum_{j=1}^{m} S_{t-j}\right) = \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d. } N(0, \sigma^2) \tag{1}$$

where the lag operator $\phi(L)$ is kth order with roots outside the unit circle, Δy_t is the first difference of log real GDP, and S_t is an unobserved Markov-switching state variable that takes on discrete values of 0 or 1 according to transition probabilities $\Pr[S_t = 0|S_{t-1} = 0] = q$ and $\Pr[S_t = 1|S_{t-1} = 1] = p$. We normalize the states by restricting $\mu_1 < 0$. If $\mu_0 + \mu_1 < 0$, then $S_t = 1$ corresponds to a 'contractionary' regime.

The key variable in our model is the summation term $\sum_{j=1}^{m} S_{t-j}$, which we denote as $\widetilde{S}_t$, hereafter. This term implies a 'bounce-back' effect if $\lambda > 0$, while Hamilton's (1989) model obtains if $\lambda = 0$. Given $\lambda > 0$, $\widetilde{S}_t$ implies that growth will be above average for the first m periods of an 'expansionary' regime.

To see how the 'bounce-back' effect works, consider Figure 1, which shows the simulated effect of a recession for both our model and Hamilton's original model. For both models, we set the underlying growth rate parameters to be $\mu_0 = 1$ and $\mu_1 = -2$. For our model, we set the 'bounce-back' coefficient to be $\lambda = 0.2$ and the length of the post-recession 'bounce-back' to $m = 6$ periods. We ignore the autoregressive parameters since, for simplicity of presentation, we abstract from the regular linear ε_t shocks in simulating the effects of a recession on output. In the bottom of the figure, the thick line represents a hypothetical time path for the state variable S_t. The shift in S_t from 0 to 1 represents a movement of the economy into a 'contractionary' regime for the four periods denoted by the shading. As the regime hits at time $t = 0$ and persists until time $t = 4$, output falls for both our model and Hamilton's model. Meanwhile, the summation

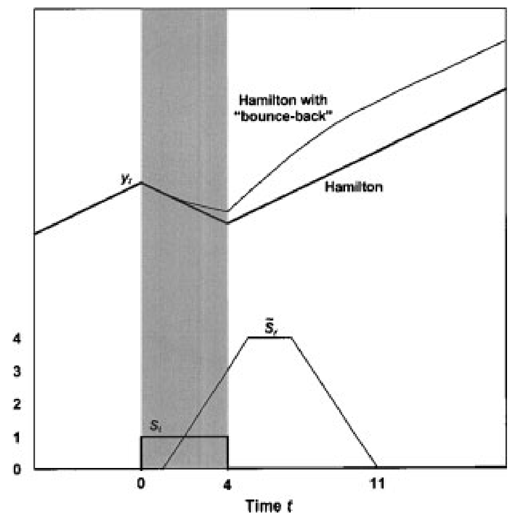

Figure 1. The 'bounce-back' effect (recession is shaded)

term $\widetilde{S}_t$ increases each period up to the length of the recession. For our model, the effect of the $\widetilde{S}_t$ term begins to offset the effect of the S_t term as the recession persists, and output starts to level off.[3] After the recession ends and S_t returns to 0, the summation term $\widetilde{S}_t$ reaches its maximum, and the level of output rises faster than average since $\lambda > 0$. This 'bounce-back' in the level of output continues for $m = 6$ periods, but its effect diminishes as the expansion persists and the $\widetilde{S}_t$ term decreases until it reaches its minimum value of 0. By contrast, Hamilton's model with $\lambda = 0$ has output rise from its trough at its regular 'expansionary' growth rate only, corresponding to a much larger permanent effect of the recession on the level of output.

We estimate the model in (1) via maximum likelihood using the filter presented in Hamilton (1989). The main added complexity is that, due to the $\widetilde{S}_t$ term, we need to keep track of more states (2^{k+m} versus 2^k, where k is the number of autoregressive terms) when constructing the likelihood function in each period. Standard errors are based on numerical second derivatives.

4. ESTIMATES FOR US GDP

The data for y_t are 100 times the log of quarterly US real GDP over the period 1947:Q1 to 2003:Q1. To keep the sample period consistent for every model considered in this paper, we set the first observation for the dependent variable Δy_t to 1949:Q1. We use the Schwarz information criterion (SIC) to select the lag length k for the autoregressive polynomial and the length m of the post-recession 'bounce-back'. We consider upper bounds of $k = 4$ and $m = 9$. For the autoregressive polynomial, we find that $k = 0$, suggesting that the nonlinear dynamics in our model are sufficient to capture most or all of the serial correlation in the data. For the post-recession 'bounce-back', we find that $m = 6$ quarters, which is consistent with the results in Table I.

Table II reports maximum likelihood estimates for the $k = 0$ and $m = 6$ case. The results are robust for similar values of k and m. The first thing to notice about the estimates is that $\mu_0 + \mu_1 < 0$, implying that $S_t = 1$ corresponds to a 'contractionary' regime. The transition probabilities also suggest that expansions are more persistent than contractions, much like the NBER reference cycle. Indeed, the top panel of Figure 2 reveals a strong correspondence between

Table II. Maximum likelihood estimates for the 'bounce-back' model

Parameter	Estimate	Standard error
μ_0	0.836	0.064
μ_1	−2.055	0.232
$\mu_0 + \mu_1$	−1.219	0.229
λ	0.319	0.050
q	0.957	0.017
p	0.695	0.101
σ	0.768	0.042
Λ	−0.412	0.898
Log-likelihood		−288.088

[3] As an alternative specification, we could have allowed the recession state variable and the 'bounce-back' term to interact such that the dynamics in the Hamilton model and our model differ only following a recession. We examine this possibility when we consider specification tests of our model in Section 5.

