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Nonlocal thermoelastic semi-infinite medium with variable thermal conductivity due to a laser short-pulse

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ABSTRACT

In this article, the thermoelastic interactions in an isotropic and homogeneous semi-infinite medium with variable thermal conductivity caused by an ultra-short pulsed laser heating based on the linear nonlocal theory of elasticity has been considered. We consider that the thermal conductivity of the material is dependent on the temperature. The surface of the surrounding plane of the medium is heated by an ultra-short pulse laser. Basic equations are solved along with the corresponding boundary conditions numerically by means of the Laplace transform technique. The influences of the rise time of the laser pulse, as well as the nonlocal parameter on thermoelastic wave propagation in the medium, have also been investigated in detail. Presented numerical results, graphs and discussions in this work lead to some important deductions. The results obtained here will be useful for researchers in nonlocal material science, low-temperature physicists, new materials designers, as well as to those who are working on the development of the theory of nonlocal thermoelasticity.

1. Introduction

Recent interest has increased with the use of heat sources such as lasers and microwaves, which have found many applications for scientific research and material processing. Lasers are used in medicine and in many important applications. The experimental application of laser ultrasound in solid materials has an extended and successful history. Laser has been widely used in experimental acoustics in the last decades because it allows contactless excitation of various thermoelastic waves [1, 2]. The so-called ultra-short lasers are those whose pulse duration varies from nanoseconds to femtoseconds. In the case of ultra-short pulsed laser heating, the high-intensity energy flow and the ultra-short duration can cause very high thermal gradients or ultra-high temperature limits [3-5].

Many theoretical and experimental studies show that thermal diffusivity and conductivity in materials should be considered as variables in material investigation. This provides researchers a motivation to capture transient nonlinear responses to generalized problems of thermoelasticity with variable thermal diffusivity and conductivity. When the temperature dependence of thermal properties is accounted for, the heat equation becomes nonlinear and its exact solution is unattainable. In recent years, thermal conductivity has been found to be closely related to the distribution of temperature. This type of temperature-dependent thermal conductivity plays a fundamental role in thermal mechanical investigations [6-12]. The thermal conductivity based on temperature has become more important in many engineering applications [13-19].

The nonlocal theory of elasticity is adopted to deal with many applications in nano-mechanics. Different studies of nanotubes and beams have been studied more widely in the context of nanostructures. The models of the nonlocal beams expected increasing attention in the early few years. In 1972, Eringen introduced the theory of nonlocal continuum mechanics [20-23], to deal with the small-scale structure problems. The theories of nonlocal continuum consider the state of stress at a point as a function of the states of strain of all points in the medium. This is not the same in classical continuum mechanics in which it assumes the state of stress at a certain point uniquely depends on the state of strain on that same point. Solutions for various problems

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support this theory [24-28]. The mechanical behaviors of nanoscale devices, particularly the failure of a nanoscale, are inevitably associated to temperature: the devices may be exposed to heat from the environment or heat produced by themselves when working. The applications of these theories have been examined extensively by many investigators (see [29]-[32]).

Classical thermoelasticity may be not appropriate to the analysis at the micro or nanoscale as the characteristic length of the structure become comparable to the internal characteristic length, for example the free-mean path, the wavelength. On the other hand, the thermoelastic coupling is rarely reported with the effect of size in thermal conductivity and deformation. Therefore, the theory of a complete size-dependent coupling of the two fields, i.e. displacement and temperature or stress and heat flux, is greatly needed, and to bridging the gap is the basic objective of this work. The theory of coupled thermoelasticity was originally proposed to describe the coupled thermoelastic mechanical behavior. However, it predicts an infinite speed of temperature propagation. Lord and Shulman [33] and Green and Lindsay [34] develop the theory of couple thermoelasticity by including the thermal relaxation time in the constituent relations.

In this work, size-dependent thermoelasticity is formulated by combining nonlocal thermoelasticity and nonlocal elasticity suggested by Eringen [20, 21] with the aids of extended irreversible thermodynamics and generalized free energy. In addition, the current work is an effort to investigate a thermoelastic problem of a semi-infinite medium loaded thermally by ultra-short pulse laser. Also, a new nonlocal model is introduced, and the parameter of thermal conductivity considers to be a function of temperature. Numerical calculations are performed in different cases to highlight the influence of the small-scale parameter, as well as the rise time of the laser pulse on the propagation of thermoelastic waves in the nonlocal medium. Also, some important explanations have been made and discussed.

