Nonlocal effective medium model for multilayered metal-dielectric metamaterials

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(Received 3 May 2011; revised manuscript received 24 July 2011; published 22 September 2011)

We study layered metal-dielectric structures, which can be considered as a simple example of nanostructured metamaterials. We analyze the dispersion properties of such structures and demonstrate that they show strong optical nonlocality due to excitation of surface plasmon polaritons. We derive a model of a nonlocal effective medium for describing the effects of strong spatial dispersion in the multilayered metal-dielectric metamaterials. We obtain analytical expressions for the components of the effective permittivity tensor which depend on the wave vector and reveal that spatial dispersion effects exist in both directions across and along the layers.

DOI: 10.1103/PhysRevB.84.115438 PACS number(s): 78.67.Pt, 78.20.Bh, 78.67.−n

I. INTRODUCTION

Multilayered metal-dielectric nanostructured metamaterials demonstrate unusual electromagnetic properties that may find useful applications. In particular, such structures amplify evanescent waves, and they can carry information about the properties of objects much smaller than the wavelength.[1–3] This property can be utilized in subwavelength imaging,[4,5] nanolithography,[6] as well as in a design of cloaking shells.[7,8] Using the model of a local effective medium, Rytov in his early paper[9] considered four cases for the propagation of electromagnetic waves in such layered systems: propagation across the layers for both transverse electric (TE) and transverse magnetic (TM) waves and propagation along the layers for both polarizations. Also, it was found that by changing the relative thicknesses of the layers and permittivity magnitudes, one may achieve very large values of effective permittivities.

The approach to introduce a local effective medium for describing one-dimensional (periodic or random) layered structures with layers much thinner than the wavelength has been developed recently in Ref. 10. However, it was shown in Refs. 11 and 12 that a majority of multilayered structures demonstrate strong spatial dispersion effects. Spatial dispersion means the existence of nonlocal electromagnetic response, i.e., the dependence of the components of the effective permittivity tensor on the wave vector.[13,14] The approach developed in Refs. 9 and 10 does not take into account such effects. To describe the effects of spatial dispersion, we need to generalize a standard homogenization theory introducing *nonlocal effects*. Such effects were observed in a number of different structures, and for some of them even an analytical description was suggested.

For example, in Ref. 15 it was shown that the so-called wire media possess strong spatial dispersion at arbitrary frequencies, including very low frequencies as compared with their period. Strong spatial dispersion of artificial metamaterials formed by split-ring resonators was demonstrated in Ref. 16 at frequencies near the resonance eigenfrequency. The numerical finite difference time domain (FDTD) method[17] was used to study parameters of nonlocal materials. It was also discovered in Ref. 18 that multilayered metal-dielectric metamaterials demonstrate strong optical nonlocality, which manifests itself in such phenomena as splitting of a TM-polarized wave into two waves, the existence of both positive and negative refraction, etc. However, an accurate analytical description of such materials remains an open problem.

In this paper, we apply the homogenization procedure earlier suggested by Silveirinha[19] to the case of multilayered metal-dielectric metamaterials. Using this theory, we derive analytical expressions for nonlocal material parameters of the effective media, which should be capable of describing analytically the effects revealed in Ref. 18. We show that spatial dispersion effects are observed for both possible directions of propagation, i.e., across and along the layers. However, the components of the permittivity tensor change differently in these two directions. We also find that when the nondiagonal components do not vanish, the optic axes may experience rotation.

II. NONLOCAL HOMOGENIZATION PROCEDURE FORMULATION

We consider the multilayered metal-dielectric metamaterial formed by the layers with permittivities ε_1, ε_2 and thicknesses d_1, d_2. Our goal is to derive analytical expressions for the permittivity of these nanostructured metamaterials, paying attention to the spatial dispersion effects. To do so, we apply the homogenization procedure suggested in Refs. 17 and 19. The procedure works in the frequency domain [the time dependence is of the form $\exp(-i\omega t)$]. It assumes that the structure is excited by an external current with a distribution

$$\mathbf{J}(x,y,z) = \mathbf{J}_0 e^{i(k_x x + k_y y + k_z z)}, \tag{1}$$

for which we have to solve Maxwell's equations and find microscopic fields $\mathbf{E},\mathbf{H}$ in each layer n (where $n = 1,2$ for our case of two layers):

$$\mathbf{E}_{\mathbf{J}}^{(n)} = i\frac{\mu_0\omega\mathbf{J} - \frac{\mathbf{k}(\mathbf{k}\cdot\mathbf{J})}{\omega\varepsilon_0\varepsilon_n}}{k^2 - \varepsilon_n\frac{\omega^2}{c^2}}, \tag{2}$$

$$\mathbf{H}_{\mathbf{J}}^{(n)} = i\frac{\mathbf{k}\cdot\mathbf{J}}{k^2 - \varepsilon_n\frac{\omega^2}{c^2}}. \tag{3}$$

Averaging of these fields over a unit cell with volume V_{cell} provides us with macroscopic fields

$$\begin{aligned}\mathbf{E}_{\mathbf{av}} &= \frac{1}{V_{\text{cell}}}\int_{\text{cell}} \mathbf{E}(\mathbf{r})e^{-i\mathbf{k}\cdot\mathbf{r}}d^3\mathbf{r},\\ \mathbf{H}_{\mathbf{av}} &= \frac{1}{V_{\text{cell}}}\int_{\text{cell}} \mathbf{H}(\mathbf{r})e^{-i\mathbf{k}\cdot\mathbf{r}}d^3\mathbf{r}.\end{aligned} \tag{4}$$

Now, it is possible to derive the effective permittivity $\overline{\overline{\varepsilon}}_{\text{eff}}$ using the following relation:

$$\mathbf{D}_{\mathbf{g},\mathbf{av}} = \varepsilon_0\mathbf{E}_{\mathbf{av}} + \mathbf{P}_{\mathbf{g}} = \overline{\overline{\varepsilon}}_{\text{eff}}(\omega,\mathbf{k})\mathbf{E}_{\mathbf{av}}, \tag{5}$$

where $\mathbf{P}_{\mathbf{g}}$ is the electric polarization induced by the external current [Eq. (1)] and $\mathbf{D}_{\mathbf{g},\mathbf{av}}$ is the electric displacement vector in the medium.

Since $\overline{\overline{\varepsilon}}_{\text{eff}}$ is a dyadic, its independent components must be calculated separately. In general, the current density from Eq. (1) excites in the metamaterial a plane wave of a given polarization. We are able to specify the direction of the external current vector $\mathbf{J}_0$ so that the excited wave will be either TE or TM polarized. Next, assuming that the wave vector lies in the xy plane, we consider sequentially the currents directed along the z, y, and x axes, which correspond to TE and TM polarizations (see Fig. 1). For each of these cases we calculate different components of $\overline{\overline{\varepsilon}}_{\text{eff}}$. In what follows, the TM polarization with the current directed along the y axis will be labeled TM_1, and the current along the x axis will be TM_2.