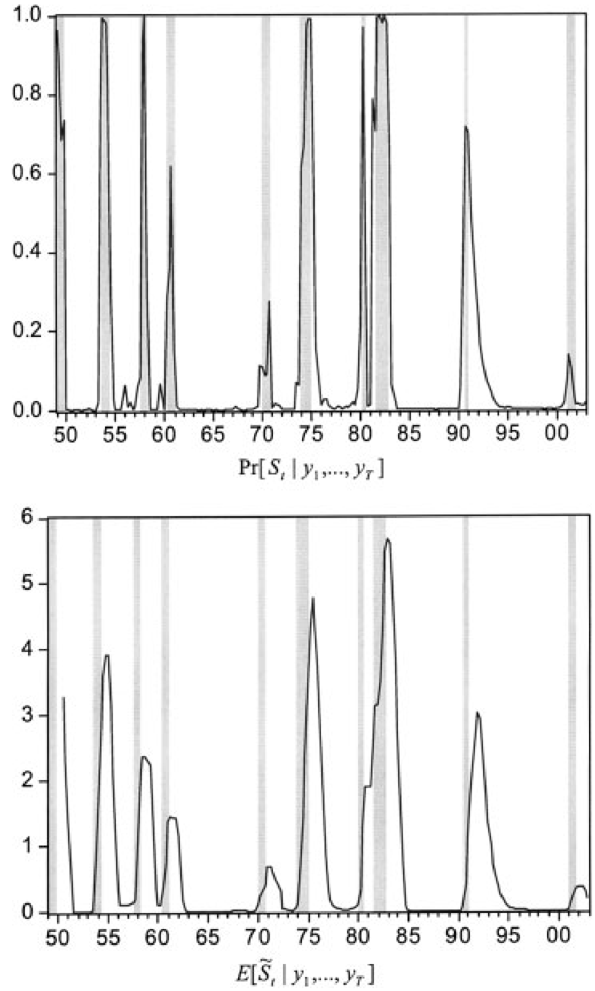

Figure 2. Smoothed inferences for S_t and $\tilde{S}_t$ (NBER recessions are shaded)

the smoothed probability of being in a contractionary regime and the NBER recession dates denoted by the shading. For eight of the ten NBER recessions in the sample, the smoothed probability spikes up above 50% immediately after the business cycle peak date established by the NBER. The 1970 and 2001 recessions are the exceptions, for these recessions the smoothed probability moves up during the NBER recession dates, but remains below 50%. Also, for seven of these eight recessions, the smoothed probability falls to close to zero around the trough date established by the NBER. Here the exception is the 1990–1991 recession, for which the smoothed probability only returns to low levels after the end of the NBER trough date.[4] Meanwhile, the bottom panel

[4] In Section 7, we discuss some modifications of our model that improve its ability to capture the 1970, 1990–1991 and 2001 recessions. We note, however, that these modifications come at the cost of losing a straightforward measure of permanent effects of recessions.

of Figure 2 displays the smoothed estimate of $\tilde{S}_t$. As in Figure 1, this term increases as the length of each contraction progresses, and declines soon after the recession is over. Again, this term and its coefficient λ determine the size of the 'bounce-back' effect. Our estimate of λ is positive, corresponding to faster growth during post-recession recoveries.

5. TESTING THE MODEL

We consider five tests of the model specification. First, we use Monte Carlo analysis to test the significance of the Markov-switching form of nonlinearity that underlies our whole analysis. Second, we use Monte Carlo analysis to examine the small sample distribution of the 'bounce-back' effect conditional on Markov switching. Third, we consider whether the 'bounce-back' effect operates during prolonged recessions, as specified in our model, or only after recessions end. Fourth, we consider whether the dynamics of the 'recovery' phase are independent of the length of the preceding recession, as in standard three-regime models. Fifth, we compare our model with a variety of other models in the ability to reproduce certain features of US business cycles.

To test Markov switching, we construct a likelihood ratio statistic for which the null hypothesis is a linear AR(2) model and the alternative is our model with $k = 2$ and $m = 6$.[5] We use Monte Carlo analysis to determine the distribution of the test statistic under the null. Specifically, we simulate 1000 series using estimates for a linear AR(2) model and compute the likelihood ratio statistic for each simulated series. A problem in conducting Monte Carlo analysis of regime-switching models is concern about local maxima and unstable estimation. To address these problems, we consider a grid search across the transition probabilities q and p when estimating the model under the alternative. Given the grid search, numerical optimization is stable and robust to different starting values, although estimation using even a very coarse grid is highly computationally intensive. We consider a grid for which q and p vary from 0.1 to 0.9 in increments of 0.1.[6] The coarseness of the grid affects the precision of the estimates and potentially reduces the power of the likelihood ratio test. However, despite any concerns about power, we are able to reject the linear model using the likelihood ratio test. Our test statistic based on a grid search using the historical data is 17.32, which has a p-value of less than 0.01 according to the Monte Carlo distribution displayed in Figure 3.[7] Thus, we strongly reject linearity for US GDP in favour of Markov switching.