We shall use the following symbolizations:

λ, μ	Lamé's constants
α_t	Thermal expansion coefficients
$\gamma = (3\lambda + 2\mu)\alpha_t$	Coupling parameter
T_0	Environment temperature
$\theta = T - T_0$	Temperature increment
T	Absolute temperature
C_E	Specific heat
$\boldsymbol{\tau}$	Nonlocal stress tensor
$\boldsymbol{\sigma}$	Local stress tensor
$\boldsymbol{\varepsilon}$	Strain tensor
$\lvert x' - x\rvert$	Euclidean distance
$\alpha(\lvert x' - x\rvert)$	Nonlocal Kernel
$\xi = e_0 a/l$	Nonlocal parameter
a	Internal characteristic length
l	external characteristic length
e_0	Adjusting constant
K	Thermal conductivity
$\boldsymbol{I}$	Identity tensor
$\boldsymbol{u}$	Displacement vector
$\boldsymbol{F}$	Body force vector
Q	Heat source
τ_0	Relaxation time
ρ	Material density
E	Young's modulus
$Oxyz$	Cartesian coordinates
∇^2	Laplacian operator
t	Time

2. Mathematical Model of Nonlocal Theory

According to the model of nonlocal elasticity suggested by Eringen [20, 21], the stress field at the point $\boldsymbol{x}$ in the elastic medium does not only depend on the field of strain on the point but also on the strains at all other points in the medium. The stress-strain-temperature relations have the form

$$\boldsymbol{\tau}(\boldsymbol{x}) = \int_V \alpha(|\boldsymbol{x}' - \boldsymbol{x}|, \xi)\, \boldsymbol{\sigma}(\boldsymbol{x}) \mathrm{d}V(\boldsymbol{x}'), \tag{1}$$

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \lambda(\mathrm{div}\, \boldsymbol{u})\boldsymbol{I} + 2\mu\boldsymbol{\varepsilon} - \gamma\theta\boldsymbol{I}, \tag{2}$$

where

$$\boldsymbol{\varepsilon} = \frac{1}{2}[\nabla\boldsymbol{u} + \nabla(\boldsymbol{u}^T)]. \tag{3}$$

Eringen [21] replaced the nonlocal constitutive equations given by the integral formulation by the gradients. Thus, by applying the differential operator $(1 - \xi^2\nabla^2)$ to both sides of Eq. (1), one may get the equivalent differential form of the nonlocal theory as

$$(1 - \xi^2\nabla^2)\boldsymbol{\tau} = \lambda(\mathrm{div}\, \boldsymbol{u})\boldsymbol{I} + 2\mu\boldsymbol{\varepsilon} - \gamma\theta\boldsymbol{I}, \tag{4}$$

which considers the size effect on the response of nanostructures.

The balance of linear momentum gives the equations of motion in the form

$$\nabla \cdot \boldsymbol{\tau} + \boldsymbol{F} = \rho\ddot{\boldsymbol{u}}. \tag{5}$$

After using Eq. (4), the equations of motion can be obtained in terms of the temperature and displacement fields as

$$(\lambda + \mu)\nabla(\nabla\boldsymbol{u}) + \mu\nabla^2\boldsymbol{u} - \gamma\nabla\theta + (1 - \xi^2\nabla^2)\boldsymbol{F} = \rho(1 - \xi^2\nabla^2)\ddot{\boldsymbol{u}}. \tag{6}$$

The differential form of the classical Fourier's law of heat conduction displays that the heat flux vector $\boldsymbol{q}$ is equal to the product of the thermal conductivity K and the negative local temperature gradient $\nabla\theta$ as follows:

$$\boldsymbol{q} = -K\nabla\theta. \tag{7}$$

The modified Fourier's law of heat conduction is constructed as [20]

$$\left(1 + \tau_0 \frac{\partial}{\partial t}\right)\boldsymbol{q} = -K\nabla\theta. \tag{8}$$

Equation (8) together with the energy equation

$$\rho C_E \frac{\partial\theta}{\partial t} + \gamma T_0 \frac{\partial}{\partial t}\big(\mathrm{div}(\mathbf{u})\big) = -\mathrm{div}(\boldsymbol{q}) + Q, \tag{9}$$

yields the generalized heat conduction equation

$$\nabla(K\nabla\theta) = \left(1 + \tau_0 \frac{\partial}{\partial t}\right)\left[\rho C_E \frac{\partial\theta}{\partial t} + \gamma T_0 \frac{\partial}{\partial t}\big(\mathrm{div}(\mathbf{u})\big) - Q\right]. \tag{10}$$

The thermal conductivity K is considered as one of the properties of the material to conduct heat. In most cases, it is treated as a constant, though this is not always true. It is known that the thermal conductivity of a material has generally varied with the temperature. But this variation can be small over a significant range of temperatures for some common materials.

3. Description of the Problem

Let us consider the problem of a half-space ($x \geq 0$) with the x-axis pointing into the medium initially unstressed with reference temperature T_0. This medium is irradiated uniformly by a laser pulse on the bounding plane ($x = 0$). Also, let us assume that body forces of the medium are neglected. The dynamic problem of the thin slim strip can be treated as a 1D problem in which all physical variables considered depend only on the space variable x and time variable t.