III. EXCITATION OF TE-POLARIZED WAVES

If we choose the current direction along the z axis, then the current density is given by $\mathbf{J} = \mathbf{J}_z e^{i(k_x x + k_y y)}$. This corresponds to the TE-polarized waves. As in this case $\mathbf{J} \perp \mathbf{k}$, and the expressions for the fields from Eqs. (2) and (3) take the

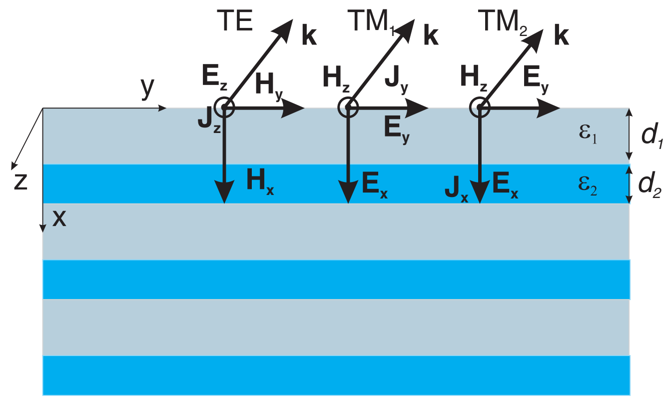

FIG. 1. (Color online) Diagram of the metamaterial under consideration and field vectors arrangements for the three differently directed currents, $\mathbf{J}_z$,$\mathbf{J}_y$, and $\mathbf{J}_x$.

following forms [the factor $\exp(ik_y y)$ is dropped]:

$$E_{\mathbf{J}_z,z}^{(1,2)}(x) = \frac{i\mu_0\omega J_z}{k^2 - \varepsilon_{1,2}\frac{\omega^2}{c^2}}e^{ik_x x}, \tag{6}$$

$$H_{\mathbf{J}_z,x}^{(1,2)}(x) = \frac{ik_y J_z}{k^2 - \varepsilon_{1,2}\frac{\omega^2}{c^2}}e^{ik_x x}, \tag{7}$$

$$H_{\mathbf{J}_z,y}^{(1,2)}(x) = \frac{ik_x J_z}{k^2 - \varepsilon_{1,2}\frac{\omega^2}{c^2}}e^{ik_x x}. \tag{8}$$

At each point inside the metamaterial the total field can be considered as a sum of the plane waves that are the solutions of the uniform Maxwell equations (i.e., the eigenmodes of the layers) and the fields created by the given external current (i.e., the source-driven fields). Let A_1^{TE}, A_2^{TE}, B_1^{TE}, and B_2^{TE} be the amplitudes of forward-propagating and backward-propagating eigenmodes in the first and the second layers, respectively, and k_{x1} and k_{x2} be the wave numbers in the same layers, expressed as $k_{x1} = \sqrt{\varepsilon_1\frac{\omega^2}{c^2} - k_y^2}$, $k_{x2} = \sqrt{\varepsilon_2\frac{\omega^2}{c^2} - k_y^2}$.

The total electric and magnetic fields must satisfy the known Maxwell's boundary conditions at the layer boundaries. Let us assume the first boundary is at $x = d_1$ and the second one is at $x = D \equiv d_1 + d_2$ (see Fig. 2). Using the continuity of the tangential components of the electric and magnetic fields at the interfaces, we obtain

$$\begin{cases} A_1^{\text{TE}}e^{ik_{x1}d_1} + B_1^{\text{TE}}e^{-ik_{x1}d_1} + E_{\mathbf{J}_z,z}^{(1)}(d_1) = A_2^{\text{TE}}e^{ik_{x2}d_1} + B_2^{\text{TE}}e^{-ik_{x2}d_1} + E_{\mathbf{J}_z,z}^{(2)}(d_1)\\ A_1^{\text{TE}}e^{ik_x D} + B_1^{\text{TE}}e^{ik_x D} + E_{\mathbf{J}_z,z}^{(1)}(D) = A_2^{\text{TE}}e^{ik_{x2}D} + B_2^{\text{TE}}e^{-ik_{x2}D} + E_{\mathbf{J}_z,z}^{(2)}(D)\\ A_1^{\text{TE}}k_{x1}e^{ik_{x1}d_1} - B_1^{\text{TE}}k_{x1}e^{-ik_{x1}d_1} + k_x E_{\mathbf{J}_z,z}^{(1)}(d_1) = A_2^{\text{TE}}k_{x2}e^{ik_{x2}d_1} - B_2^{\text{TE}}k_{x2}e^{-ik_{x2}d_1} + k_x E_{\mathbf{J}_z,z}^{(2)}(d_1)\\ A_1^{\text{TE}}k_{x1}e^{ik_x D} - B_1^{\text{TE}}k_{x1}e^{ik_x D} + k_x E_{\mathbf{J}_z,z}^{(1)}(D) = A_2^{\text{TE}}k_{x2}e^{ik_{x2}D} - B_2^{\text{TE}}k_{x2}e^{-ik_{x2}D} + k_x E_{\mathbf{J}_z,z}^{(2)}(D) \end{cases}. \tag{9}$$

The first two equations from the system in Eq. (9) represent the continuity of the tangential electric field at the interface between the first and second layer in a unit cell and at the boundary between the two neighboring unit cells, respectively. The last two equations represent the continuity of the tangential magnetic field at the same interfaces, where the magnetic field is expressed using Maxwell's equations as $H_y = \frac{k_x E_z}{\mu_0\omega}$.

We are interested in the electromagnetic fields that (on average) have the same phase advance inside the structure as defined by the external current. Thus, if the phase of the external source advances along Ox as $e^{ik_x x}$, then, in

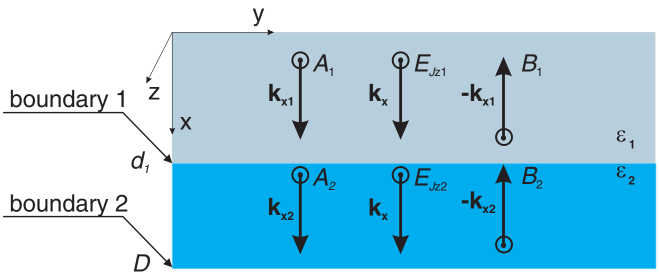

FIG. 2. (Color online) Boundary conditions definition for the electromagnetic field in the multilayered metamaterial.

accordance with the Floquet theorem for periodic structures, the electric and magnetic fields at the plane $x = D$ inside the structure must differ from the same fields at the plane $x = 0$ only by the phase factor $e^{ik_x D}$. When combined with the boundary conditions in Eq. (9), this Floquet periodicity condition results in a system of linear equations for the unknown wave amplitudes that is formulated and solved in Appendix A.

We notice that for the computed amplitudes the following symmetry transformation is valid: Replacing $k_{x1} \to -k_{x1}$ implies $A_1^{\mathrm{TE}} \to B_1^{\mathrm{TE}}$; similarly, $k_{x2} \to -k_{x2}$ implies $A_2^{\mathrm{TE}} \to B_2^{\mathrm{TE}}$. This is a simple x inversion, while replacing $\varepsilon_1 \to \varepsilon_2$ (which implies $k_{x1} \to k_{x2}$), and $d_1 \to d_2$, $k_x \to -k_x$ gives us

$$\begin{aligned} A_1^{\mathrm{TE}} &\to B_2^{\mathrm{TE}} e^{ik_{x2}D}, \\ B_1^{\mathrm{TE}} &\to A_2^{\mathrm{TE}} e^{-ik_{x2}D}, \\ A_2^{\mathrm{TE}} &\to B_1^{\mathrm{TE}} e^{-ik_{x1}D}, \\ B_2^{\mathrm{TE}} &\to A_1^{\mathrm{TE}} e^{ik_{x1}D}. \end{aligned} \tag{10}$$

Such a transformation is equivalent to rotating the system 180 degrees about the z-axis, as one can see from Fig. 3, because under this rotation $A_1 \to B_2 e^{ik_{x2}D}$.

It should be mentioned that the current $\mathbf{J} = \mathbf{J}_z e^{ik_x d_1}$ occurring in $\Delta E^{(1,2)}_{\mathbf{J}_z,z}$ must be kept invariant under the mentioned transformation. For that reason there is no need to replace $k_x \to -k_x$ and $d_1 \to d_2$ in the expressions for the current.

