Journal of Productivity Analysis, 19, 5–32, 2003


# Nonparametric Dynamic Production Analysis and the Theory of Cost

ELVIRA SILVA
*Faculty of Economics of Porto, University of Porto, Research Center on Labor and Firm Economics (CETE), Rua Dr. Roberto Frias, 4200 Porto, Portugal*

SPIRO E. STEFANOU*
*Department of Agricultural Economics and Rural Sociology, The Pennsylvania State University, Armsby Building, University Park, PA 16802*

*Abstract*

While the dynamic theory of production provides little insight towards identifying a specific functional form for the firm's technology, dynamic production analysis has been explored traditionally in a parametric framework. A nonparametric dynamic dual cost approach to production analysis is developed in this article. Recovering technological information from intertemporal cost minimizing behavior is possible without imposing a parametric functional form on the firm's technology. Nonparametric tests to analyze the structure of a dynamic technology are presented from a dynamic cost minimizing perspective. The empirical implementation of these tests is illustrated for a balanced panel data set of Pennsylvania dairy operators during the time period 1986–1992.

**JEL classification:** D21, C61, D92, Q12

**Keywords:** dynamic cost approach, nonparametric production analysis

## 1. Introduction

While the dynamic theory of production provides little insight towards identifying a specific functional form for the firm's technology, dynamic production analysis has been explored traditionally in a parametric framework. The parametric approach analyzes the production structure by explicitly or implicitly specifying a functional form for the technology of the firm. Consequently, some production structure characteristics are imposed *a priori* on the data rather than formulated as empirical hypotheses.

The evolution towards flexible functional forms for the production, cost or profit functions reflects the analyst's perceived ignorance of the production process and the need to impose as few restrictions as possible on the data. Characterization of the production structure may be different across functional forms and, consequently, policy implications derived from

* Corresponding author.

empirical studies on the production structure can vary. The vulnerability of the empirical results to the *ad hoc* specification of the technology motivates investigation into a free-functional form methodology.

Considerable effort has been devoted to the theoretical development of the nonparametric approach to the static theory of production leading to a proliferation of empirical work using this approach. While Farrell (1957) is the forerunner of the nonparametric revealed preference approach to production analysis, Afriat (1972) and Hanoch and Rothschild (1972) lay the foundation for the more complete primal/dual nonparametric characterizations of production behavior by developing a set of inequalities that must be satisfied by observed prices, input decisions and output realizations that would be generated by optimizing (cost minimizing and profit maximizing) behavior. Computationally convenient tests developed by Diewert and Parkan (1983) and Varian (1984) broadened the power of the revealed preference approach to production analysis.

The static models of production are based on the firm's ability to adjust instantaneously and ignore the intertemporal linkage of production decisions. Sluggish adjustment in some factors of production is present in many industries. The weakness underlying the static theory of production in explaining how some inputs are gradually adjusted has led to the development of the dynamic models of production where the impact of current production decisions constrain or enhance future production possibilities.

The adjustment-cost model of the firm provides a consistent dynamic theoretical framework to analyze the firm's behavior and the underlying production technology. The adjustment-cost approach is developed initially by Eisner and Strotz (1963) and further elaborated, among others, by Lucas (1967), and Treadway (1969, 1970). Important theoretical contributions to the deterministic analysis of the dynamics of economic adjustment include the presentation of the dynamic dual approach (McLaren and Cooper, 1980), the discussion of the aggregation of quasi-fixed factors (Epstein, 1983; Blackorby and Schworm, 1983) and the aggregation of technologies across firms to model an aggregate technology (Blackorby and Schworm, 1982).

Two observed directions in the production literature contribute to this research. First, the theoretical and empirical relevance of the dynamic models of production provide a more complete description of decision making. However, the empirical implementation of dynamic models of production decision making exploit parametric methods, almost exclusively. Second, the free-functional form feature of nonparametric methods and the application of these methods in the context of the static theory of production provide a useful starting point for the dynamic generalization of production analysis. These directions motivate the construction of a unified nonparametric approach to the dynamic theory of production from a cost perspective.

A nonparametric dynamic dual cost framework to production analysis is developed based on a generalization of the nonparametric static dual cost approach proposed by Varian (1984). Recovering technological information from intertemporal cost minimizing behavior is possible without imposing implicitly a parametric functional form on the firm's production technology. Nonparametric tests to check for consistency of a data series with dynamic cost minimizing behavior and to analyze the structure of a dynamic technology are developed. The empirical implementation of these nonparametric tests is illustrated for a balanced panel data set of Pennsylvania dairy operators during the time period 1986–1992.

## 2. Dynamic Cost Minimization and Recovery of Technological Information

The dynamic cost function reflects the properties of the underlying technology. A well-behaved technology can be described by a family of input requirement sets or by a production function satisfying some regularity conditions.

Let $y(t)$ denote the maximum amount of output a firm can produce at time $t$, given the $m$ dimensional vector of variable inputs $x(t)$, the $o$-vector of gross investments $I(t)$, and the $o$-vector of initial capital stocks $k(t)$ at time $t$. Let $V(y(t):k(t))$ represent the input requirement set for $y(t)$ given the initial capital stock vector $k(t)$.

A well-behaved technology can be described by a family of input requirement sets, $\{V(y(t):k(t))\}$, satisfying the following properties

A.1 Closeness: Any sequence of points in $V(y(t):k(t))$ converges to a point in this set.