Given this evidence of Markov switching, the next issue is whether the 'bounce-back' effect is statistically significant. The t-statistic for $H_0 : \lambda = 0$ is 6.4, which is highly significant using standard asymptotic critical values. However, there is a possible concern about whether relying on the standard critical values for this test is appropriate. Hess and Iwata (1997b) argue that Beaudry and Koop's (1993) 'current-depth-of-recession' variable is nonstationary. Thus, the estimate for its coefficient may have a nonstandard distribution. In our case, however, the $\tilde{S}_t$ term will be stationary since S_t is stationary and $\tilde{S}_t$ is the sum of a finite number of lags of S_t. Of course, given the persistence of the $\tilde{S}_t$ term, the small sample distribution may be very different to the asymptotic distribution. To address this concern, we conduct another Monte Carlo experiment. For our data generating process, we use Hamilton's (1989) original estimated model, for which $\lambda = 0$. We estimate our model allowing $\lambda \neq 0$ for each simulation and calculate t-statistics for

[5] We choose an AR(2) specification based on SIC lag selection under the null hypothesis of no Markov switching.
[6] Even with such a coarse grid and a 1.9 GHz processor, the experiment takes over 1000 CPU hours.
[7] The 10%, 5% and 1% critical values for the test of Markov switching are 10.1, 11.8 and 16.3, respectively.

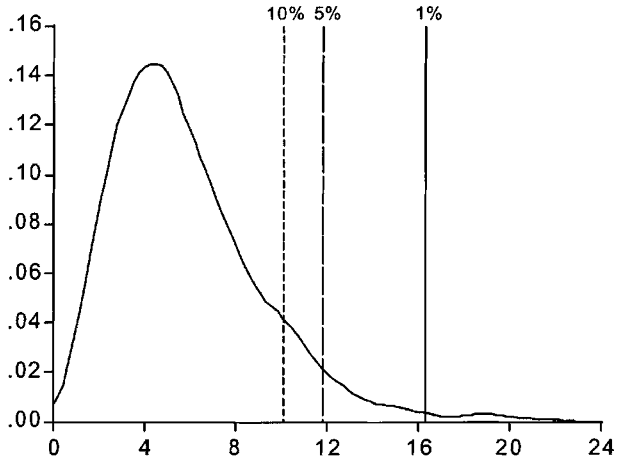

Figure 3. Monte Carlo distribution of test statistic for Markov switching (smoothed density generated by the Epanechnikov kernel)

the null hypothesis $H_0 : \lambda = 0$. Table III reports critical values for our experiment based on 1000 simulations and sample sizes of $T = 200$ and $T = 500$.[8] The critical values are larger than the standard normal case, reflecting a small sample distortion. However, our estimate of λ remains significant at the 1% level, even using the $T = 200$ distribution. Thus, conditional on Markov switching, there is strong evidence for a 'bounce-back' effect.

A more subtle specification issue is whether the $\tilde{S}_t$ term operates during the course of a recession or only takes hold after the recession is over. Specifically, in a prolonged recession, is there a levelling off of output, as displayed in Figure 1? To examine this possibility, we estimate a more general model that includes both the 'bounce-back' term $\tilde{S}_t$ and an interaction term between $\tilde{S}_t$ and $(1 - S_t)$. We find that the coefficient on the 'bounce-back' term is largely unchanged, while the coefficient on the interaction term is highly insignificant (the t-statistic is -0.12). Thus, the levelling off of output during a prolonged recession appears to be an important aspect of business cycle dynamics.

Another subtle specification issue is whether the recovery phase is strongly linked to the length of the preceding recession, as specified in our model, or whether it is a distinct third regime that is independent of the severity of the preceding recession. To examine this, we augment our

Table III. Monte Carlo results for testing the 'bounce-back' effect

p-Value	Critical values		
	$T = 200$	$T = 500$	$N(0,1)$
0.01	5.00	3.16	2.57
0.05	3.09	2.19	1.96
0.10	2.45	1.85	1.64

Note: Critical values are based on 1000 simulations.

[8] Each experiment takes over 400 CPU hours using a 1.9 GHz processor.

'bounce-back' model with a third regime. The three-regime model with a 'bounce-back' effect is

$$\phi(L)\left(\Delta y_t - \mu_0 - \mu_1 S_{1t} - \mu_2 S_{2t} - \lambda \sum_{j=1}^{m} S_{1t-j}\right) = \varepsilon_t, \quad \varepsilon_t \sim \text{ i.i.d.} N(0, \sigma^2) \tag{2}$$

where $S_{1t} = 1$ if $S_t = 2$ and 0 otherwise, $S_{2t} = 1$ if $S_t = 3$ and 0 otherwise, and S_t is an unobserved Markov-switching state variable that takes on discrete values of 1, 2, 3 according to transition probabilities $\Pr[S_t = i | S_{t-1} = j]$, which are summarized by the following transition probability matrix with i defined by the row and j defined by the column:

$$\begin{bmatrix} q & 0 & 1-r \\ 1-q & p & 0 \\ 0 & 1-p & r \end{bmatrix}$$

Following Sichel (1994) and Boldin (1996), these transition probabilities are constrained such that the three regimes corresponding to expansion, recession and recovery always occur in that order. The parameter r denotes the probability of remaining in the 'recovery' regime, during which the underlying expansionary growth rate of μ_0 is augmented by μ_2. Table IV reports estimates for the three-regime model with a 'bounce-back' effect. Interestingly, there appears to be both faster growth in the third regime ($\mu_2 > 0$) and a significant 'bounce-back' effect ($\lambda > 0$). However, the third regime is reasonably persistent and the growth rate in the expansionary regime is smaller than for our model. Thus, the third regime appears to be allowing for slower growth in the latter stages of an expansion, rather than capturing faster growth in the recovery. Instead, the faster growth recovery is captured by the 'bounce-back' effect. Meanwhile, the third regime is not statistically significant. The likelihood ratio statistic comparing the three-regime model to our model is 4.00. Based on a $\chi^2(3)$ distribution, which likely overstates the significance due to the presence of nuisance parameters, the p-value for this test statistic is 0.26. Therefore, we do not include a third regime in our main model.

The final test of our model involves comparing linear and Markov-switching models in terms of their ability to reproduce certain features of the US business cycle. Specifically, we simulate data series for each model where parameters are set to their maximum likelihood values and the length of each simulated series is the same as our sample period. We then evaluate the extent to which

Table IV. Maximum likelihood estimates for the three-regime model with a 'bounce-back' effect

Parameter	Estimate	Standard error
μ_0	0.670	0.098
μ_1	−1.830	0.231
μ_2	0.484	0.154
$\mu_0 + \mu_1$	−1.160	0.214
$\mu_0 + \mu_2$	1.154	0.134
λ	0.260	0.057
q	0.933	0.027
p	0.678	0.102
r	0.897	0.043
σ	0.742	0.041
Log-likelihood		−286.081

the simulated series produce recessions and expansions that are similar in character to those in the historical data. This technique for evaluating time-series models of aggregate output has been used in several recent studies, including Hess and Iwata (1997a), Galvão (2000) and Harding and Pagan (2002a).

For comparison with our 'bounce-back' model, we consider three linear models: an AR(1), an AR(2) and an MA(1). In terms of these three models, the SIC favours the AR(2) specification, although Hess and Iwata (1997a) find that the AR(1) performs best in capturing business cycle features. We also consider two alternative Markov-switching models, namely Hamilton's original two-regime model without a 'bounce-back' effect and a three-regime model without a 'bounce-back' effect. These two models correspond to versions of (1) and (2) where $\lambda = 0$. The various models are evaluated in terms of their ability to replicate the average length and depth of recessions. Also, we consider two features of US business cycles related to a post-recession 'bounce-back'. The first is a higher growth rate in the early stages of a recovery, which we measure using the average growth rate in the four quarters following a business cycle trough. The second is a relationship between the severity of a recession and the strength of the subsequent recovery, which we measure using the correlation between the depth of a recession and the growth rate in the four quarters following the trough.

In order to measure these business cycle features, we need an algorithm to identify peaks and troughs in a given data series. For this purpose, we use Harding and Pagan's (2002a) extension of the Bry and Boschan (1971) algorithm to the analysis of quarterly data. Operationally, the algorithm has three steps. First, we identify peaks and troughs as local maxima and minima. Specifically, a particular quarter is determined to be a peak (trough) if the level of the series is higher (lower) than in the previous and subsequent two quarters. Second, we ensure that peaks and troughs alternate by selecting the highest (lowest) of multiple peaks (troughs). Third, we apply censoring rules ensuring that business cycle phases last a minimum of two quarters and complete cycles a minimum of five quarters. Harding and Pagan (2002b) show that when this algorithm is applied to US real GDP, it provides a chronology of peak and trough dates very close to that established by the NBER.[9]

Table V compares the mean values and standard deviations of the various business cycle features obtained from 10,000 simulations for each model with the corresponding business cycle features for US real GDP. Consistent with Hess and Iwata (1997a) and Harding and Pagan (2002a), we find that the Markov-switching models provide no obvious improvement over linear models in replicating the length of recessions. However, the Markov-switching models appear to do somewhat better in reproducing the depth of recessions. Meanwhile, the three-regime model and our 'bounce-back' model are clearly better at reproducing the rapid growth that tends to follow a business cycle trough. The fourth and fifth columns of Table V show that average growth in US real GDP in the year following a business cycle trough is nearly 5%, compared to an annualized 3.3% average growth rate for the full sample. The linear and Hamilton models miss most of this faster growth. By contrast, the three-regime and 'bounce-back' models generate rapid growth in the year following a business cycle trough. This is consistent with Galvão (2000), who finds that models incorporating a high growth recovery phase following recessions are better able to replicate all business cycle stylized facts than linear and two-regime nonlinear models.

[9] Hess and Iwata (1997a) define business cycles differently. They label any switch between positive and negative growth, no matter how short lived, to be a business cycle turning point. For US real GDP, their methodology identifies twice as many turning points as reported by the NBER.