For 1D problems, the displacement field has the form

$$u_x = u(x,t), \quad u_y = u_z = 0. \tag{11}$$

Equation (5) also gives the stress components

$$\left(1 - \xi^2 \frac{\partial^2}{\partial x^2}\right) \tau_{xx} = (\lambda + 2\mu) \frac{\partial u}{\partial x} - \gamma\theta. \tag{12}$$

With the help of Eqs. (6) and (12), we get the equation of motion as

$$\rho \left(1 - \xi^2 \frac{\partial^2}{\partial x^2}\right) \frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial \theta}{\partial x}. \tag{13}$$

The heat conduction equation, Eq. (10), with ignoring the heat source is now expressed as

$$\frac{\partial}{\partial x}\left(K \frac{\partial \theta}{\partial x}\right) = \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \left[\frac{K}{k} \frac{\partial \theta}{\partial t} + \gamma T_0 \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right)\right], \tag{14}$$

where $k = \rho C_E / K$ denotes the thermal diffusivity.

The aim of this study is to consider the influence of temperature dependency of thermal conductivity. In this case, the other elastic and thermal parameters still constants. So, the thermal conductivity K and its corresponding thermal diffusivity k are supposed to vary with temperature θ according to the following linear relation [35]:

$$K = K(\theta) = K_0(1 + K_1\theta), \tag{15}$$

where K_0 is the constant thermal conductivity of the material (independent of the temperature θ) and K_1 represents a non-positive small constant. The other material properties of the domain, such as density ρ and specific heat capacity C_E are assumed to be constant.

Now we introduce the following mapping [36]

$$K_0\varphi = \int_0^\theta (1 + K_1\tau) \mathrm{d}\tau, \tag{16}$$

where φ is a new function that represents the heat conduction. Using Eq. (15) to obtain [36]

$$\varphi = \theta \left(1 + \tfrac{1}{2} K_1 \theta\right). \tag{17}$$

Differentiating Eq. (16) with respect to x, we obtain

$$K_0 \frac{\partial \varphi}{\partial x} = K \frac{\partial \theta}{\partial x}. \tag{18}$$

In the same manner, the differentiating of the mapping with respect to time t yields

$$K_0 \frac{\partial \varphi}{\partial t} = K \frac{\partial \theta}{\partial t}. \tag{19}$$

Using Eqs. (18) and (19), the modified model of heat equation become

$$K_0 \frac{\partial^2 \varphi}{\partial x^2} = \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \left[\frac{K_0}{k} \frac{\partial \varphi}{\partial t} + \gamma T_0 \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right)\right], \tag{20}$$

Substituting from Eqs. (17) and (18) into Eqs. (12) and (13), we obtain

$$\left(1 - \xi^2 \frac{\partial^2}{\partial x^2}\right) \tau_{xx} = (\lambda + 2\mu) \frac{\partial u}{\partial x} - \frac{\gamma K_0}{K(\theta)} \varphi, \tag{21}$$

$$\rho \left(1 - \xi^2 \frac{\partial^2}{\partial x^2}\right) \frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \frac{\gamma K_0}{K(\theta)} \frac{\partial \varphi}{\partial x}. \tag{22}$$

Without losing any generality, we can approximate the thermal conductivity as $K(\theta) \approx K(T_0)$ for linearity which is constant depending on the reference temperature T_0. Therefore, one gets

$$\left(1 - \xi^2 \frac{\partial^2}{\partial x^2}\right) \tau_{xx} = (\lambda + 2\mu) \frac{\partial u}{\partial x} - \frac{\gamma}{(1 + K_1 T_0)} \varphi. \tag{23}$$

$$\rho \left(1 - \xi^2 \frac{\partial^2}{\partial x^2}\right) \frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \frac{\gamma}{(1 + K_1 T_0)} \frac{\partial \varphi}{\partial x}. \tag{24}$$

Considering the following dimensionless quantities

$$\begin{gathered}\{x', u'\} = c_0\omega_0\{x, u\}, \ \{t', \tau'_0\} = c_0^2\omega_0\{t, \tau_0\}, \ \xi' = c_0^2\omega_0^2\xi, \\ \theta' = \frac{\theta}{T_0}, \quad \varphi' = \frac{\varphi}{T_0}, \quad \tau'_{xx} = \frac{\tau_{xx}}{\lambda + 2\mu}, \quad c_0 = \frac{\lambda + 2\mu}{\rho}, \quad \omega_0 = \frac{1}{k}.\end{gathered} \tag{25}$$

The governing equations, Eqs. (20)-(22), may be finally written as (dropping the primes)

$$\left(1 - \xi^2 \frac{\partial^2}{\partial x^2}\right) \tau_{xx} = \frac{\partial u}{\partial x} - a\varphi, \tag{26}$$

$$\left(1 - \xi^2 \frac{\partial^2}{\partial x^2}\right) \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - a \frac{\partial \varphi}{\partial x}, \tag{27}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = \left(1 + \tau_0 \frac{\partial}{\partial \mathrm{t}}\right) \left[\frac{\partial \varphi}{\partial t} + \varepsilon \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right)\right], \tag{28}$$

where

$$a = \frac{\gamma T_0}{(\lambda + 2\mu)(1 + K_1 T_0)}, \quad \varepsilon = \frac{\gamma}{\rho C_E}. \tag{29}$$

The aim of this work is to determine the displacement, the temperature and the nonlocal thermal stress at the surface for the medium described by Eqs. (26)-(28).