A.2 Nestedness in $y(t)$: If $(x(t), I(t)) \in V(y(t):k(t))$ and $y(t) \geq y'(t)$, then $(x(t), I(t)) \in V(y'(t):k(t))$.

A.3 Positive monotonicity in $x(t)$: If $(x(t), I(t)) \in V(y(t):k(t))$ and $x'(t) \geq x(t)$, then $(x'(t), I(t)) \in V(y(t):k(t))$.

A.4 Negative monotonicity in $I(t)$: If $(x(t), I(t)) \in V(y(t):k(t))$ and $I(t) \geq I'(t)$, then $(x(t), I'(t)) \in V(y(t):k(t))$.

A.5 Convexity in $(x(t), I(t))$: $\forall(x(t), I(t)), (x'(t), I'(t)) \in V(y(t):k(t))$, and $\forall\mu \in [0,1](\mu x(t) + (1-\mu)x'(t), \mu I(t) + (1-\mu)I'(t)) \in V(y(t):k(t))$.

A.6 Reverse Nestedness in $k(t)$: If $(x(t), I(t)) \in V(y(t):k(t))$ and $k'(t) \geq k(t)$, then $(x(t), I(t)) \in V(y(t):k'(t))$.

Alternatively, a production function can be used to describe the technology. The usual regularity conditions assumed for the production function, $F(x(t), I(t), k(t))$, are: (B.1) $F_x, F_k > 0$, (B.2) $F_I < 0$, (B.3) $F(x(t), I(t), k(t))$ is concave in $(x(t), I(t), k(t))$, and (B.4) its domain is bounded and open.

Assumption A.4 and convexity of $V(y(t):k(t))$ in $I(t)$, or, alternatively assumption B.2 and concavity of $F(x(t), I(t), k(t))$ in $I(t)$ reflect the existence of internal adjustment costs as a reduction in physical output. The adjustment cost hypothesis states current additions to the capital stock are output decreasing (assumption A.4 or B.2) and the more rapidly the quasi-fixed factors are adjusted the greater the cost (convexity of $V(y(t):k(t))$, or concavity of $F(x(t), I(t), k(t))$ in $I(t)$). Convexity of $V(y(t):k(t))$ in $I(t)$ (or, concavity of $F(.)$ in $I(t)$) leads to sluggish adjustment in the quasi-fixed factors since it implies an increasing marginal cost of adjustment. However, current additions to the capital stock increase output in the future by increasing the future stock of capital (assumption A.6, or $F_k > 0$).

Theorems 1, 12 and 13 in Varian (1984, pp. 581 and 591) are generalized to a dynamic framework. Generalization of these theorems establishes a nonparametric dynamic dual cost framework to production analysis. In particular, generalization of theorem 1 provides a testable necessary and sufficient condition that can be used to test the consistency of a particular data set with intertemporal cost minimizing behavior. If the data available are consistent with intertemporal cost minimization, the dynamic version of theorems 12 and 13 provides a means to recover technological information from intertemporal cost minimizing behavior.

### *2.1. Nonparametric Test for Dynamic Cost Minimization*

Consider the following data set

$$S^c = \{(y^i(t), x^i(t), I^i(t), k^i(t), w^i(t), c^i(t)); \quad i = 1, \ldots, n; t = 1, \ldots, T\}$$

representing the observed behavior. The data series, $S^c$, provides $nT$ observations on the output level $y^i(t)$ and the associated $m$-vectors of perfectly variable inputs $x^i(t)$, the $o$-vectors of gross investments $I^i(t)$ and the $o$-vectors of the initial capital stocks $k^i(t)$. $w^i(t)$ and $c^i(t)$ represent the perfectly variable input price vector and the quasi-fixed factor price vector, respectively for observation $i$ at time $t$.

*Definition 1.* A family of input requirement sets cost-rationalizes the data set $S^c$ if $(x^i, I^i)$, $i = 1, \ldots, n$, solves the following problem (the Hamilton-Jacobi-Bellman equation or dynamic programming equation (DPE))[1]

$$rW(w^i, c^i, y^i, k^i) = \min_{x,I} \left\{ w^{i'}x + c^{i'}k^i + W_k^{i'}(I - \delta k^i) : (x, I) \in V(y^i : k^i) \right\}$$

or, equivalently, if

$$w^{i'}x + W_k^{i'}I \geq w^{i'}x^i + W_k^{i'}I^i; \quad \text{for all } (x, I) \in V(y^i : k^i), \quad i = 1, \ldots, n$$

where $rW(w^i, c^i, y^i, k^i)$ is a flow version of the intertemporal cost, $W_k^i = W_k(w^i, c^i, y^i, k^i)$ is the vector of the shadow-value of capital, and

$$W(w^i, c^i, y^i, k^i) = \min_{x,I} \left\{ \int_0^\infty e^{-rt}[w^{i'}x + c^{i'}K^i]\, dt : \dot{K}^i = I - \delta K^i;\ K^i(0) = k^i; \right.$$
$$\left. (x, I) \in V(y^i : K^i) \right\}$$

with $r$ being the discount rate, $\dot{K} = dK/dt$ the vector of net investment and $\delta$ the depreciation rate.[2]

THEOREM 1 *The following conditions are equivalent: (1) There exists a family of nested and reverse nested input requirement sets, $\{V(y : k)\}$, cost-rationalizing the data set $S^c$. (2) If $y^j \geq y^i$, $k^i \geq k^j$, then $w^{i'}x^j + W_k^{i'}I^j \geq w^{i'}x^i + W_k^{i'}I^i$ for all $i$ and $j$. (3) There exists a family of nontrivial, closed, convex input requirement sets, positive monotonic in $x$ and negative monotonic in $I$, cost-rationalizing the data set $S^c$.*[3]

Theorem 1 states condition (2) is necessary and sufficient for the data set $S^c$ to be consistent with intertemporal cost minimizing behavior. Thus, if condition (2) holds at all data points, $S^c$ is said to be consistent with intertemporal cost minimizing behavior. Condition (2) is denoted hereafter as the Weak Axiom of Dynamic Cost Minimization (WADCM).