It is known that the macroscopically averaged fields $\langle \mathbf{D} \rangle$ and $\langle \mathbf{E} \rangle$ in the case of an anisotropic medium are connected by the permittivity tensor in the following manner:

$$\langle \mathbf{D} \rangle = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \langle \mathbf{E} \rangle. \tag{11}$$

Since we consider TE polarization in this chapter, $\langle \mathbf{D} \rangle = [0,0,\langle D_z \rangle]^T$ and $\langle \mathbf{E} \rangle = [0,0,\langle E_z \rangle]^T$. Subsequently, we are able to find out ε_{zz}:

$$\varepsilon_{zz} = \frac{\langle D_z \rangle}{\langle E_z \rangle} = \frac{\varepsilon_1 \langle E_z^{(1)} \rangle + \varepsilon_2 \langle E_z^{(2)} \rangle + \left[\varepsilon_1 E^{(1)}_{\mathbf{J}_z,z}(d_1) d_1 + \varepsilon_2 E^{(2)}_{\mathbf{J}_z,z}(d_1) d_2 \right] e^{-ik_x d_1}}{\langle E_z^{(1)} \rangle + \langle E_z^{(2)} \rangle + \left[E^{(1)}_{\mathbf{J}_z,z}(d_1) d_1 + E^{(2)}_{\mathbf{J}_z,z}(d_1) d_2 \right] e^{-ik_x d_1}}, \tag{12}$$

where expressions for averaged electric fields $\langle E_z^{(1)} \rangle, \langle E_z^{(2)} \rangle$ are derived in Appendix B and the averaged electric displacement is calculated from the following integral:

$$\langle D_z \rangle = \frac{1}{D} \int_0^D \varepsilon E_z e^{-ik_x x} dx. \tag{13}$$

IV. EXCITATION OF TM-POLARIZED WAVES

Now, let the currents be directed along y and x axes (with the corresponding current densities $\mathbf{J}_y$ and $\mathbf{J}_x$). This case corresponds to the excitation of the TM-polarized waves in our structure. Similarly to the case of the TE waves, after imposing the boundary and the periodicity conditions we obtain an inhomogeneous system of linear equations that is solved in Appendix C. This allows us to obtain analytical expressions for amplitudes of excited waves $A_1^{\mathrm{TM1,2}}$, $A_2^{\mathrm{TM1,2}}$, $B_1^{\mathrm{TM1,2}}$, and $B_2^{\mathrm{TM1,2}}$ for the two cases under consideration.

Expressions for the averaged fields $\langle E_x \rangle^{\mathrm{TM1,2}}, \langle E_y \rangle^{\mathrm{TM1,2}}$ and electric displacement $\langle D_x \rangle^{\mathrm{TM1,2}}, \langle D_y \rangle^{\mathrm{TM1,2}}$ in the layers are given in Appendix D.

Substitution of $\langle \mathbf{D} \rangle^{\mathrm{TM1}} = [\langle D_x \rangle^{\mathrm{TM1}}, \langle D_y \rangle^{\mathrm{TM1}}, 0]^T$, $\langle \mathbf{E} \rangle^{\mathrm{TM1}} = [\langle E_x \rangle^{\mathrm{TM1}}, \langle E_y \rangle^{\mathrm{TM1}}, 0]^T$ and $\langle \mathbf{D} \rangle^{\mathrm{TM2}} = [\langle D_x \rangle^{\mathrm{TM2}}, \langle D_y \rangle^{\mathrm{TM2}}, 0]^T$, $\langle \mathbf{E} \rangle^{\mathrm{TM2}} = [\langle E_x \rangle^{\mathrm{TM2}}, \langle E_y \rangle^{\mathrm{TM2}}, 0]^T$ into Eq. (11) separates it into two subsystems formed by two equations with four unknown quantities ε_{xx}, ε_{xy}, ε_{yx}, ε_{yy},

$$\begin{aligned} \begin{bmatrix} \langle D_x \rangle^{\mathrm{TM1}} \\ \langle D_y \rangle^{\mathrm{TM1}} \end{bmatrix} &= \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{bmatrix} \begin{bmatrix} \langle E_x \rangle^{\mathrm{TM1}} \\ \langle E_y \rangle^{\mathrm{TM1}} \end{bmatrix}, \\ \begin{bmatrix} \langle D_x \rangle^{\mathrm{TM2}} \\ \langle D_y \rangle^{\mathrm{TM2}} \end{bmatrix} &= \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{bmatrix} \begin{bmatrix} \langle E_x \rangle^{\mathrm{TM2}} \\ \langle E_y \rangle^{\mathrm{TM2}} \end{bmatrix}. \end{aligned} \tag{14}$$

This system of equations can be reduced to two independent systems of equations with two unknowns,

$$\begin{bmatrix} \langle E_x \rangle^{\mathrm{TM1}} & \langle E_y \rangle^{\mathrm{TM1}} \\ \langle E_x \rangle^{\mathrm{TM2}} & \langle E_y \rangle^{\mathrm{TM2}} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{xy} \end{bmatrix} = \begin{bmatrix} \langle D_x \rangle^{\mathrm{TM1}} \\ \langle D_x \rangle^{\mathrm{TM2}} \end{bmatrix}, \tag{15}$$

$$\begin{bmatrix} \langle E_x \rangle^{\mathrm{TM1}} & \langle E_y \rangle^{\mathrm{TM1}} \\ \langle E_x \rangle^{\mathrm{TM2}} & \langle E_y \rangle^{\mathrm{TM2}} \end{bmatrix} \begin{bmatrix} \varepsilon_{yx} \\ \varepsilon_{yy} \end{bmatrix} = \begin{bmatrix} \langle D_y \rangle^{\mathrm{TM1}} \\ \langle D_y \rangle^{\mathrm{TM2}} \end{bmatrix}. \tag{16}$$

Solving (15) and (16) together, we obtain components of the dielectric permittivity,

$$\begin{aligned} \varepsilon_{xx} &= \frac{\langle D_x \rangle^{\mathrm{TM1}} \langle E_y \rangle^{\mathrm{TM2}} - \langle D_x \rangle^{\mathrm{TM2}} \langle E_y \rangle^{\mathrm{TM1}}}{\Delta_\varepsilon}, \\ \varepsilon_{xy} &= \frac{\langle D_x \rangle^{\mathrm{TM2}} \langle E_x \rangle^{\mathrm{TM1}} - \langle D_x \rangle^{\mathrm{TM1}} \langle E_x \rangle^{\mathrm{TM2}}}{\Delta_\varepsilon}, \end{aligned}$$

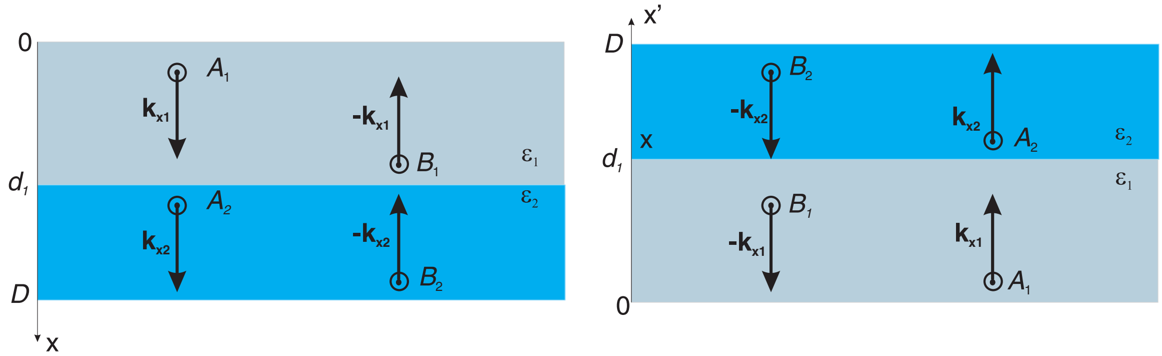

FIG. 3. (Color online) Schematic of the symmetry transformation between the amplitudes.

$$\varepsilon_{yx} = \frac{\langle D_y\rangle^{\mathrm{TM1}}\langle E_y\rangle^{\mathrm{TM2}} - \langle D_y\rangle^{\mathrm{TM2}}\langle E_y\rangle^{\mathrm{TM1}}}{\Delta_\varepsilon},$$
$$\varepsilon_{yy} = \frac{\langle D_y\rangle^{\mathrm{TM2}}\langle E_x\rangle^{\mathrm{TM1}} - \langle D_y\rangle^{\mathrm{TM1}}\langle E_x\rangle^{\mathrm{TM2}}}{\Delta_\varepsilon}, \qquad (17)$$

where $\Delta_\varepsilon = \langle E_x\rangle^{\mathrm{TM1}}\langle E_y\rangle^{\mathrm{TM2}} - \langle E_x\rangle^{\mathrm{TM2}}\langle E_y\rangle^{\mathrm{TM1}}$.