4. Solution of the Problem

The closed form solution of the governing and constitutive equations can be possible by adapting the Laplace transform technique. Applying Laplace transform defined by the relation

$$\bar{f}(x,t)=\int_0^\infty f(x,t)\mathrm{e}^{-st}\mathrm{d}t, \tag{30}$$

under the initial conditions

$$\theta(x,0)=\frac{\partial\theta(x,0)}{\partial t}=0=u(x,0)=\frac{\partial u(x,0)}{\partial t}, \tag{31}$$

one gets the transformed field equations, Eqs. (26)-(28), as

$$\left(1-\xi^2\frac{\mathrm{d}^2}{\mathrm{d}x^2}\right)\bar{\tau}_{xx}=\frac{\mathrm{d}\bar{u}}{\mathrm{d}x}-a\bar{\varphi}, \tag{32}$$

$$s^2\left(1-\xi^2\frac{\mathrm{d}^2}{\mathrm{d}x^2}\right)\bar{u}=\frac{\mathrm{d}^2\bar{u}}{\mathrm{d}x^2}-a\frac{\mathrm{d}\bar{\varphi}}{\mathrm{d}x}, \tag{33}$$

$$\frac{\mathrm{d}^2\bar{\varphi}}{\mathrm{d}x^2}=s(1+\tau_0 s)\left(\bar{\varphi}+\varepsilon\frac{\mathrm{d}\bar{u}}{\mathrm{d}x}\right). \tag{34}$$

If we eliminate the function $\bar{\varphi}$ between Eqs. (33) and (34), we have

$$\left(\frac{\mathrm{d}^4}{\mathrm{d}x^4}-m_1\frac{\mathrm{d}^2}{\mathrm{d}x^2}+m_2\right)\bar{u}(x)=0, \tag{35}$$

where the coefficients m_1 and m_2 are given by

$$m_1=\frac{s^2+\alpha_1\alpha_2+a\varepsilon\alpha_2}{\alpha_1},\quad m_2=\frac{s^2\alpha_2}{\alpha_1},$$
$$\alpha_1=1+s^2\xi^2,\quad \alpha_2=s(1+\tau_0 s). \tag{36}$$

The solution of Eqs. (35) under the interface conditions ($\bar{u}(x)$ and $\bar{\theta}(x)$ bounded as $x\to\infty$) can be represented as

$$\bar{u}(x)=\sum_{n=1}^2 C_n\mathrm{e}^{-k_n x}, \tag{37}$$

where C_n $(n=1,2)$ are parameters depending on s. In Eq. (37), k_1 and k_2 are the roots of the characteristic equation

$$k^4-m_1k^2+m_2=0, \tag{38}$$

In the same way, if we eliminate $\bar{u}$ between Eqs. (33) and (34), we obtain

$$\left(\frac{\mathrm{d}^4}{\mathrm{d}x^4}-m_1\frac{\mathrm{d}^2}{\mathrm{d}x^2}+m_2\right)\bar{\varphi}(x)=0. \tag{39}$$

Once again, the solution of the differential equation, Eq. (39), can be written as

$$\bar{\varphi}(x)=\sum_{n=1}^2\beta_n C_n\mathrm{e}^{-k_n x}, \tag{40}$$

where the compatibility between $\bar{\varphi}(x)$ and $\bar{u}(x)$ and Eq. (33), gives

$$\beta_n=-\frac{\alpha_1k_n^2-s^2}{\alpha_1 k_n},\quad n=1,2. \tag{41}$$

Using Eqs. (33) and (37) in Eq. (32), the stress component $\bar{\tau}_{xx}$ can be deduced as

$$\bar{\tau}_{xx}(x)=\sum_{n=1}^2\gamma_n C_n\mathrm{e}^{-k_n x}, \tag{42}$$

where

$$\gamma_n=\frac{-k_n-a}{1-\xi^2k_n^2},\quad n=1,2. \tag{43}$$

With the expression of $\bar{\varphi}$, the temperature $\bar{\theta}$ can be determined by solving Eq. (14) as

$$\bar{\theta}=\frac{-1+\sqrt{1-2K_1\bar{\varphi}}}{K_1}. \tag{44}$$

5. Application to the Problem

To obtain the parameters C_n $(n=1,2)$, one can apply the boundary conditions on the bounding plane $x=0$ of the supposed medium.