The WADCM is checked under two hypotheses: (1) observations are perfect measurements and (2) presence of measurement errors in the data. In case (1), the nonparametric test is deterministic. A stochastic test of the type proposed by Varian (1995) is conducted assuming (2).

***2.1.1. Deterministic Test for Intertemporal Cost Minimizing Behavior*** Assuming observations are perfect measurements, the WADCM depends on directly observed variables, $(w^i, x^i, I^i, y^i, k^i)$, and on the unobservable shadow value of capital, $W_k^i$. The shadow value of capital underlying the observed production and investment decisions of the firm, denoted as the behavioral shadow value of capital, can be estimated using a nonparametric regression method (e.g., the kernel estimation method).[4] Given the behavioral shadow value of capital, $W_k^{bi}$, for each observation, the WADCM can be checked for the data series $S^c$ as follows

$$\text{If } y^j \geq y^i, k^i \geq k^j, \text{ then } w^{i'} x^j + W_k^{bi'} I^j \geq w^{i'} x^i + W_k^{bi'} I^i; \text{ for all } i \text{ and } j.$$

By theorem 1, $S^c$ is said to be consistent with intertemporal cost minimizing behavior if and only if the WADCM holds at all data points. This test is deterministic since no probability assessments are implied and is a diagnostic checking whether the data are fully consistent with the intertemporal cost minimization hypothesis.

***2.1.2. Stochastic Test for Intertemporal Cost Minimizing Behavior*** For the data series $S^c$, define the null hypothesis as the "true" data that is consistent with intertemporal cost minimizing behavior. Assuming only input demand data is measured with error, the observed quantity variables $x^i$ and $I^i$ can be related to the "true" variables as follows

$$\begin{aligned} x_l^i &= \xi_l^i + \varepsilon_{vl}^i, \quad l = 1, \ldots, m; \\ I_h^i &= \eta_h^i + \varepsilon_{qh}^i, \quad h = 1, \ldots, o; \end{aligned} \tag{1}$$

$i = 1, \ldots, n$. $\xi_l^i$ is the "true" input demand of the variable input $l$ at observation $i$, $\eta_h^i$ is the "true" gross investment of the quasi-fixed factor $h$ at observation $i$ and $\varepsilon_{vl}^i$ and $\varepsilon_{qh}^i$ are random errors assumed to be independently and identically distributed as $N(0, \sigma^2)$.

Given the assumptions in (1), the WADCM depends on observed variables $(w^i, y^i, k^i)$ and a set of unobservable variables, $(\xi^i, \eta^i, W_k^i)$. The WADCM can be checked by running the following quadratic programming problem

$$S = \min_{\zeta_l^i, \eta_h^i, W_{k_h}^i} \left\{ \sum_{i=1}^{n} \left[ \sum_{l=1}^{m} \left(\zeta_l^i - x_l^i\right)^2 + \sum_{h=1}^{o} \left(\eta_h^i - I_h^i\right)^2 \right] : \zeta^i \geq 0; \eta^i \geq 0; \right.$$
$$\left. w^{i'} \zeta^j + W_k^{i'} \eta^j \geq w^{i'} \zeta^i + W_k^{i'} \eta^i, y^j \geq y^i, k^i \geq k^j \right\}. \tag{2}$$

Note that in problem (2), the shadow value of capital is one of the variables to be determined while testing for dynamic cost minimizing behavior. Rejection of the null hypothesis occurs when $S/\sigma^2 > C_\alpha$, or $\sigma^2 < S/C_\alpha$, where $C_\alpha$ is the $\alpha\%$ critical value from the $\chi^2$ table for $n(m+o)$ degrees of freedom. Define $\overline{\sigma^2} = S/C_\alpha$ as the critical value of $\sigma^2$, whose value is obtained after solving (2). If the error variance of the input demand data is known and less than $\overline{\sigma^2}$, the null hypothesis is rejected. Thus, the stochastic test in (2) provides a range for the error variance over which the data set $S^c$ is consistent with intertemporal cost minimizing behavior.

### *2.2. Inner and Outer Bounds on the Technology*

Consider the data set $S^c$ and assume consistency with intertemporal cost minimizing behavior. Let $\{V(y:k)\}$ be the "true" family of input requirement sets that cost-rationalizes $S^c$. By theorem 1, $\{V(y:k)\}$ represents a well-behaved technology and satisfies properties A.1–A.6.

A family of input requirement sets can be constructed from the data set $S^c$, without assuming a parametric functional form on the production technology. However, a family of input requirement sets satisfying properties A.1–A.6 and constructed from observed data is not unique. In fact, there are many families of input requirement sets satisfying the aforementioned properties that can be derived from a finite number of data points (Varian, 1984). Nevertheless, outer and inner bounds can be derived on the technological possibilities underlying the data set $S^c$. Two families of input requirement sets, $\{V_I(y:k)\}$ and $\{V_O(y:k)\}$, provide the tightest inner and outer bounds on the "true" technology underlying the data set $S^c$, respectively; i.e., $V_I(y:k) \subset V(y:k) \subset V_O(y:k)$ for all $y$ and $k$.