V. NUMERICAL RESULTS AND DISCUSSIONS

The analytical theory developed above allows for a complete analysis of the effects of temporal and spatial dispersion in periodic structures formed by dielectric layers. In this section we discuss the peculiarities in the behavior of the spatially dispersive dielectric permittivity $\overline{\overline{\varepsilon}}(\omega,\mathbf{k})$ in such media, with an emphasis on its wave vector dependence.

In the following numerical examples we consider a periodic structure shown in Fig. 1 that is formed by metallic and dielectric layers with thicknesses $d_1 = 0.6D$ and $d_2 = 0.4D$, respectively, where D is the period of the structure (in the actual calculations all dimensions and wavelengths are normalized to the period of the structure). The permittivity of the dielectric layer is $\varepsilon_1 = 4.6$. The metal is modeled by a plasmonic permittivity of the Drude form (the loss is neglected): $\varepsilon_2 = 1 - \omega_{\mathrm{p}}^2/\omega^2$, with the plasma frequency $\omega_{\mathrm{p}} = 2\pi c/\lambda_{\mathrm{p}}$, where $\lambda_{\mathrm{p}} = 4D$. The angular frequency is chosen equal to $\omega = 2\pi c/\lambda$ with $\lambda = 8D$.

In Figs. 4–7 the variations in the components of the dielectric permittivity tensor $\overline{\overline{\varepsilon}}(\omega,\mathbf{k})$ are shown as functions of the normalized wave vector components. As one may see, the effective dielectric permittivity of the considered layered metamaterial, in general, changes rather significantly with $\mathbf{k}$. For instance, from Fig. 4 it is seen that the absolute value of the permittivity component normal to the layers may be orders of magnitude greater at the middle of the first Brillouin zone (where $\mathbf{k} = 0$) when compared to the same at relatively large wave numbers along the y axis (when $\tilde{k}_y \approx 1$). Such a very pronounced spatial dispersion of the xx component of the dielectric permittivity is not predicted by the commonly used quasistatic models of layered dielectric media.

The variation in the tangential components of the permittivity tensor are less pronounced but can be also clearly seen in Figs. 5 and 6. Another feature that is noticeable from these figures is that the dependence of the yy and zz components of the permittivity tensor on the wave vector is different for these two components. At moderate $\tilde{k}_y$, ε_{yy} decreases with the increase of $\tilde{k}_x$, while ε_{zz} increases with $\tilde{k}_x$ under the same conditions. It is also seen that the slope of change of ε_{yy} with $\tilde{k}_x$ may switch sign depending on the value of $\tilde{k}_y$. In contrast, the dependencies of these permittivity components on $\tilde{k}_y$ are similar: Both ε_{yy} and ε_{zz} generally decrease with $\tilde{k}_y$ in the range of wave numbers of our interest.

Another very peculiar feature of the studied layered metamaterial is that the direction of the main axes of the permittivity tensor vary with the wave vector. This may lead to an interesting effect in which the optical axes of a metamaterial crystal may experience rotation depending on the conditions under which a sample of such medium is excited. Indeed, an external excitation may fix the rate of change of the fields in space (at least, along some selected directions), fixing in this way certain components of the wave vector. As can be seen from Fig. 7, there may exist combinations of $\tilde{k}_x$ and $\tilde{k}_y$ for which the off-diagonal components of the permittivity tensor are different from zeros. This corresponds to a change in the directions of the main axes of the permittivity tensor, which,

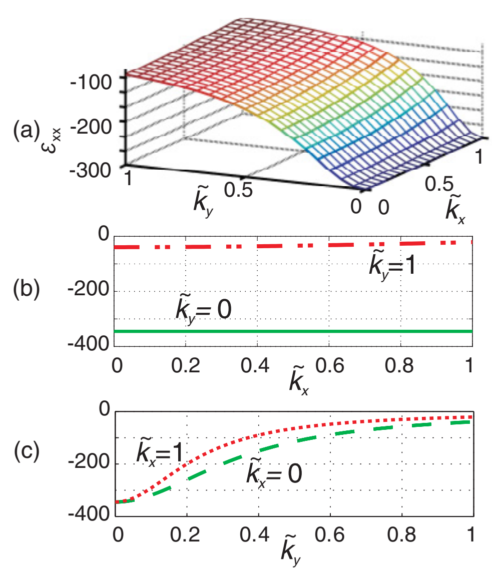

FIG. 4. (Color online) Permittivity across layers, ε_{xx}, as a function of the normalized wave vector components, $\tilde{k}_x = k_x D/\pi$, $\tilde{k}_y = k_y D/\pi$: (a) surface plot, (b) cross sections at $\tilde{k}_y = 0$ (solid curve) and $\tilde{k}_y = 1$ (dash-dotted curve), and (c) cross sections at $\tilde{k}_x = 0$ (dashed curve) and $\tilde{k}_x = 1$ (dotted curve).

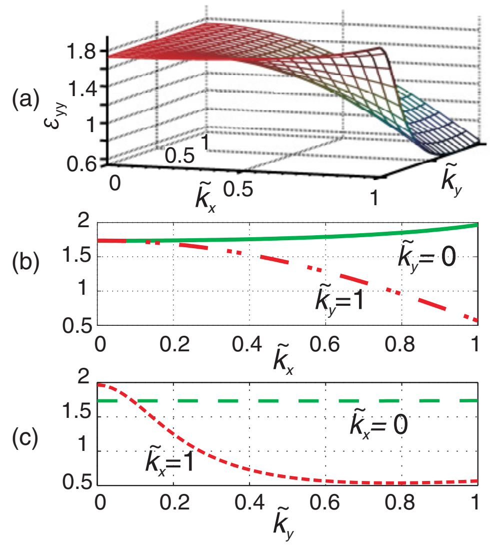

FIG. 5. (Color online) Permittivity along layers, ε_{yy}, as a function of the normalized wave vector components, $\tilde{k}_x = k_x D/\pi, \tilde{k}_y = k_y D/\pi$: (a) surface plot, (b) cross sections at $\tilde{k}_y = 0$ (solid curve) and $\tilde{k}_y = 1$ (dash-dotted curve), and (c) cross sections at $\tilde{k}_x = 0$ (dashed curve) and $\tilde{k}_x = 1$ (dotted curve).

for example, may be observable as a rotation of an optical axis of the crystal.

We would like to stress that the developed analytical homogenization model is applicable under the regimes of both strong and weak spatial dispersion (e.g., it is applicable to photonic crystals). This follows from the fact that the homogenization procedure that we use in this work may be understood as an *exact averaging* that in the end results in correct coefficients connecting the amplitudes of the lowest-order Bloch waves in a periodic structure [for example, the

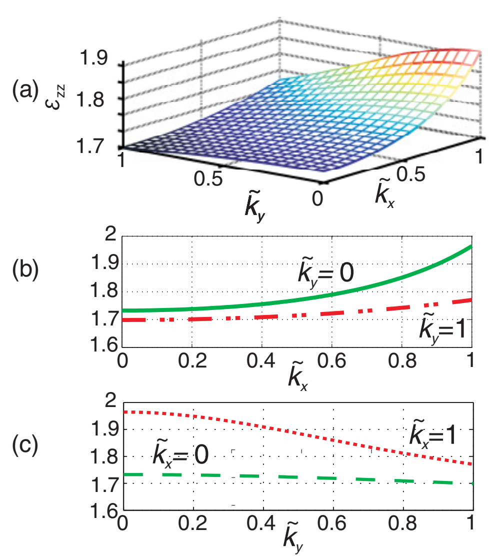

FIG. 6. (Color online) Permittivity along layers, ε_{zz}, as a function of the normalized wave vector components, $\tilde{k}_x = k_x D/\pi, \tilde{k}_y = k_y D/\pi$: (a) surface plot, (b) cross sections at $\tilde{k}_y = 0$ (solid curve) and $\tilde{k}_y = 1$ (dash-dotted curve), and (c) cross sections at $\tilde{k}_x = 0$ (dashed curve) and $\tilde{k}_x = 1$ (dotted curve).