We consider the boundary $x=0$ satisfies the following relation

$$u(x,t)=0\quad\text{on}\quad x=0. \tag{45}$$

For $t>0$ the boundary surface $x=0$ is under a dimensionless pulse heat flux $q(x,t)$ of constant intensity q_0. That is

$$\frac{\partial\theta}{\partial x}=q(x,t)=q_0 f(t)\quad\text{on}\quad x=0, \tag{46}$$

such that $f(t)$ is exponentially decaying function form [37] defined as follows,

$$f(t)=\frac{t^2}{16t_p^2}\mathrm{e}^{-t/t_p}\quad\text{on}\quad x=0, \tag{47}$$

where t_p is the pulse rise time of heat flux.

Using Eq. (18), we get

$$\frac{\partial\varphi}{\partial x}=\frac{K_0q_0}{(1+\mathrm{K}_1T_0)}f(t)\quad\text{on}\quad x=0. \tag{48}$$

If we apply Laplace transform on the boundary conditions, Eqs. (30)-(32), we get

$$\bar{u}(0,s)=0,$$
$$\frac{\partial\bar{\varphi}(0,s)}{\partial x}=\frac{K_0q_0t_p}{8(1+\mathrm{K}_1T_0)(1+st_p)^3}=\bar{G}(s), \tag{49}$$

So, Eqs. (37) and (40), with the aid of the boundary conditions appeared above, become

$$\sum_{n=1}^2 C_n=0, \tag{50}$$

$$\sum_{n=1}^2 k_n\beta_n C_n=-\bar{G}(s). \tag{51}$$

The solution of the above system of linear equations gives the unknown parameters C_n $(n=1,2)$ as

$$C_n=(-1)^n\frac{\bar{G}(s)}{k_1\beta_1-k_2\beta_2}.$$

(52)

To determine the studied fields in the physical domain, the Riemann-sum approximation method is used to obtain the numerical results (Honig and Hirdes [38]).

6. Numerical Results

To illustrate and compare the analytical results obtained in the previous sections, we now demonstrate a numerical example, which represent the distributions of thermodynamic temperature θ, displacement u, and nonlocal stress component τ_{xx}. For numerical computations, the material is specified as copper. The relevant material parameters necessary to be known are given in Table 1 [32].

Table 1. Mechanical and thermoelastic properties of the rod ($T_0 = 293$ K).

Material properties	**Value**
Thermal conductivity K (W m^{-1}K^{-1})	386
Young's modulus E (GPa)	128
Density ρ (kg m^{-3})	8953
Thermal expansion α_t (K^{-1})	1.78×10^{-5}
Specific heat C_E (J kg^{-1}K^{-1})	384.56
Poisson's ratio ν	0.36

The results are represented graphically in Figs. 1-16 at different positions of x. The computations are carried out for the wide range of x $(0 \leq x \leq 10)$ at small value of time $t = 0.2$. The field quantities such as the temperature, the strain, the nonlocal stress and the displacement distributions depend not only on the space coordinate x and time t, but also depend on the nonlocal parameter ξ, relaxation time τ_0. Numerical calculation is made in four cases.

6.1. Effect of nonlocal parameter

In this case, some parametric studies are performed to estimate the effect of the nonlocal parameter ξ on the responses of the material, like the temperature, displacement, strain and nonlocal stress. The results are presented graphically in Figs. 1-4 for $t = 0.2$, $\tau_0 = 0.3$ and $t_p = 0.1$. In this case, we notice that when the nonlocal parameter ξ; vanishing ($\xi = 0$) indicates the old situation (local model of elasticity) while other values ($\xi = 0.1, 0.3$) indicate the nonlocal theories of elasticity and thermoelasticity.

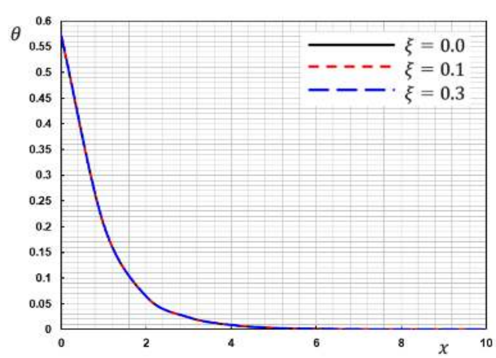

Figure 1. The temperature θ with different nonlocal parameter ξ.

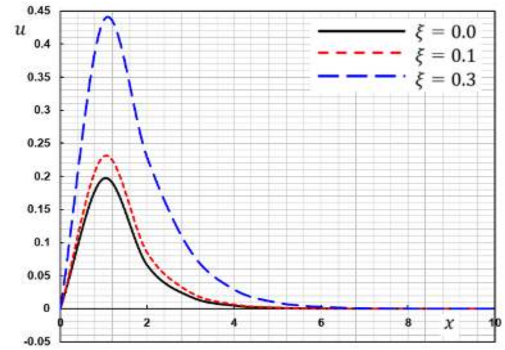

Figure 2. The displacement u with different nonlocal parameter ξ.