$V_I(y:k)$ can be constructed as the convex monotonic hull of $(x^i, I^i)$ such that $y^i \geq y$ and $k \geq k^i$; i.e.,

$$V_I(y:k) = com\{z^i + e^i : y^i \geq y; k^i \leq k; z^i = (x^i, I^i); e^i = \left(e_x^i, -e_I^i\right) \geq 0\} \tag{3}$$

if such $y^i$ and $k^i$ exist and $V_I(y:k) = \emptyset$, otherwise. In the proof of theorem 1, $V_I(y:k)$ is shown to cost-rationalize the data and theorem 2 establishes that $\{V_I(y:k)\}$ is the tightest inner bound to $\{V(y:k)\}$.

THEOREM 2 *The following statements are true: (1) $\{V_I(y:k)\}$ cost-rationalizes the data set $S^c$. (2) Let $\{V(y:k)\}$ be a family of closed, convex input requirement sets, positive monotonic in x and negative monotonic in I, cost-rationalizing the data set $S^c$. Then, $V(y:k) \supset V_I(y:k)$ for all $y \leq y^m$ and $k \leq k^m$; where $y^m$ and $k^m$ are the largest observed output level and initial capital stock vector, respectively. (3) Let $V'(y:k)$ be a closed, convex input requirement set, positive monotonic in x and negative monotonic in I, which is strictly contained in $V_I(y:k)$; i.e., $V'(y:k) \subset V_I(y:k)$. Then, $V'(y:k)$ cannot cost-rationalize the data set $S^c$.*

Proceeding in a similar way as Afriat (1972) and Diewert and Parkan (1983) in the static framework, the tightest inner bound in (3) can be constructed as[5]

$$V_I(y:k) = \left\{(x, I) : x \geq \sum_{j=1}^{n} \lambda^j x^j; I \leq \sum_{j=1}^{n} \lambda^j I^j; y^j \geq y; k^j \leq k; \right.$$
$$\left. \sum_{j=1}^{n} \lambda^j = 1; \lambda^j \in \mathbb{R}^+, \forall j \right\}. \tag{4}$$

Generalization to the dynamic framework of the outer bound proposed by Varian (1984) in the static context leads to

$$V_O(y:k) = \left\{(x, I) : w^{i'}x + W_k^{i'}I \geq w^{i'}x^i + W_k^{i'}I^i;\ y \geq y^i;\ k \leq k^i\right\}. \tag{5}$$

Theorem 3 establishes that $\{V_O(y:k)\}$ is the tightest outer bound to $\{V(y:k)\}$.

THEOREM 3 *The following statements are true: (1) $\{V_O(y:k)\}$ cost-rationalizes the data set $S^c$. (2) If $\{V(y:k)\}$ is a family of input requirement sets that cost-rationalizes the data set $S^c$, then $V_O(y:k) \supset V(y:k)$ for $y \leq y^m$ and $k \leq k^m$. (3) If $V'(y:k)$ is an input requirement set strictly containing $V_O(y:k)$ (i.e., $V'(y:k) \supset V_O(y:k)$) then $V'(y:k)$ cannot rationalize the data set $S^c$.*

Figure 1 illustrates the case of one variable input and one quasi-fixed factor. The inner and outer bounds on the "true" production technology are constructed from two observed data points $z^*$ and $z'$. The inner bound, $V_I(y:k)$, is determined by (4) as the intersection of closed upper half-spaces. The hyperplane determining the half-spaces are generated by convex combinations of observed input data points with an output level at least as great as $y$ and an initial capital stock vector less than or equal to $k$. Assuming the output level associated with the two observed data points is at least as great as $y$ and the underlying initial capital stock vector is less than or equal to $k$, the inner bound is the area on and above the line segments $[a, z', z^*, b]$. Similarly, the outer bound is determined by (5) as the intersection of closed upper half-spaces. The hyperplane determining the half-spaces is an isocost plane (or isocost line for the case of only two inputs) with slope equal to $(-W_k^i/w^i)$. In Figure 1, the outer bound is defined as the area on and above the line segments $[c, d, e]$.

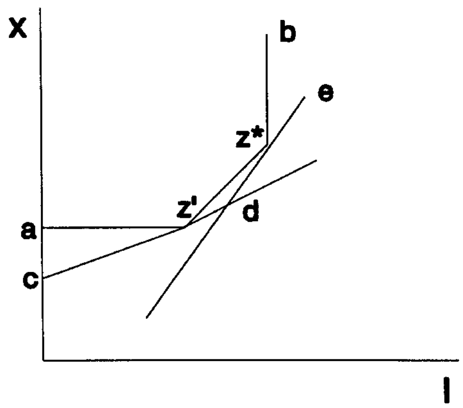


*Figure 1.* Inner and outer bounds.

### 2.3. *Dynamic Undercost and Overcost Functions*

The dynamic cost function reflects the properties of the underlying production technology by the duality theory. Given the inner and outer bounds on the technological possibilities underlying the data series $S^c$, lower and upper bounds on the dynamic cost function can be established.