Table V. Reproducing business cycles

Model	Business cycle features				
	Average length of recession	Average depth of recession	Average post-recession growth	Average growth	Correlation between depth and recovery
AR(1)	3.26	−1.73	3.98	3.31	0.02
	(0.61)	(0.43)	(0.70)	(0.39)	(0.36)
AR(2)	3.47	−1.81	3.82	3.29	0.03
	(0.70)	(0.49)	(0.72)	(0.43)	(0.38)
MA(1)	3.00	−1.52	4.07	3.33	0.01
	(0.54)	(0.36)	(0.66)	(0.34)	(0.37)
Hamilton	3.86	−2.11	3.83	3.30	0.04
	(0.91)	(0.56)	(0.73)	(0.42)	(0.40)
Three-regime	3.75	−2.01	5.19	3.27	−0.15
	(0.87)	(0.56)	(0.89)	(0.36)	(0.39)
'Bounce-back'	3.11	−2.26	5.24	3.26	−0.37
	(0.55)	(0.57)	(0.90)	(0.21)	(0.39)
Observed data	3.33	−2.13	4.96	3.33	−0.73

Note: Mean values for 10,000 simulated series are reported. Standard deviations based on the 10,000 simulations are reported in parentheses. For average length of recessions, the reported values refer to number of quarters. For average depth of recession, the reported values refer to the percentage change in the series from peak to trough. For average post-recession growth and average growth, the reported numbers are annualized percentages. Post-recession growth is growth in the four quarters following a business cycle trough. Parameters for the various models are based on estimates using US real GDP from 1949:Q1 to 2003:Q1.

Is there any advantage of our 'bounce-back' model over the three-regime Markov-switching specification? The final column of Table V shows that US real GDP displays a substantial negative correlation between growth rates in a particular recession and its subsequent recovery. Only the three-regime and 'bounce-back' models are successful at generating any negative correlation in the simulation experiment. However, the 'bounce-back' model performs considerably better on this dimension than the three-regime model, generating nearly half of the large observed correlation.[10]

6. ARE US RECESSIONS PERMANENT?

Given such strong support for our model, there is a question of what the model implies about the permanent effects of recessions on the level of output. Hamilton (1989) provides a useful measure of the long-run effects of recessions in the context of his model. He considers the expected difference in the long-run level of output given that the economy is currently in a 'contractionary' regime versus an 'expansionary' regime:

$$\lim_{j\to\infty}\{E[y_{t+j}|S_t = 1, \Omega_{t-1}] - E[y_{t+j}|S_t = 0, \Omega_{t-1}]\}$$

[10] It is curious that the three-regime model generates any such correlation since the model specification contains no obvious link between recession severity and subsequent growth. This correlation appears to be a result of the business cycle dating algorithm, which identifies two types of recessions in the three-regime model. The first is generated by the regime switching, and tends to be relatively deep and is followed by the rapid growth phase. The second is generated by the symmetric error term, and tends to be relatively shallow and is not followed by the rapid growth phase. When combined, these two types of recessions yield a negative correlation between recession severity and subsequent growth. However, for each particular type of recession, there should be no correlation.

where $\Omega_{t-1} = \{S_{t-1} = 0, S_{t-2} = 0, \ldots; y_{t-1}, y_{t-2}, \ldots\}$. For the model in (1), this limit, which we denote as Λ hereafter, has a closed-form expression:

$$\Lambda = (\mu_1 + m\lambda)/(2 - q - p)$$

Returning to Table II, the estimated value for Λ is -0.412, or just under a 0.5% permanent drop in the level of GDP. By contrast, Hamilton's estimates imply a 4.5% permanent drop.

It is interesting to compare our finding to what has been reported in classic studies, including Nelson and Plosser (1982), Campbell and Mankiw (1987) and Stock and Watson (1988), on the long-run effects of shocks to the level of output using linear ARIMA models. For example, consider the following linear autoregressive model of the first differences of output:

$$\phi(L)(\Delta y_t - \mu) = \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d.}(0, \sigma^2) \tag{3}$$

where the lag operator $\phi(L)$ is kth order with roots outside the unit circle. For this model, the long-run response of output to a unit shock, $\varepsilon_t = 1$, is the following closed-form expression:

$$\lim_{j\to\infty} \{E[y_{t+j}|\varepsilon_t = 1] - E[y_{t+j}|\varepsilon_t = 0]\} = \frac{1}{\phi(1)}$$

The literature reports estimates of this expression that are uniformly large. For example, Stock and Watson (1988) survey the literature and report a range of estimates between 1.6 for lag order $k = 1$ and 0.9 for lag order $k = 24$. These estimates are consistent with what we find for the regular linear ε_t shocks since our preferred lag order $k = 0$ corresponds to an implied long-run multiplier of 1. However, linear models restrict the dynamics to be the same for all shocks. Thus, linear models imply that recessions have large permanent effects on output. By contrast, we find that not all shocks have the same dynamic effects on output. In particular, recessionary shocks appear to have nonlinear dynamics that imply much smaller long-run effects on output.

7. A STRUCTURAL BREAK IN BUSINESS CYCLE VOLATILITY AND THE ROLE OF DEPTH

As Figure 2 demonstrates, the recession dates established by the NBER are closely matched by the contractionary regimes identified by the model. The exceptions to this are the 1970 and 2001 recessions, for which there is little evidence of a contractionary regime, and the 1990–1991 recession, for which the end of the contractionary regime is after the trough date established by the NBER. In this section, we discuss some possible reasons for these exceptions, and present two modifications to the model that improve its ability to capture the NBER-dated recessions.

One explanation for the inability of the model in (1) to match all NBER recessions is that it ignores a reduction in the volatility of the US business cycle since the mid-1980s. Kim and Nelson (1999b) and McConnell and Perez-Quiros (2000) have shown that Hamilton's (1989) Markov-switching model is better able to detect NBER recessions once this structural change is accounted for. Thus, it is possible that the failure of the model to capture certain recessions is a consequence of ignoring this structural change.