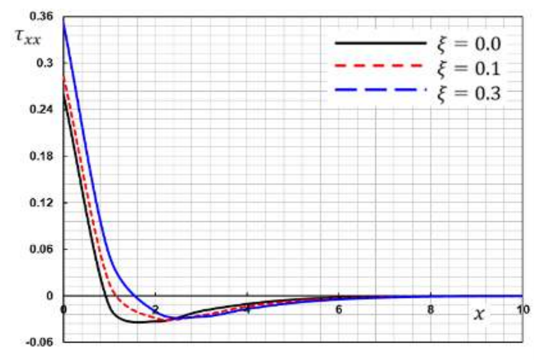

Figure 3. The nonlocal stress τ_{xx} with different nonlocal parameter ξ.

From all these figures, it is evident that all curves coincident when x tends to infinity, all physical fields satisfy the boundary conditions. Thus, the obtained solution is limited to a finite area of space and does not spread to infinity. This is not in the case of the coupled theory of thermoelasticity, where the solution extends to infinity rapidly, suggesting an infinite velocity in the propagation of waves. It is obtained from Figs. 2-4 that the parameter ξ has a great effect on the distributions of displacement, nonlocal stress and strain. The waves reach the steady state depending on the value of the parameter ξ. It is noted that the temperature θ small dependent on non-local parameter variation.

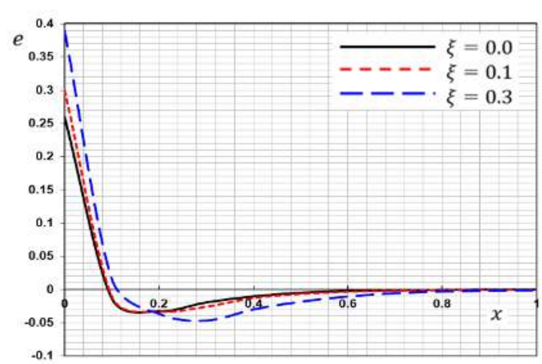

Figure 4. The strain distributions e with different nonlocal parameter ξ.

6.2. Influence of the variation of thermal conductivity

In this case, the influence of temperature dependency of thermal conductivity is investigated. We take into consideration three different values of the parameter $K_1 = 0.0, -0.001, -0.002$, while the other parameters have been taken as $\xi = 0.1$, $t_p = 0.1$, and the parameter $\tau_0 = 0.1$. When $K_1 = 0.0$, indicates the old situation (constant thermal conductivity). The distributions temperature θ and the displacement u are shown in Figs. 5 and 6 while the nonlocal stress τ_{xx} and strain e distributions are shown in Figs. 7 and 8. The parameter K_1 have noticeable effects on all the profiles of the studied fields. As shown in Fig. 5, as in most thermoelastic materials the thermal conductivity decreases with increasing temperature [39]. This state is usually ignored by most other researchers who consider thermal conductivity is temperature independent (constant).

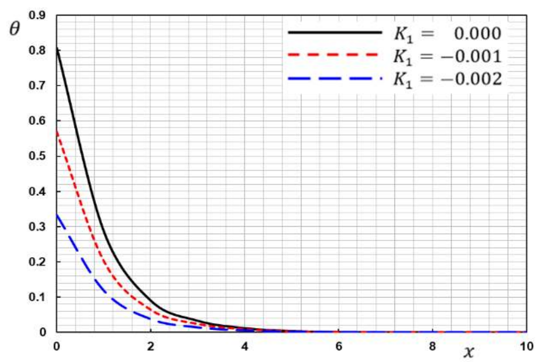

Figure 5. The effect of the variation of thermal conductivity on temperature θ.

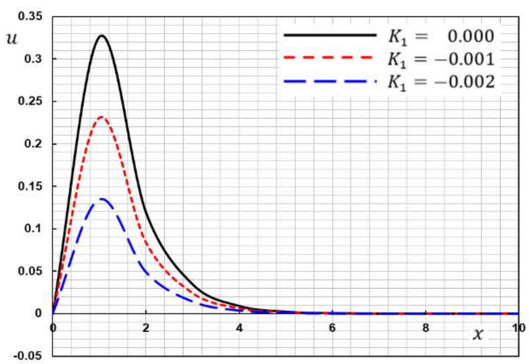

Figure 6. The effect of the variation of thermal conductivity on displacement u.

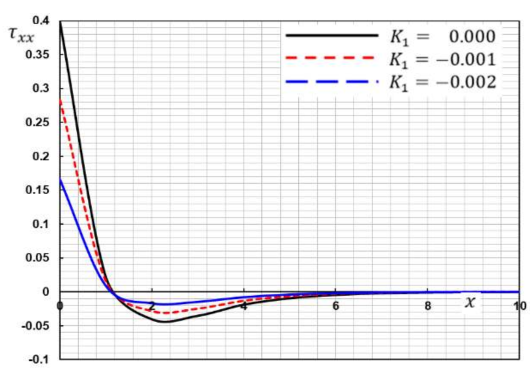

Figure 7. The effect of the variation of thermal conductivity on nonlocal stress τ_{xx}.