Define the flow version of the intertemporal cost minimization problem in $V_I(y:k)$ as

$$rW(w^i, c^i, y^i, k^i, V_I) = \min_{x,I} \left\{ w^{i'}x + c^{i'}k^i + W_k^{i'}(I - \delta k^i) : (x, I) \in V_I(y^i : k^i) \right\}$$

or equivalently,

$$rW(w^i, c^i, y^i, k^i, V_I) = \min_{x,I,\lambda} \left\{ w^{i'}x + c^{i'}k^i + W_k^{i'}(I - \delta k^i) : x \geq \sum_{j=1}^{n} \lambda^j x^j; \right.$$
$$\left. I \leq \sum_{j=1}^{n} t^j I^j;\ y^j \geq y^i;\ k^j \leq k^i;\ \sum_{j=1}^{n} \lambda^j = 1;\ \lambda^j \in \mathbb{R}^+, \forall j \right\}. \tag{6}$$

Similarly, the flow version of the dynamic cost minimization problem in $V_O(y:k)$ is defined as

$$rW(w^i, c^i, y^i, k^i, V_O) = \min_{x,I} \left\{ w^{i'}x + c^{i'}k^i + W_k^{i'}(I - \delta k^i) : (x, I) \in V_O(y^i : k^i) \right\}$$

or, equivalently,

$$rW(w^i, c^i, y^i, k^i, V_O) = \min_{x,I} \left\{ w^{i'}x + c^{i'}k^i + W_k^{i'}(I - \delta k^i) : \right.$$
$$\left. w^{i'}x + W_k^{i'}I \geq w^{i'}x^i + W_k^{i'}I^i;\ y \geq y^i;\ k \leq k^i \right\}. \tag{7}$$

The dynamic cost functions in (6) and (7), respectively, are the upper and lower bounds on the “true” intertemporal cost function; i.e.,

$$rW(w^i, c^i, y^i, k^i, V_O) \leq rW(w^i, c^i, y^i, k^i, V) \leq rW(w^i, c^i, y^i, k^i, V_I) \tag{8}$$

for all $i \in S^c$. These weak inequalities are derived from the relation between the three input requirement sets; i.e., $V_I(y:k) \subset V(y:k) \subset V_O(y:k)$ for all $y$ and $k$. The dynamic cost functions in (6) and (7) are called the dynamic overcost and undercost functions, respectively.

## 3. Dynamic Cost Minimization and a Constant Returns to Scale Technology

The regularity conditions on the firm's production technology discussed previously (A.1–A.6 for a family of input requirement sets, or B.1–B.4 for the production function) are general properties common to any well-behaved technology. The structure of a well-behaved production technology can be described by more restrictive properties such as constant returns to scale (CRS).

A nonparametric test for CRS is presented from a dynamic dual cost approach and implementation of this nonparametric test in empirical work is discussed. If the data series is consistent with a CRS technology and intertemporal cost minimizing behavior, a CRS production function can be derived from the observed data without assuming implicitly a parametric functional form for the firm's production technology.

### 3.1. *Nonparametric Test for Constant Returns to Scale*

Theorem 5 in Varian (1984, p. 585) is generalized to a dynamic framework. The dynamic version of this theorem provides a testable necessary and sufficient condition that can be used to check whether the data set $S^c$ is consistent with a CRS technology and intertemporal cost minimizing behavior.

*Definition 2.* A production function $F(x, I, k)$ is said to cost-rationalize the data set $S^c$ if

$$y^i = F(x^i, I^i, k^i),\; F(x, I, k) \geq y^i,\; k^i \geq k \Rightarrow w^{i'} x + W_k^{i'} I \geq w^{i'} x^i + W_k^{i'} I^i; \quad i = 1, \ldots, n.$$

THEOREM 4 *The following conditions are equivalent: (1) There exists a linearly homogeneous production function cost-rationalizing the data set $S^c$. (2) $\frac{w^{j'} x^i + W_k^{j'} I^i}{w^{j'} x^j + W_k^{j'} I^j} \geq \frac{y^i}{y^j}$, $k^j \geq k^i$; for all $j = 1, \ldots, n$. (3) There exists a linearly homogeneous, continuous, concave production function, positive monotonic in $x$ and negative monotonic in $I$, cost-rationalizing the data set $S^c$.*

Theorem 4 states condition (2) is necessary and sufficient for the data set $S^c$ to be consistent with a CRS technology. In addition, a production function satisfying properties B.1–B.4 and the linear homogeneity property can be generated nonparametrically from the data set $S^c$.

**3.1.1. *Deterministic Test for Constant Returns to Scale*** Assuming no measurement errors in the data, condition (2) depends on directly observed variables ($w^i, x^i, I^i, y^i, k^i$) and on the unobservable variable, $W_k^i$. Given the behavioral shadow value of capital, condition (2) can be checked for all observations in $S^c$.[6] Since this test is deterministic, failure of this condition to hold for all $i \in S^c$ implies inconsistency with CRS and intertemporal cost minimization.

If the data set did not include market input prices, the existence of a valid CRS technology could also be tested using only quantity data. In this case, run the following linear programming problem

$$\min_{p_v^i, p_k^i, W_k^i} \Big\{ \gamma : \Lambda^1 \varphi^1 + v^1 k^1 = y^1,\; y^1 = \min_{i \in S^c} y^i;\; \Lambda^i \varphi^j \geq \Lambda^i \varphi^i,\; y^j \geq y^i,\; k^i \geq k^j;$$
$$\Lambda^l (\varphi^i / y^i) + v^l k^i - \Lambda^i (\varphi^i / y^i) - v^i k^i + \gamma \geq 0,\; i \neq l;\; p_v^i \geq 0;\; p_k^i \geq 0;\; \gamma \geq 0 \Big\} \quad (9)$$

where $\Lambda^i = (p_v^i, W_k^i)$, $\varphi^i = (x^i, I^i)$, $v^i = (p_k^i - \delta W_k^i)$, and $p_v^i$ and $p_k^i$ are the variable input price vector and the quasi-fixed input price vector, respectively, $i = 1, \ldots, n$. The first constraint is a normalization that does not bias the result of the test. The data set $S^c$ is said to be consistent with a valid CRS technology if and only if $\gamma = 0$. If $\gamma > 0$, $\gamma$ is an index of the violation of CRS.