To analyse the role of the apparent reduction in business cycle volatility in explaining our results, we consider a model that allows for a structural break in model parameters related to business cycle volatility. Specifically, as in Kim and Nelson (1999b), we allow for a one-time change in the drift parameters μ_0 and μ_1 and the standard deviation parameter σ. The breakpoint is set at 1984:Q1, the date established by both Kim and Nelson (1999b) and McConnell and Perez-Quiros (2000). All other model parameters are assumed to be constant over the entire sample period.

Table VI reports the parameter estimates for a 'bounce-back' model with a structural break. For comparison purposes, we set the autoregressive lag length $k = 0$ and the length of the 'bounce-back' $m = 6$. The estimates suggest large changes in μ_0, μ_1 and σ corresponding to a reduction in volatility. In particular, the standard deviation of ε_t shocks falls by half from 0.9% to 0.4%, and there is a reduction in the gap between the drift parameters from 1.9% to 1.2%. Also, the average growth rate in recessions increases from −0.8% to −0.3%. Figure 4 shows that allowing for this structural break significantly improves the ability of the model to capture both the 1970 and 2001 recessions, with the smoothed probability of recession now rising above 50% for both recessions. However, the model has a difficult time identifying the end of the two recessions that occur after the structural break.

In addition to being affected by a reduction in volatility, the two most recent recessions have been followed by relatively weak recoveries. It is likely that our model, which predicts rapid

Table VI. Maximum likelihood estimates for the 'bounce-back' model with a structural break

Parameter	Estimate	Standard error
$\mu_{0,1949:Q1-1983:Q4}$	1.084	0.114
$\mu_{0,19841:Q1-2001:Q1}$	0.873	0.069
$\mu_{1,1949:Q1-1983:Q4}$	−1.826	0.245
$\mu_{1,1984:Q1-2003:Q1}$	−1.123	0.160
λ	0.154	0.035
q	0.936	0.024
p	0.862	0.055
$\sigma_{1949:Q1-1983:Q4}$	0.930	0.070
$\sigma_{1984:Q1-2003:Q1}$	0.428	0.037
Log-likelihood		−269.662

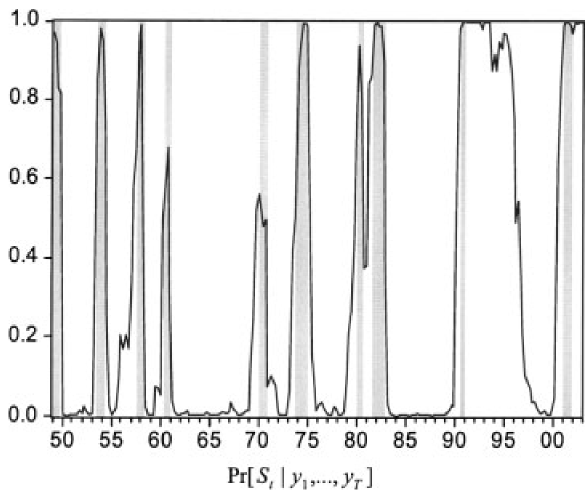

Figure 4. Smoothed inferences for S_t given a structural break (NBER recessions are shaded)

growth following the end of a recession, is overstating the length of the estimated contractionary regimes as a way to account for this weak growth. Indeed, according to Figure 4, the 1990s contractionary regime did not end until the onset of fast growth in the late 1990s. One reason that our model might suggest such different timing than the NBER for the 1990s recession is that it implicitly scales the size of the 'bounce-back' effect by the length of the preceding recession, while it may actually be more closely linked to the severity or depth of the recession.[11] Length and depth of recessions are obviously related. However, the link between the two may have weakened since the structural break in GDP volatility reduced the depth of recessions.

To examine whether differences in the depth of specific recessions can explain our results, we consider the following modified version of our model:

$$\phi(L)\left(\Delta y_t - \mu_0 - \mu_1 S_t - \lambda \sum_{j=1}^{m} S_{t-j}\Delta y_{t-j}\right) = \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d.}\, N(0,\sigma^2) \tag{4}$$

where each lagged state S_{t-j} in the summation term interacts with the corresponding lagged change in output Δy_{t-j}.[12] This modification implicitly scales the size of the post-recession 'bounce-back' by the depth of the recession. That is, given two recessions of equal length but different cumulative declines in output, the deeper recession is predicted to have the larger 'bounce-back' effect.

Table VII reports the results for the model with a structural break and depth as the determinant of the size of the post-recession 'bounce-back' effect. Again for comparison purposes, we set the autoregressive lag length $k = 0$ and the length of the 'bounce-back' $m = 6$. The parameter estimates are qualitatively similar to those reported in Table II and VI.[13] However, Figure 5 demonstrates that the model with a structural break and depth is able to capture the two most recent recessions. This finding suggests that the decreased severity of the two most recent recessions helps explain their subsequent slow recoveries.