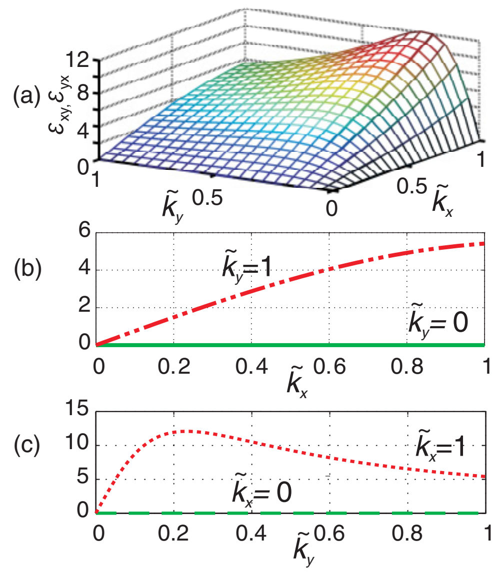

FIG. 7. (Color online) Nondiagonal components, $\varepsilon_{xy}, \varepsilon_{yx}$, of the permittivity tensor as functions of the normalized wave vector components, $\tilde{k}_x = k_x D/\pi, \tilde{k}_y = k_y D/\pi$: (a) surface plot, (b) cross sections at $\tilde{k}_y = 0$ (solid curve) and $\tilde{k}_y = 1$ (dash-dotted curve), and (c) cross sections at $\tilde{k}_x = 0$ (dashed curve) and $\tilde{k}_x = 1$ (dotted curve).

obtained tensor $\overline{\overline{\varepsilon}}(\omega,\mathbf{k})$ relates the amplitudes of such waves of the electric field and the displacement field in a metamaterial].

Nevertheless, in practice it is sometimes beneficial to reduce the complete frequency- and wave-vector-dependent permittivity tensor to a number of *spatially local* parameters that depend only on the frequency. When the spatial dispersion is weak, this can be done as developed in Ref. 17. Under this approach one considers the behavior of the permittivity tensor $\overline{\overline{\varepsilon}}(\omega,\mathbf{k})$ in the vicinity of the middle point of the first Brillouin zone and then expands it in a power series with respect to the components of the wave vector. It may be shown that (in most cases) a second-order expansion allows for introduction of the standard bianisotropic model in which a material is described by four frequency-dependent material tensors. An application of this method to the layered metal-dielectric structure will be the topic of a future work.

VI. CONCLUSIONS

We have developed a homogenization theory and derived the effective parameters for multilayered metal-dielectric metamaterials, taking into account the effects of nonlocality and strong spatial dispersion. We have analyzed the dependencies of the components of the nonlocal permittivity tensor on the wave vector and found that the components parallel to the layers differ from each other. In addition, we have revealed that the nondiagonal components do not vanish in general, and this means that the optical axis may experience a rotation. We believe that our result may be useful for the analytical description of multilayered metamaterials with arbitrary permittivities of the layers, and they may help in describing novel effects in such structures by employing the improved effective parameters.

ACKNOWLEDGMENTS

The authors acknowledge support from the Ministry of Education and Science of the Russian Federation, Dynasty Foundation (Russia), the Engineering and Physical Sciences Research Council (UK), and the Australian Research Council (Australia).

APPENDIX A: AMPLITUDES OF TE-POLARIZED WAVES

The system of linear equations for the amplitudes of the TE-polarized waves can be written in a matrix form as

$$\mathbf{M}\mathbf{X} = \mathbf{B}, \tag{A1}$$

where

$$\mathbf{X} = \begin{bmatrix} A_1^{\mathrm{TE}} \\ B_1^{\mathrm{TE}} \\ A_2^{\mathrm{TE}} \\ B_2^{\mathrm{TE}} \end{bmatrix} \tag{A2}$$

is the vector of the unknown field amplitudes,

$$\mathbf{M} = \begin{bmatrix} e^{ik_{x1}d_1} & e^{-ik_{x1}d_1} & -e^{ik_{x2}d_1} & -e^{-ik_{x2}d_1} \\ e^{ik_x D} & e^{ik_x D} & -e^{ik_{x2}D} & -e^{-ik_{x2}D} \\ k_{x1}e^{ik_{x1}d_1} & -k_{x1}e^{-ik_{x1}d_1} & -k_{x2}e^{ik_{x2}d_1} & k_{x2}e^{-ik_{x2}d_1} \\ k_{x1}e^{ik_x D} & -k_{x1}e^{ik_x D} & -k_{x2}e^{ik_{x2}D} & k_{x2}e^{-ik_{x2}D} \end{bmatrix} \tag{A3}$$

is the matrix of the system, and

$$\mathbf{B} = \begin{bmatrix} \Delta E_{\mathbf{J}_z,z} \\ \Delta E_{\mathbf{J}_z,z}e^{ik_x d_2} \\ k_x \Delta E_{\mathbf{J}_z,z} \\ k_x \Delta E_{\mathbf{J}_z,z}e^{ik_x d_2} \end{bmatrix} \tag{A4}$$

is a column vector composed of the terms that depend on the external currents in the structure. In these terms, $\Delta E_{\mathbf{J}_z,z} = E^{(2)}_{J_z,z}(d_1) - E^{(1)}_{J_z,z}(d_1)$, where $E^{(1,2)}_{J_z,z}(d_1)$ is given by Eq. (6).

The determinant of the system matrix $\mathbf{M}$ (A1) can be written in the following form:

$$\mathrm{Det}\,\mathbf{M} = 4e^{ik_x D}\big\{\sin(k_{x1}d_1)\sin(k_{x2}d_2)\big(k_{x1}^2 + k_{x2}^2\big) + 2k_{x1}k_{x2}[\cos(k_x D) - \cos(k_{x1}d_1)\cos(k_{x2}d_2)]\big\}.$$

Assuming Det $\mathbf{M} \neq 0$ it is possible to solve the system and to obtain the following amplitudes:

$$\begin{aligned} A_1^{\mathrm{TE}} &= \frac{2e^{ik_x D}}{\mathrm{Det}\,\mathbf{M}}\Delta E_{\mathbf{J}_z,z}\big(i\sin(k_{x2}d_2)[-1 + e^{-id_1(k_{x1}+k_x)}]\big(k_{x2}^2 + k_{x1}k_x\big) \\ &\quad + k_{x2}(k_{x1}+k_x)\{e^{-iD(k_{x1}+k_x)+ik_{x1}d_2} + e^{id_2k_x} - \cos(k_{x2}d_2)[1 + e^{-id_1(k_{x1}+k_x)}]\}\big), \\ B_1^{\mathrm{TE}} &= -\frac{2e^{ik_x D}}{\mathrm{Det}\,\mathbf{M}}\Delta E_{\mathbf{J}_z,z}\big(i\sin(k_{x2}d_2)[-1 + e^{id_1(k_{x1}-k_x)}]\big(k_{x2}^2 - k_{x1}k_x\big) \\ &\quad - k_{x2}(k_{x1}-k_x)\{e^{iD(k_{x1}-k_x)-ik_{x1}d_2} + e^{id_2k_x} - \cos(k_{x2}d_2)[1 + e^{id_1(k_{x1}-k_x)}]\}\big), \\ A_2^{\mathrm{TE}} &= \frac{2e^{iD(k_x-k_{x2})}}{\mathrm{Det}\,\mathbf{M}}\Delta E_{\mathbf{J}_z,z}\big\{i\sin(k_{x1}d_1)[-1 + e^{id_2(k_{x2}+k_x)}]\big(k_{x1}^2 + k_xk_{x2}\big) \\ &\quad - k_{x1}(k_{x2}+k_x)[e^{iD(k_{x2}+k_x)-ik_{x2}d_1} + e^{-id_1k_x} - \cos(k_{x1}d_1)[1 + e^{id_2(k_{x2}+k_x)}]]\big\}, \\ B_2^{\mathrm{TE}} &= -\frac{2e^{iD(k_x+k_{x2})}}{\mathrm{Det}\,\mathbf{M}}\Delta E_{\mathbf{J}_z,z}\big\{i\sin(k_{x1}d_1)[-1 + e^{-id_2(k_{x2}-k_x)}]\big(k_{x1}^2 - k_{x2}k_x\big) \\ &\quad - k_{x1}(k_x-k_{x2})[e^{-iD(k_{x2}-k_x)+ik_{x2}d_1} + e^{-id_1k_x} - \cos(k_{x1}d_1)[1 + e^{-id_2(k_{x2}-k_x)}]]\big\}, \end{aligned} \tag{A5}$$

When Det $\mathbf{M} = 0$ it is clear from Eq. (A5) that the wave amplitudes tend to infinity. This happens at the specific frequencies and wave numbers that correspond to the eigenmodes.

APPENDIX B: AVERAGING OF TE-POLARIZED WAVES

The z component of the electric field E_z of a TE-polarized wave can be averaged over a period of the structure, $D = d_1 + d_2$, as follows:

$$\langle E_z\rangle = \frac{1}{D}\int_0^D E_z e^{-ik_x x}dx = \frac{1}{D}\big\{\big\langle E_z^{(1)}\big\rangle + \big\langle E_z^{(2)}\big\rangle + \big[E^{(1)}_{\mathbf{J}_z,z}(d_1)d_1 + E^{(2)}_{\mathbf{J}_z,z}(d_1)d_2\big]e^{-ik_x d_1}\big\}, \tag{B1}$$

where

$$\begin{aligned} \big\langle E_z^{(1)}\big\rangle &= \int_0^{d_1} E_z^{(1)}e^{-ik_x x}dx = A_1^{\mathrm{TE}}\frac{[e^{id_1(k_{x1}-k_x)} - 1]}{i(k_{x1}-k_x)} + B_1^{\mathrm{TE}}\frac{[e^{-id_1(k_{x1}+k_x)} - 1]}{-i(k_{x1}+k_x)} = \frac{8\Delta E_{\mathbf{J}_z,z}e^{-ik_x d_1 + ik_x D}}{\mathrm{Det}\,\mathbf{M}\big(k_{x1}^2 - k_x^2\big)}\big\{\sin(k_{x2}d_2)[\cos(k_x d_1) \\ &\quad - \cos(k_{x1}d_1)]k_{x1}\big(k_{x2}^2 + k_x^2\big) + \sin(k_{x1}d_1)[\cos(k_x d_2) - \cos(k_{x2}d_2)]k_{x2}\big(k_{x1}^2 + k_x^2\big) \\ &\quad + [-\sin(k_x D) + \sin(k_x d_2)\cos(k_{x1}d_1) + \cos(k_{x2}d_2)\sin(k_x d_1)]2k_{x1}k_{x2}k_x\big\}, \end{aligned} \tag{B2}$$

$$\langle E_z^{(2)}\rangle = \int_{d_1}^{D} E_z^{(2)} e^{-ik_x x} dx = A_2^{\mathrm{TE}} \frac{[e^{iD(k_{x2}-k_x)} - e^{id_1(k_{x2}-k_x)}]}{i(k_{x2}-k_x)} + B_2^{\mathrm{TE}} \frac{[e^{-iD(k_{x2}+k_x)} - e^{-id_1(k_{x2}+k_x)}]}{-i(k_{x2}+k_x)}$$
$$= -\frac{8\Delta E_{\mathbf{J}_z,z} e^{-ik_x d_1 + ik_x D}}{\mathrm{Det}\ \mathbf{M}\left(k_{x2}^2 - k_x^2\right)} \big\{ \sin(k_{x1}d_1)[\cos(k_x d_2) - \cos(k_{x2}d_2)]k_{x2}\left(k_{x1}^2 + k_x^2\right) + \sin(k_{x2}d_2)[\cos(k_x d_1)$$
$$- \cos(k_{x1}d_1)]k_{x1}\left(k_{x2}^2 + k_x^2\right) + [-\sin(k_x D) + \sin(k_x d_1)\cos(k_{x2}d_2) + \cos(k_{x1}d_1)\sin(k_x d_2)]2k_{x2}k_{x1}k_x\big\}, \tag{B3}$$

and $E_{\mathbf{J}_z,z}^{(1,2)}$ can be found from Eq. (6).

APPENDIX C: AMPLITUDES OF TM-POLARIZED WAVES

The system of linear equations for the amplitudes of the TM-polarized waves, when the current is directed along the y axis, can be written in matrix form as follows:

$$\mathbf{MX} = \mathbf{B}, \tag{C1}$$

where

$$\mathbf{X} = \begin{bmatrix} A_1^{\mathrm{TM1}} \\ B_1^{\mathrm{TM1}} \\ A_2^{\mathrm{TM1}} \\ B_2^{\mathrm{TM1}} \end{bmatrix} \tag{C2}$$

is the vector of the unknown field amplitudes,

$$\mathbf{M} = \begin{bmatrix} e^{ik_{x1}d_1} & e^{-ik_{x1}d_1} & -e^{ik_{x2}d_1} & -e^{-ik_{x2}d_1} \\ e^{ik_x D} & e^{ik_x D} & -e^{ik_{x2}D} & -e^{-ik_{x2}D} \\ \frac{k_{x1}}{\varepsilon_1} e^{ik_{x1}d_1} & \frac{-k_{x1}}{\varepsilon_1} e^{-ik_{x1}d_1} & \frac{-k_{x2}}{\varepsilon_2} e^{ik_{x2}d_1} & \frac{k_{x2}}{\varepsilon_2} e^{-ik_{x2}d_1} \\ \frac{k_{x1}}{\varepsilon_1} e^{ik_x D} & \frac{-k_{x1}}{\varepsilon_1} e^{ik_x D} & \frac{-k_{x2}}{\varepsilon_2} e^{ik_{x2}D} & \frac{k_{x2}}{\varepsilon_2} e^{-ik_{x2}D} \end{bmatrix} \tag{C3}$$

is the system matrix, and

$$\mathbf{B} = \begin{bmatrix} \Delta H_{\mathbf{J}_y,z} \\ \Delta H_{\mathbf{J}_y,z} e^{ik_x d_2} \\ \varepsilon_0 \omega \Delta E_{\mathbf{J}_y,y} \\ \varepsilon_0 \omega \Delta E_{\mathbf{J}_y,y} e^{ik_x d_2} \end{bmatrix} \tag{C4}$$

is the column vector containing information about the external currents in the structure.