***3.1.2. Stochastic Test for Constant Returns to Scale*** A stochastic test can be conducted to account for the possibility of measurement errors in the data. Consider the data series $S^c$ and define the null hypothesis as the "true" data is consistent with intertemporal cost minimizing behavior and a CRS technology. Assuming only input demand data is measured with error, define the observed demand for each input as in (1). Run the following quadratic programming problem for the set $S^c$

$$S = \min_{\zeta_l^i, \eta_h^i, W_{k_h}^i} \Bigg\{ \sum_{i=1}^{n} \left[ \sum_{l=1}^{m} \left(\zeta_l^i - x_l^i\right)^2 + \sum_{h=1}^{o} \left(\eta_h^i - I_h^i\right)^2 \right] : \Delta^1 \psi^1 + \upsilon^1 k^1 = y^1,\ y^1 = \min_{i \in S^c} y^i;$$
$$\Delta^i \psi^j \geq \Delta^i \psi^i,\ y^j \geq y^i,\ k^i \geq k^j;\ \Delta^i(\psi^l/y^l) + \upsilon^i k^l \geq \Delta^l(\psi^l/y^l) + \upsilon^l k^l,\ i \neq l;\ \psi^i \geq 0 \Bigg\} \tag{10}$$

where $\Delta^i = (w^i, W_k^i)$, $\upsilon^i = (c^i - \delta W_k^i)$, and $\psi^i = (\zeta^i, \eta^i)$, $i = 1, \ldots, n$. The first constraint is a normalization that does not bias the result of the test. This test is conducted in a similar way as the stochastic test in (2). The stochastic test in (10) provides a range for the error variance over which the set $S^c$ is consistent with intertemporal cost minimization and a CRS technology.

### *3.2. Inner and Outer Bounds on the CRS Technology*

Consider the set $S^c$ and assume consistency with a CRS technology and intertemporal cost minimizing behavior. A linearly homogeneous production function satisfying properties B.1–B.4 can be generated from the observed data without imposing a parametric functional form on the technology. Similarly, a family of input requirement sets satisfying properties A.1–A.6 and the linear homogeneity property can also be derived, in a nonparametric fashion, from $S^c$.

Let $\{V_c(y : k)\}$ be the "true" family of CRS input requirement sets underlying the data series $S^c$. By theorems 2 and 3, inner and outer bounds can be established on $V_c(y : k)$. The tightest inner bound on the CRS production technology underlying the set $S^c$, $V_{cI}(y : k)$, is generated in a similar way as $V_I(y : k)$ in (4) by deleting the constraint that the sum of the $\lambda$'s is equal to one. The tightest outer bound, $V_{cO}(y : k)$, is generated as follows

$$V_{cO}(y : k) = \left\{(x, I) : w^{i'} x(y^i/y) + W_k^{i'} I(y^i/y) \geq w^{i'} x^i + W_k^{i'} I^i;\ k \leq k^i\right\}. \tag{11}$$

Bounds on the "true" CRS production function can then be constructed. Define the CRS overproduction and underproduction functions, respectively, as

$$\begin{aligned} F^+(x, I, k) &= \max\{y : (x, I) \in V_{cI}(y : k)\}, \\ F^-(x, I, k) &= \max\{y : (x, I) \in V_{cO}(y : k)\}. \end{aligned} \tag{12}$$

Using the relation between the input requirement sets, one can bound the "true" CRS production function as follows

$$F^{-}(x, I, k) \leq F(x, I, k) \leq F^{+}(x, I, k). \tag{13}$$

### *3.3. Dynamic Undercost and Overcost Functions for a CRS Technology*

The "true" dynamic cost function is related with the "true" technology by the duality relations. Given the inner and outer bounds on the "true" CRS technology, upper and lower bounds can be established on the "true" dynamic cost function.

Define the flow version of the intertemporal cost minimization problem in $V_{cI}(y : k)$ as follows

$$rW(w^i, c^i, y^i, k^i, V_{cI}) = \min_{x,I} \left\{ w^{i'}x + c^{i'}k^i + W_k^{i'}(I - \delta k^i) : (x, I) \in V_{cI}(y^i : k^i) \right\}.$$

This minimization problem can be expressed as the one in (6) by deleting the constraint that the sum of the $\lambda$'s is equal to one.

Proceeding in a similar fashion, the flow version of the intertemporal cost minimization problem in $V_{cO}(y : k)$ is defined as

$$rW(w^i, c^i, y^i, k^i, V_{cO}) = \min_{x,I} \left\{ w^{i'}x + c^{i'}k^i + W_k^{i'}(I - \delta k^i) : (x, I) \in V_{cO}(y^i : k^i) \right\}$$

which is equivalent to the following problem

$$\begin{aligned} rW(w^i, c^i, y^i, k^i, V_{cO}) = \min_{x,I} \big\{ & w^{i'}x + c^{i'}k^i + W_k^{i'}(I - \delta k^i) : w^{i'}x(y^i/y) \\ & + W_k^{i'}I(y^i/y) \geq w^{i'}x^i + W_k^{i'}I^i; k \leq k^i \big\}. \end{aligned} \tag{14}$$

Upper and lower bounds on the "true" dynamic cost function under CRS are established using the relation between the input requirement sets; i.e.,

$$rW(w^i, c^i, y^i, k^i, V_{cO}) \leq rW(w^i, c^i, y^i, k^i, V_c) \leq rW(w^i, c^i, y^i, k^i, V_{cI}) \tag{15}$$

for all $i \in S^c$. The upper and lower bounds are called the dynamic overcost and undercost functions, respectively, under CRS.