Table VII. Maximum likelihood estimates for the 'bounce-back' model with a structural break and depth

Parameter	Estimate	Standard error
$\mu_{0,1949:Q1-1983:Q4}$	1.191	0.141
$\mu_{0,19841:Q1-2001:Q1}$	0.927	0.063
$\mu_{1,1949:Q1-1983:Q4}$	−1.719	0.293
$\mu_{1,1984:Q1-2003:Q1}$	−0.963	0.167
λ	−0.303	0.105
q	0.926	0.025
p	0.759	0.076
$\sigma_{1949:Q1-1983:Q4}$	0.887	0.069
$\sigma_{1984:Q1-2003:Q1}$	0.436	0.038
Log-likelihood		−269.422

[11] To be precise, our model captures length up to the upper bound equal to m, the length of the post-recession 'bounce-back' period. However, since the longest post-war recession in the US is six quarters and our model selection procedure picks $m = 6$, the summation term can be said to capture length.

[12] Note that this modification, while apparently simple, makes calculation of the long-run effects of recessions much more difficult due to the interaction of two random variables. A closed-form solution for the long-run effect is not available.

[13] Note that the 'bounce-back' coefficient is not directly comparable. However, since the estimates in Table II suggest that average GDP growth is close to −1% in a recession, the 'bounce-back' parameter for the depth model should be of a similar magnitude, but opposite sign.

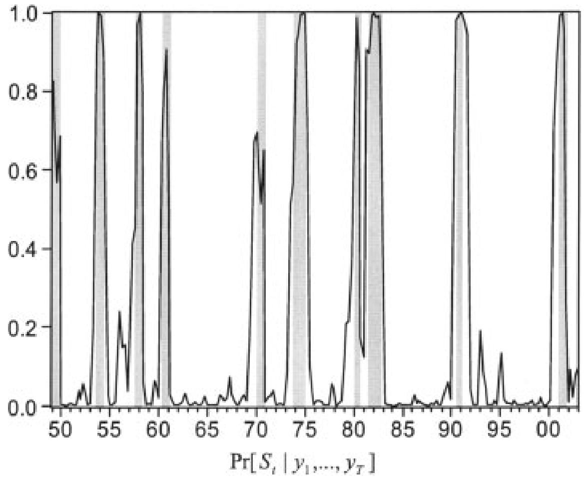

Figure 5. Smoothed inferences for S_t given a structural break and depth (NBER recessions are shaded)

8. INTERNATIONAL EVIDENCE

Are the regime-switching dynamics that we find in the US data a common feature of business cycle fluctuations in other countries? If so, is a 'bounce-back' effect an important component of these dynamics? To help answer these questions, we apply the model in (1) to quarterly real GDP data obtained from the OECD for Australia, Canada, France, Germany, Italy and the United Kingdom. Due to data limitations and the fact that several of these countries display significant slowdowns in trend productivity growth some time during the early 1970s, we consider the international evidence for a sample period beginning in 1973:Q1. We also re-estimate the model for the United States over this shorter sample period to provide a benchmark for comparison. For all countries, we set the autoregressive lag length $k = 2$ and the length of the 'bounce-back' $m = 6$. This provides a common model specification for cross-country comparisons.

We begin by testing the statistical significance of the Markov-switching form of nonlinearity. To do so, we compare the 'bounce-back' model with a null hypothesis of a linear AR(2) model by performing the grid search likelihood ratio test presented in Section 5. The evidence for Markov switching provided by this test is mixed. For French and Italian real GDP, the evidence is weak. The respective likelihood ratio statistics are 6.3 and 4.1, with corresponding p-values of 0.36 and 0.66 according to the distribution presented in Figure 3. For Canadian real GDP, the results are somewhat stronger. The test statistic is 9.1, with a p-value of 0.15. For the remaining countries, the evidence is much stronger. For both Australian and German real GDP, the likelihood ratio statistics are just over 12 and the p-values are less than 0.05. For UK real GDP, the test statistic is 19.2, with a p-value of less than 0.01. However, in the case of German real GDP, the nonlinearity appears to be related to higher frequency movements, rather than business cycle phases. In particular, the estimated transition probability for the low growth regime is just 0.1, corresponding to an expected duration of this regime of just 1.1 quarters. Thus, in examining the permanent effects of recessions, we exclude France, Germany and Italy from analysis since there is no evidence of Markov switching at business cycle frequencies for these countries.

Having tested for nonlinearity, the remaining countries that we consider are Australia, Canada, the United Kingdom and the United States. Figure 6 displays the smoothed probability of being in a 'contractionary' regime and the real GDP series for these countries. In every case, the 'contractionary' regime corresponds to declines in the level of output. The results for the United States closely match those for the same part of the longer sample period displayed in Figure 2.

Table VIII reports estimates of the average length of a recession $1/(1-p)$, the 'bounce-back' coefficient λ, and our measure of the permanent effect of recessions Λ. The estimates of $1/(1-p)$ suggest that recessions have lasted longer in Canada and the United Kingdom than in Australia and the United States. There is wide variation in the estimates of the 'bounce-back' coefficient λ. Australia and the United States have positive and relatively large 'bounce-back' effects, while the estimates for Canada and the United Kingdom are close to zero.[14] The results for Λ, which summarizes the permanent effects of recessions, reflect the length of recessions and the size of the 'bounce-back' effect. For Australia and the United States, Λ is estimated to be fairly small, on the order of 1% to 1.5%. Meanwhile, for Canada and the United Kingdom, the estimated long-run effect is much larger, 4.5% in Canada and 5.5% in the United Kingdom.[15]

One caveat for these results is that, given $m = 6$, our model cannot capture a post-recession 'bounce-back' that occurs later in expansions. However, even if a delayed recovery is driving the results for Canada and the United Kingdom, it still suggests that the welfare costs of recessions are much higher than in Australia and the United States. Indeed, while many explanations for the differing dynamics are possible, the longer duration and persistence of recessions in these countries is suggestive of theories of hysteresis used to explain their higher levels of unemployment (see, for example, Blanchard and Summers, 1986). That is, the 'bounce-back' effect in Australia and the United States could reflect greater flexibility in labour markets.