The expressions for the source terms are the following: $\Delta H_{\mathbf{J}_y,z} = H_{\mathbf{J}_y,z}^{(2)}(d_1) - H_{\mathbf{J}_y,z}^{(1)}(d_1)$, $\Delta E_{\mathbf{J}_y,y} = E_{\mathbf{J}_y,y}^{(2)}(d_1) - E_{\mathbf{J}_y,y}^{(1)}(d_1)$. In turn, the electric and magnetic fields induced by the source are found from Eqs. (2) and (3):

$$E_{\mathbf{J}_y,x}^{(1,2)}(d_1) = \frac{-ik_x k_y J_y e^{ik_x d_1}}{\omega \varepsilon_0 \varepsilon_{1,2}\left(k^2 - \varepsilon_{1,2}\frac{\omega^2}{c^2}\right)}, \tag{C5}$$

$$E_{\mathbf{J}_y,y}^{(1,2)}(d_1) = iJ_y e^{ik_x d_1} \frac{-k_y^2 + \varepsilon_{1,2}\frac{\omega^2}{c^2}}{\omega \varepsilon_0 \varepsilon_{1,2}\left(k^2 - \varepsilon_{1,2}\frac{\omega^2}{c^2}\right)}, \tag{C6}$$

$$H_{\mathbf{J}_y,z}^{(1,2)}(d_1) = \frac{ik_x J_y e^{ik_x d_1}}{k^2 - \varepsilon_{1,2}\frac{\omega^2}{c^2}}. \tag{C7}$$

Assuming Det $\mathbf{M} \neq 0$ we solve this system by Gaussian elimination and obtain the following result:

$$A_1^{\mathrm{TM1}} = \frac{2e^{-ik_{x1}D}}{\mathrm{Det}\ \mathbf{M}} \big\{ i\sin(k_{x2}d_2)[e^{iD(k_{x1}+k_x)} - e^{id_2(k_{x1}+k_x)}]\big[-\Delta H_{\mathbf{J}_y,z}\left(k_{x2}^2\varepsilon_1^2 + k_{x2}k_{x1}\varepsilon_2\varepsilon_1\right)$$
$$+ \varepsilon_0\varepsilon_1\varepsilon_2\omega\Delta E_{\mathbf{J}_y,y}(k_{x2}\varepsilon_1 - k_{x1}\varepsilon_2)\big] + \Delta H_{\mathbf{J}_y,z}k_{x2}k_{x1}\varepsilon_2\varepsilon_1(e^{ik_x d_2} - e^{-ik_{x2}d_2})[e^{iD(k_{x1}+k_x)} - e^{id_2(k_{x1}+k_{x2}}]$$
$$- \varepsilon_0\varepsilon_1\varepsilon_2\omega\Delta E_{\mathbf{J}_y,y}k_{x2}\varepsilon_1(e^{id_2k_{x2}} - e^{id_2k_x})[e^{iD(k_{x1}+k_x)} - e^{id_2(k_{x1}-k_{x2})}]\big\},$$

$$B_1^{\mathrm{TM1}} = -\frac{2e^{ik_{x1}D}}{\mathrm{Det}\ \mathbf{M}} \big\{ i\sin(k_{x2}d_2)[e^{iD(-k_{x1}+k_x)} - e^{id_2(-k_{x1}+k_x)}]\big[-\Delta H_{\mathbf{J}_y,z}\left(k_{x2}^2\varepsilon_1^2 - k_{x2}k_{x1}\varepsilon_2\varepsilon_1\right)$$
$$+ \varepsilon_0\varepsilon_1\varepsilon_2\omega\Delta E_{\mathbf{J}_y,y}(k_{x2}\varepsilon_1 + k_{x1}\varepsilon_2)\big] - \Delta H_{\mathbf{J}_y,z}k_{x2}k_{x1}\varepsilon_2\varepsilon_1(e^{ik_x d_2} - e^{-ik_{x2}d_2})[e^{iD(-k_{x1}+k_x)} - e^{id_2(-k_{x1}+k_{x2})}]$$
$$- \varepsilon_0\varepsilon_1\varepsilon_2\omega\Delta E_{\mathbf{J}_y,y}k_{x2}\varepsilon_1(e^{id_2k_{x2}} - e^{id_2k_x})[e^{iD(-k_{x1}+k_x)} - e^{id_2(-k_{x1}-k_{x2})}]\big\},$$

$$A_2^{\mathrm{TM1}} = -\frac{2}{\mathrm{Det}\,\mathbf{M}}\{i\,\sin(k_{x1}d_1)[e^{iD(-k_{x2}-k_x)} - e^{id_1(-k_{x2}-k_x)}]\big[\Delta H_{\mathbf{J}_y,z}\big(k_{x1}^2\varepsilon_2^2 - k_{x1}k_{x2}\varepsilon_1\varepsilon_2\big)$$
$$+\varepsilon_0\varepsilon_1\varepsilon_2\omega\Delta E_{\mathbf{J}_y,y}(k_{x1}\varepsilon_2 + k_{x2}\varepsilon_1)\big] + \Delta H_{\mathbf{J}_y,z}k_{x1}k_{x2}\varepsilon_1\varepsilon_2(e^{-ik_xd_1} - e^{-ik_{x1}d_1})[e^{iD(-k_{x2}-k_x)}$$
$$-e^{id_1(-k_{x2}+k_{x1})}] - \varepsilon_0\varepsilon_1\varepsilon_2\omega\Delta E_{\mathbf{J}_y,y}k_{x1}\varepsilon_2(e^{id_1k_{x1}} - e^{-id_1k_x})[e^{iD(-k_{x2}-k_x)} - e^{id_1(-k_{x2}-k_{x1})}]\},$$
$$B_2^{\mathrm{TM1}} = \frac{2}{\mathrm{Det}\,\mathbf{M}}\{i\,\sin(k_{x1}d_1)[e^{iD(k_{x2}-k_x)} - e^{id_1(k_{x2}-k_x)}]\big[\Delta H_{\mathbf{J}_y,z}\big(k_{x1}^2\varepsilon_2^2 + k_{x1}k_{x2}\varepsilon_1\varepsilon_2\big)$$
$$+\varepsilon_0\varepsilon_1\varepsilon_2\omega\Delta E_{\mathbf{J}_y,y}(k_{x1}\varepsilon_2 - k_{x2}\varepsilon_1)\big] - \Delta H_{\mathbf{J}_y,z}k_{x1}k_{x2}\varepsilon_1\varepsilon_2(e^{-ik_xd_1} - e^{-ik_{x1}d_1})[e^{iD(k_{x2}-k_x)} - e^{id_1(k_{x2}+k_{x1})}]$$
$$-\varepsilon_0\varepsilon_1\varepsilon_2\omega\Delta E_{\mathbf{J}_y,y}k_{x1}\varepsilon_2(e^{id_1k_{x1}} - e^{-id_1k_x})[e^{iD(k_{x2}-k_x)} - e^{id_1(k_{x2}-k_{x1})}]\}. \quad \text{(C8)}$$

Here $\mathrm{Det}\,\mathbf{M} = 4e^{ik_xD}\{\sin(k_{x1}d_1)\sin(k_{x2}d_2)(k_{x1}^2\varepsilon_2^2 + k_{x2}^2\varepsilon_1^2) + 2k_{x1}k_{x2}\varepsilon_1\varepsilon_2[\cos(k_xD) - \cos(k_{x1}d_1)\cos(k_{x2}d_2)]\}$. The symmetry transformation discussed above for the case of the TE polarization in Eq. (10) also holds for the TM polarization, with an additional substitution in the expression for the amplitudes: $H_{\mathbf{J}_y,z}^{(1)} \to H_{\mathbf{J}_y,z}^{(2)}$. As in the case of TE polarization, the external current that occurs in $H_{\mathbf{J}_y,z}^{(1)}$ and $H_{\mathbf{J}_y,z}^{(2)}$ must be kept invariant, and there is no need to substitute $k_x \to -k_x$ and $d_1 \to d_2$ in the expression for the current.