## 4. Dynamic Cost Minimization and a Homothetic Technology

A nonparametric test for homotheticity and intertemporal cost minimizing behavior is presented followed by the discussion of its implementation in empirical work. If the data series is consistent with the homotheticity property and dynamic cost minimizing behavior, a homothetic technology can be derived from the observed data without assuming implicitly a parametric functional form on the firm's technology.

### *4.1. Nonparametric Test for Homotheticity*

Theorem 7 in Varian (1984, p. 586) is generalized to a dynamic framework. The dynamic version of this theorem provides a necessary and sufficient condition that can be used to check the consistency of a particular data set with a homothetic production function and intertemporal cost minimizing behavior.

*Definition 3.* A production function $F(x, I, k)$ is homothetic if there exists a monotonic function $M(\cdot)$ such that $F(x, I, k) = M(h(x, I, k))$, where $h(\cdot)$ is a homogeneous function of degree 1 in $(x, I)$.

THEOREM 5 *The following conditions are equivalent: (1) There exists a homothetic production function cost-rationalizing the data set $S^c$. (2) There exist numbers $h^i$ which are increasing in $y^i$ (i.e., $y^i > y^j$ implies $h^i > h^j$) such that $\frac{w^{j\prime} x^i + W_k^{j\prime} I^i}{w^{j\prime} x^j + W_k^{j\prime} I^j} \geq \frac{h^i}{h^j}$, $k^j \geq k^i$, $j = 1, \ldots, n$. (3) There exists a continuous, quasi-concave, homothetic production function, positive monotonic in $x$ and negative monotonic in $I$, $M(h(x, I, k))$, cost-rationalizing the data set $S^c$, with $h(x, I, k)$ being a concave function.*

Theorem 5 generates a testable necessary and sufficient condition (condition (2)) for consistency of the data set with a homothetic production function and intertemporal cost minimizing behavior. If the data set can be generated by a homothetic production function cost-rationalizing the data, it can be generated by a well-behaved homothetic production function. By the same theorem, a well-behaved homothetic production function can be constructed from the observed data in a nonparametric fashion.

***4.1.1. Deterministic Test for Homotheticity*** Assuming no measurement errors in the data, condition (2) in theorem 5 depends on observed variables, $(w^i, x^i, I^i, k^i)$, and on the unobserved variables, $(W_k^i, h^i)$. Construction of a mathematical programming problem is necessary to check whether condition (2) holds for all $i \in S^c$. Given the behavioral shadow value of capital, run the following linear mathematical programming problem for the data set $S^c$[7]

$$\min_{h^i}\{\beta : \vartheta^1 \Omega^1 = h^1; \vartheta^i \Omega^j - h^i \geq 0, y^j \geq y^i, k^i \geq k^j;$$
$$\vartheta^j \Omega^i - h^i + \beta \geq 0, i \neq j; \beta \geq 0; h^i \geq 0\} \tag{16}$$

where $\Omega^i = (x^i, I^i)$, $\vartheta^i = (w^i, W_k^{bi})$, with $W_k^{bi}$ being the behavioral shadow-value of capital, $\Omega^1$ is the input vector associated with $y^1 = \min y^i$, $i = 1, \ldots, n$, and all the other variables are defined as before.

The data set $S^c$ is consistent with intertemporal cost minimizing behavior and a homothetic technology if and only if $\beta = 0$ for each data point. A strictly positive $\beta$ implies inconsistency of $S^c$ with homotheticity, assuming consistency with intertemporal cost minimization.

If the data set did not include market input prices, the existence of a valid homothetic technology could also be tested using only quantity data. Run the following linear programming problem

$$\min_{p_v^i, p_k^i, W_k^i} \{\alpha : \Lambda^1\varphi^1 = e^{y^1}; \Lambda^i\varphi^j \geq \Lambda^i\varphi^i, y^j \geq y^i, k^i \geq k^j;$$
$$\Lambda^i\varphi^j + v^i k^j - \Lambda^j\varphi^j - v^j k^j + \alpha \geq 0, i \neq j; p_v^i \geq 0, p_k^i \geq 0, \alpha \geq 0\} \quad (17)$$

where $e^{yi} = h^i, i = 1, \ldots, n$, and all the other variables are defined as in (9). The first constraint is a normalization that does not bias the result of the test. A valid homothetic technology exists if and only if $\alpha = 0$. If $\alpha > 0$, $\alpha$ is an index of violation of homotheticity.