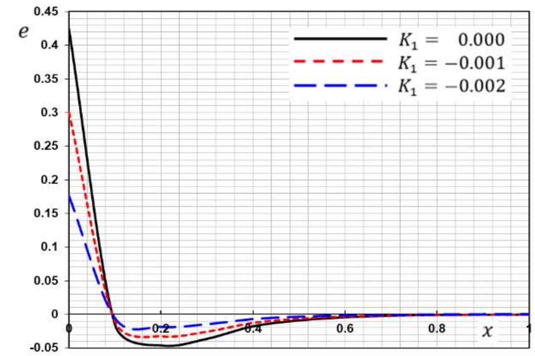

Figure 8. The effect of the variation of thermal conductivity on strain distributions e.

Also, it is noticed that the strain and nonlocal stress show an increase nature to increase or decrease amplitude with respect to distance x due to the presence of the parameter K_1. Figure 6 suggests that the parameter K_1 acts to increase the values of displacement u. This means the effects of variable material properties on thermoelastic response are mainly reflected in the distributions of displacement, temperature and stresses [40].

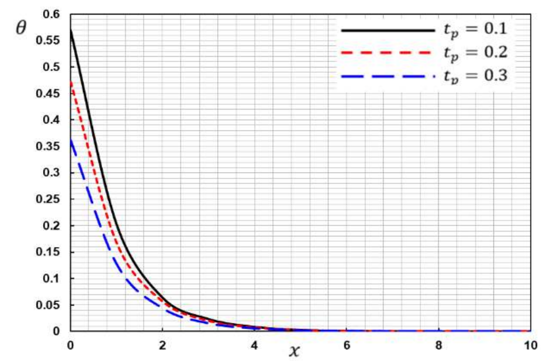

Figure 9. The effect of the pulse rises time of heat flux t_p on temperature θ.

6.3. The effect of the pulse rises time of heat flux

The third case is to investigate how the dimensionless displacement, temperature and nonlocal stress vary with different values of the pulse rise time of heat flux $t_p = 0.1, 0.2, 0.3$ when other parameters remain constants. The numerical results are obtained and presented graphically in Figs. 9-12. From these figures it is observed that the nature of variations of all the field variables for the duration of the pulse parameter t_p is significantly different. It can be observed that the pulse rise time t_p has a great effect on the displacement, temperature, strain and nonlocal stress distributions. Our results agree with those discussed in the literature for axially symmetric laser sources [41-43].

Also, we can conclude that increasing in the value of the duration of the pulse parameter t_p causes decreasing in the values of the displacement, temperature and strain which is obvious in the peak points of the curves. The increasing on the value of the pulse rise time of heat flux t_p causes increases in the values of the nonlocal stress field which is obvious in the starting points of the curves.

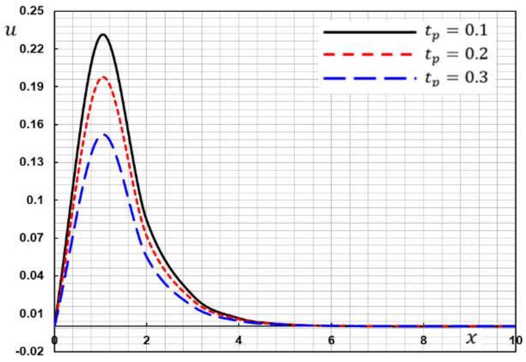

Figure 10. The effect of the pulse rises time of heat flux t_p on displacement u.

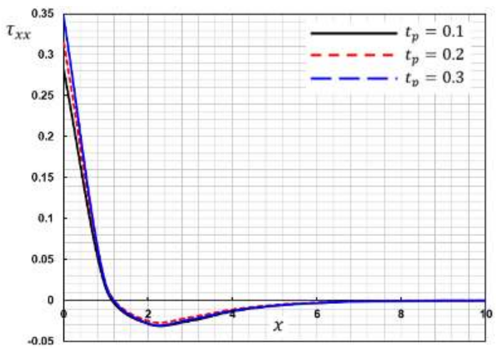

Figure 11. The effect of the pulse rises time of heat flux t_p on nonlocal stress τ_{xx}.

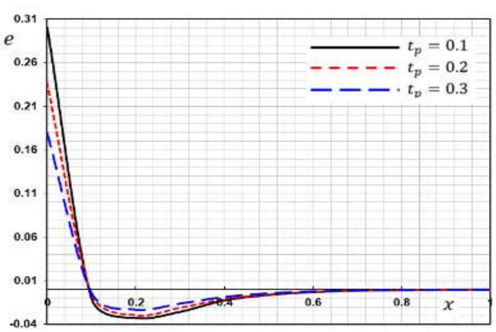

Figure 12. The effect of the pulse rises time of heat flux t_p on strain distributions e.