Table VIII. International comparison

Country	Estimate of $1/(1-p)$	Estimate of λ	Estimate of Λ
Australia	2.79	0.15	−1.57
	(0.89)	(0.05)	(0.71)
Canada	3.86	0.05	−4.58
	(2.01)	(0.07)	(2.54)
United Kingdom	4.51	0.00	−5.34
	(2.33)	(0.00)	(2.35)
United States	3.13	0.24	−1.27
	(1.61)	(0.09)	(1.68)

Note: Standard errors are in parentheses.

[14] Note that the estimate for the United Kingdom is equal to zero to the second decimal place. However, we normalize the 'bounce-back' coefficient to be non-negative for the United Kingdom. The normalization is necessary because a model with a negative 'bounce-back' coefficient and short-lived contractionary regimes is observationally equivalent to a model with no 'bounce-back' effect and more persistent recessionary regimes. Given the normalization of the regimes, there is no similar observational equivalence for models with positive 'bounce-back' coefficients.

[15] Mills and Wang (2002) also investigate the extent to which recessions are transitory in real GDP data for Canada and the United Kingdom. Consistent with our results, they find that regime shifts in the trend component are sufficient to explain Canadian recessions. However, they find that recessions in the United Kingdom are largely contained in the transitory component of real GDP. The difference in their findings could be due to their structural modelling strategy, in which they make explicit correlation assumptions between trend and cycle components. Also, they consider a somewhat different sample period.

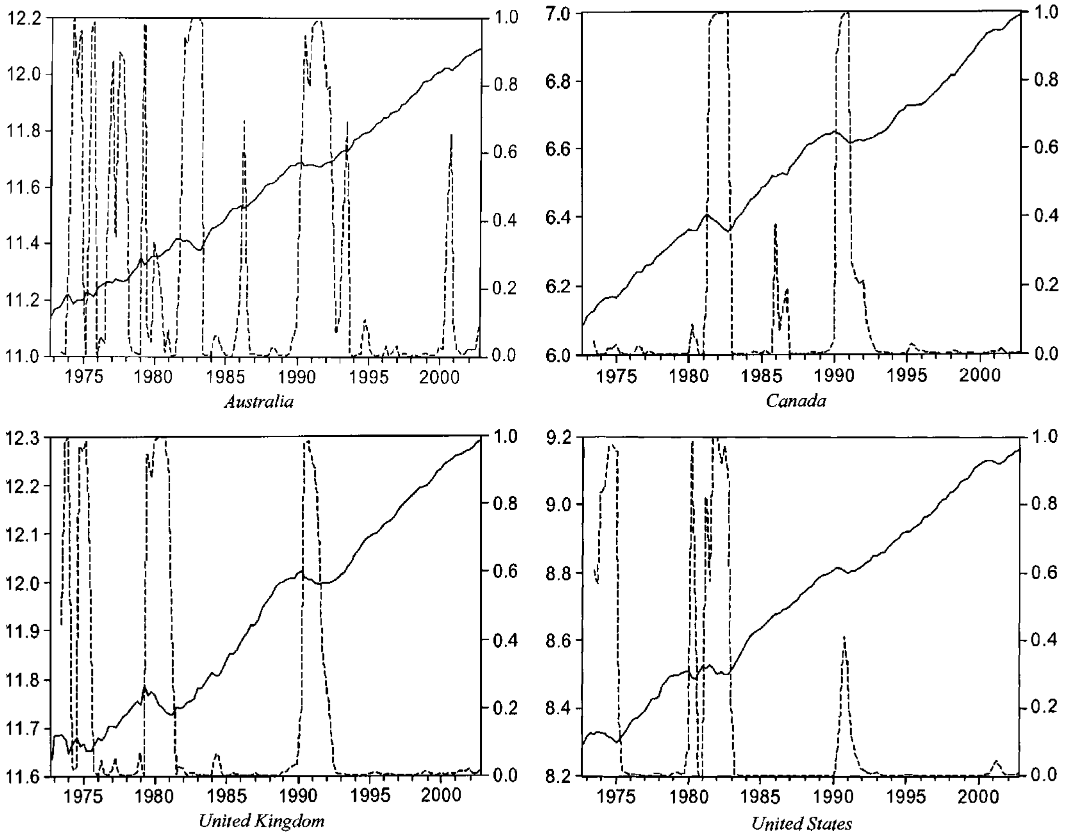

Figure 6. Smoothed probability of S_t (right axis) and log real GDP (left axis)

9. CONCLUSIONS

In summary, we find that the permanent effects of recessions in the United States are substantially less than suggested by Hamilton (1989) and most linear models (e.g. Nelson and Plosser, 1982; Campbell and Mankiw, 1987; Stock and Watson, 1988). Instead, we find evidence of a large 'bounce-back' effect during the recovery phase of the business cycle. Finally, when our model is applied to international data, the 'bounce-back' effect is smaller, corresponding to larger permanent effects of recessions for other countries.

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