The case when the current vector is directed along the x axis differs from the case considered above in the expression for vector $\mathbf{B}$,

$$\mathbf{B} = \begin{bmatrix} \Delta H_{\mathbf{J}_x,z} \\ \Delta H_{\mathbf{J}_x,z}e^{ik_xd_2} \\ \varepsilon_0\omega\Delta E_{\mathbf{J}_x,y} \\ \varepsilon_0\omega\Delta E_{\mathbf{J}_x,y}e^{ik_xd_2} \end{bmatrix}, \quad \text{(C9)}$$

where $\Delta H_{\mathbf{J}_x,z} = H_{\mathbf{J}_x,z}^{(2)}(d_1) - H_{\mathbf{J}_x,z}^{(1)}(d_1)$, $\Delta E_{\mathbf{J}_x,y} = E_{\mathbf{J}_x,y}^{(2)}(d_1) - E_{\mathbf{J}_x,y}^{(1)}(d_1)$, and the electric and magnetic fields induced by the external current in this case are

$$E_{\mathbf{J}_x,x}^{(1,2)}(d_1) = i\,J_xe^{ik_xd_1}\frac{-k_x^2 + \varepsilon_{1,2}\frac{\omega^2}{c^2}}{\omega\varepsilon_0\varepsilon_{1,2}\big(k^2 - \varepsilon_{1,2}\frac{\omega^2}{c^2}\big)}, \quad \text{(C10)}$$

$$E_{\mathbf{J}_x,y}^{(1,2)}(d_1) = \frac{-ik_xk_yJ_xe^{ik_xd_1}}{\omega\varepsilon_0\varepsilon_{1,2}\big(k^2 - \varepsilon_{1,2}\frac{\omega^2}{c^2}\big)}, \quad \text{(C11)}$$

$$H_{\mathbf{J}_x,z}^{(1,2)}(d_1) = -\frac{ik_yJ_xe^{ik_xd_1}}{k^2 - \varepsilon_{1,2}\frac{\omega^2}{c^2}}. \quad \text{(C12)}$$

The amplitudes A_1^{TM2}, A_2^{TM2}, B_1^{TM2}, and B_2^{TM2} can be obtained from Eq. (C8) by replacing $\Delta H_{\mathbf{J}_y,z}$ and $\Delta E_{\mathbf{J}_y,y}$ with $\Delta H_{\mathbf{J}_x,z}$ and $\Delta E_{\mathbf{J}_x,y}$.

APPENDIX D: AVERAGING OF TM-POLARIZED WAVES

When the current is directed along the y axis, the components of the electric field of the TM-polarized waves averaged over a period of the structure verify

$$\langle E_x\rangle^{\mathrm{TM1}} = \frac{1}{D}\int_0^D E_xe^{-ik_xx}dx = \frac{1}{D}\{\langle E_x^{(1)}\rangle + \langle E_x^{(2)}\rangle + \big[E_{\mathbf{J}_y,x}^{(1)}(d_1)d_1 + E_{\mathbf{J}_y,x}^{(2)}(d_1)d_2\big]e^{-ik_xd_1}\}, \quad \text{(D1)}$$

$$\langle E_y\rangle^{\mathrm{TM1}} = \frac{1}{D}\int_0^D E_ye^{-ik_xx}dx = \frac{1}{D}\{\langle E_y^{(1)}\rangle + \langle E_y^{(2)}\rangle + \big[E_{\mathbf{J}_y,y}^{(1)}(d_1)d_1 + E_{\mathbf{J}_y,y}^{(2)}(d_1)d_2\big]e^{-ik_xd_1}\}, \quad \text{(D2)}$$

where the components in each layer take the form

$$\langle E_x^{(1)}\rangle^{\mathrm{TM1}} = \int_0^{d_1} E_x^{(1)}e^{-ik_xx}dx = -\frac{k_y}{\varepsilon_0\varepsilon_1\omega}\left\{A_1^{\mathrm{TM1}}\frac{[e^{id_1(k_{x1}-k_x)} - 1]}{i(k_{x1}-k_x)} + B_1^{\mathrm{TM1}}\frac{[e^{-id_1(k_{x1}+k_x)} - 1]}{-i(k_{x1}+k_x)}\right\}, \quad \text{(D3)}$$

$$\langle E_x^{(2)}\rangle^{\mathrm{TM1}} = \int_{d_1}^D E_x^{(2)}e^{-ik_xx}dx = -\frac{k_y}{\varepsilon_0\varepsilon_2\omega}\left\{A_2^{\mathrm{TM1}}\frac{[e^{iD(k_{x2}-k_x)} - e^{id_1(k_{x2}-k_x)}]}{i(k_{x2}-k_x)} + B_2^{\mathrm{TM1}}\frac{[e^{-iD(k_{x2}+k_x)} - e^{-id_1(k_{x2}+k_x)}]}{-i(k_{x2}+k_x)}\right\}, \quad \text{(D4)}$$

$$\langle E_y^{(1)}\rangle^{\mathrm{TM1}} = \int_0^{d_1} E_y^{(1)}e^{-ik_xx}dx = \frac{k_{x1}}{\varepsilon_0\varepsilon_1\omega}\left\{A_1^{\mathrm{TM1}}\frac{[e^{id_1(k_{x1}-k_x)} - 1]}{i(k_{x1}-k_x)} - B_1^{\mathrm{TM1}}\frac{[e^{-id_1(k_{x1}+k_x)} - 1]}{-i(k_{x1}+k_x)}\right\}, \quad \text{(D5)}$$

$$\left\langle E_y^{(2)}\right\rangle^{\mathrm{TM1}} = \int_{d_1}^{D} E_y^{(2)} e^{-ik_x x} dx = \frac{k_{x2}}{\varepsilon_0 \varepsilon_2 \omega} \left\{ A_2^{\mathrm{TM1}} \frac{[e^{iD(k_{x2}-k_x)} - e^{id_1(k_{x2}-k_x)}]}{i(k_{x2}-k_x)} - B_2^{\mathrm{TM1}} \frac{[e^{-iD(k_{x2}+k_x)} - e^{-id_1(k_{x2}+k_x)}]}{-i(k_{x2}+k_x)} \right\}, \tag{D6}$$

while the field components in each layer as the functions of $\mathbf{J}_y$ are given in Eqs. (C10), (C11).

The components of the macroscopic electric displacement $\langle \mathbf{D} \rangle^{\mathrm{TM1}}$ verify that

$$\langle D_x \rangle^{\mathrm{TM1}} = \frac{1}{D} \int_0^D \varepsilon E_x e^{-ik_x x} dx = \frac{1}{D} \left\{ \varepsilon_1 \left\langle E_x^{(1)} \right\rangle + \varepsilon_2 \left\langle E_x^{(2)} \right\rangle + \left[\varepsilon_1 E_{\mathbf{J}_y,x}^{(1)}(d_1) d_1 + \varepsilon_2 E_{\mathbf{J}_y,x}^{(2)}(d_1) d_2 \right] e^{-ik_x d_1} \right\}, \tag{D7}$$

$$\langle D_y \rangle^{\mathrm{TM1}} = \frac{1}{D} \int_0^D \varepsilon E_y e^{-ik_x x} dx = \frac{1}{D} \left\{ \varepsilon_1 \left\langle E_y^{(1)} \right\rangle + \varepsilon_2 \left\langle E_y^{(2)} \right\rangle + \left[\varepsilon_1 E_{\mathbf{J}_y,y}^{(1)}(d_1) d_1 + \varepsilon_2 E_{\mathbf{J}_y,y}^{(2)}(d_1) d_2 \right] e^{-ik_x d_1} \right\}. \tag{D8}$$

The expressions for the averaged fields $\langle E_x^{(1)} \rangle^{\mathrm{TM2}}$, $\langle E_x^{(2)} \rangle^{\mathrm{TM2}}$, $\langle E_y^{(1)} \rangle^{\mathrm{TM2}}$, and $\langle E_y^{(2)} \rangle^{\mathrm{TM2}}$ can be obtained by replacing the amplitudes A_1^{TM1}, A_2^{TM1}, B_1^{TM1}, and B_2^{TM1} with A_1^{TM2}, A_2^{TM2}, B_1^{TM2}, and B_2^{TM2} in Eqs. (D3)–(D6), respectively.

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