6.4. The influences of the non-Fourier heat conduction

The effects of the relaxation time τ_0 on the non-dimensional temperature, the strain, the nonlocal stress and the displacement distributions with location x are depicted in Figures 13-16. It is noted that $\tau_0 = 0$ reduces the non-Fourier heat conduction model to the Fourier model. The effect of relaxation time τ_0 is very much obvious near the application point of the pulse heat flux $q(x,t)$ and it dies out when spatial coordinates are increased. Numerical results show that the non-Fourier heat conduction model has great effects on the transient temperature, the strain, the nonlocal stress and the displacement fields. Furthermore, it is observed that, the increasing on the value of the relaxation time parameter τ_0 causes decreasing in the values of the displacement which is obvious in the peak points of the curves. The nonlocal stress intensity for non-Fourier is higher than that for the classical Fourier model [44]. It is clearly indicated that the decay of temperature becomes sharp as τ_0 becomes smaller. We can conclude that the rapidity of the heat propagation is greater for smaller τ_0 [45].

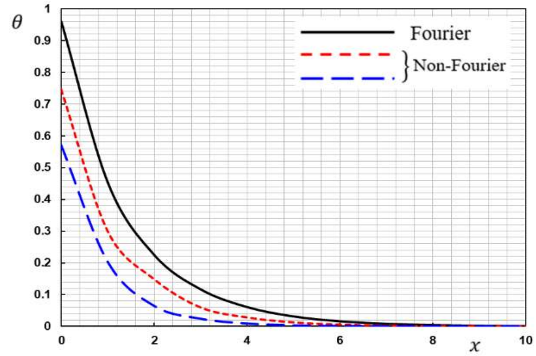

Figure 13. The temperature θ for non-Fourier heat conduction.

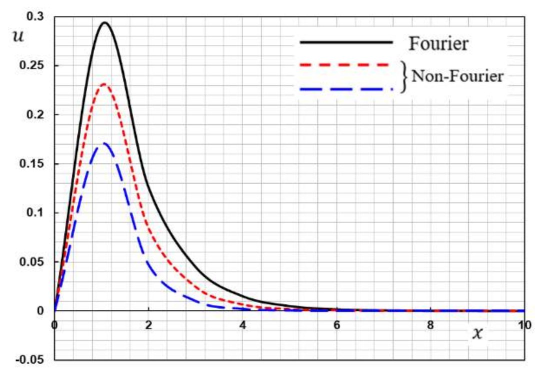

Figure 14. The displacement u for non-Fourier heat conduction.

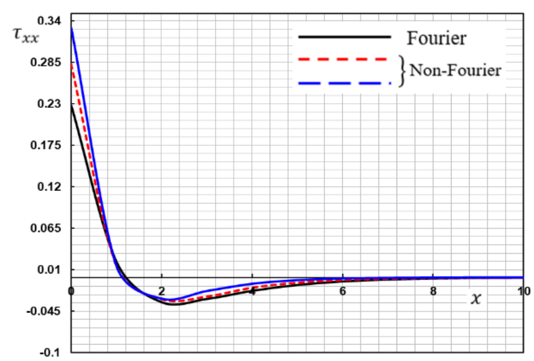

Figure 15. The nonlocal stress τ_{xx} for non-Fourier heat conduction.

7. Conclusions

This work is concerned with the thermoelastic behaviors with variable thermal material properties. The problem is in the context of non-Fourier heat conduction and Eringen's nonlocal elasticity model. The governing equations with variable thermal properties were solved by means of the Laplace transform technique. The results are demonstrated graphically to explain the effect of nonlocal parameter, the variation of thermal conductivity, the pulse rise time of heat flux and the relaxation time. The results indicate that the field quantities such as the temperature, the strain, the nonlocal stress and the displacement fields depend not only on the space coordinate x and time t, but also depend on the nonlocal parameter, the variation of thermal conductivity, the pulse rise time and the relaxation time.

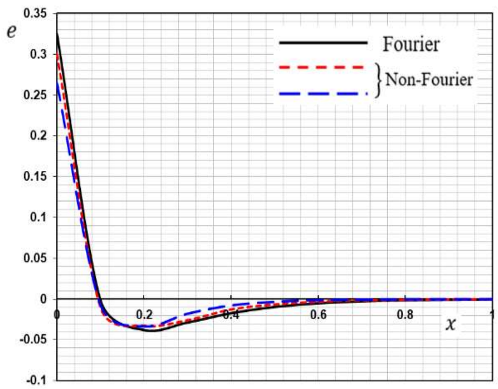

Figure 16. The strain distributions e for non-Fourier heat conduction.

The results obtained here will be useful for researchers in nonlocal material science, low-temperature physicists, new materials designers, as well as to those who are working on the development of the theory of nonlocal thermoelasticity.

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