***4.1.2. Stochastic Test for Homotheticity*** Define the null hypothesis as the "true" data is consistent with intertemporal cost minimizing behavior and the homotheticity property of the production function. Assuming only input demand data is measured with error, define the observed demand for each input as in (1). Run the following quadratic programming problem for the data set $S^c$

$$S = \min_{\zeta_l^i, \eta_h^i, W_{k_h}^i} \left\{ \sum_{i=1}^{n} \left[ \sum_{l=1}^{m} \left(\zeta_l^i - x_l^i\right)^2 + \sum_{h=1}^{o} \left(\eta_h^i - I_h^i\right)^2 \right] : \Delta^1\psi^1 = e^{y^1};\right.$$
$$\left.\Delta^i\psi^j \geq \Delta^i\psi^i, y^j \geq y^i, k^i \geq k^j; \Delta^i\psi^l + \upsilon^i k^l \geq \Delta^l\psi^l + \upsilon^l k^l, i \neq l; \psi^i \geq 0 \right\} \quad (18)$$

where all variables are defined as in (10) and (17). Again, the first constraint is a normalization that does not bias the result of the test.

The stochastic test (18) provides a range for the error variance over which the $S^c$ is consistent with intertemporal cost minimizing behavior and a homothetic technology. If the error variance of the input quantity data is known and falls in this interval, $S^c$ is said to be consistent with intertemporal cost minimizing behavior and the homotheticity property of the production function.

### *4.2. Inner and Outer Bounds on the Homothetic Technology*

Assume $S^c$ is consistent with intertemporal cost minimization and homotheticity. A well-behaved homothetic production function can be constructed in a nonparametric fashion from the observed data.

Using definition 3, let $F(x, I, k) = M(h(x, I, k))$ be the "true" homothetic production function underlying $S^c$, where $h(x, I, k)$ is a homogeneous function of degree 1 in $(x, I)$. Using theorems 2 and 3, inner and outer bounds on the "true" homogeneous technology can be constructed. Construct the tightest inner and outer bounds, respectively, as follows[8]

$$V_{hI}(h : k) = \left\{ (x, I) : x \geq \sum_{j=1}^{n} \lambda^j x^j; I \leq \sum_{j=1}^{n} \lambda^j I^j; h^j \geq h; k^j \leq k; \lambda^j \in \mathbb{R}^+, \forall j \right\} \quad (19)$$

and

$$V_{hO}(h : k) = \left\{(x, I) : w^{i'} x(h^i/h) + W_k^{i'} I(h^i/h) \geq w^{i'} x^i + W_k^{i'} I^i; k^i \geq k\right\}. \quad (20)$$

Define the upper and lower bounds on $h(x, I, k)$ as follows

$$\begin{aligned} h^{+}(x, I, k) &= \max\{h : (x, I) \in V_{hI}(h : k)\}; \\ h^{-}(x, I, k) &= \max\{h : (x, I) \in V_{hO}(h : k)\}. \end{aligned} \tag{21}$$

Since $V_{hI}(h : k) \subset V_h(h : k) \subset V_{hO}(h : k)$, then $h^{-}(x, I, k) \leq h(x, I, k) \leq h^{+}(x, I, k)$.

Consider a monotonic transform $M(\cdot)$ mapping $h(x, I, k)$ into output. The upper and lower bounds on $h(x, I, k)$ can be used to construct bounds on the "true" homothetic production function $F(x, I, k)$. Define the overproduction and underproduction functions, respectively, as follows

$$\begin{aligned} F^{+}(x, I, k) &= \mathrm{M}(h^{+}(x, I, k)); \\ F^{-}(x, I, k) &= \mathrm{M}(h^{-}(x, I, k)). \end{aligned} \tag{22}$$

where $F^{-}(x, I, k) \leq F(x, I, k) \leq F^{+}(x, I, k)$.

### 4.3. *Dynamic Undercost and Overcost Functions for a Homothetic Technology*

By construction, the "true" dynamic cost function is related to the "true" technology. Bounds on the "true" dynamic cost function are obtained by using the homothetic underproduction and overproduction functions in (22).

The upper and lower bounds on the "true" dynamic cost function are, respectively

$$rW(w^i, c^i, y^i, k^i, F^{+}) = \max_{x,I} \left\{ w^{i'} x + c^{i'} k^i + W_k^{i'}(I - \delta k^i) : y^i \leq F^{+}(x, I, k^i) \right\} \tag{23}$$

and

$$rW(w^i, c^i, y^i, k^i, F^{-}) = \min_{x,I} \left\{ w^{i'} x + c^{i'} k^i + W_k^{i'}(I - \delta k^i) : y^i \leq F^{-}(x, I, k^i) \right\}. \tag{24}$$

The dynamic cost functions in (23) and (24) are called the dynamic overcost and undercost functions for a homothetic technology, respectively, and possess the following relation

$$rW(w^i, c^i, y^i, k^i, F^{-}) \leq rW(w^i, c^i, y^i, k^i, F) \leq rW(w^i, c^i, y^i, k^i, F^{+}) \tag{25}$$

for all $i \in S^c$.

## 5. Data

A panel data set of 60 Pennsylvania dairy operators is available for the time period 1986–1992 from the Pennsylvania Farm Bureau (PFB). Information on each farm and for each year is available on the following variables: (1) total pounds of milk sold, (2) the price of milk sold, (3) milk revenue, (4) other farm income, (5) total revenue, (6) herd size of milking cows, (7) hired labor hours/year, (8) hired labor expenses, (9) miscellaneous variable expenses, (10) family labor hours/year, (11) value of land, (12) value of buildings, (13) value of machinery and equipment, (14) value of livestock, (15) value of total assets, (16) total debts, and (17) total interest.

This panel of dairy farms used consists of dairy operators with herd size ranging between 40 and 100 cows with positive profit in all seven years. In addition, these farms derive at least 80 percent of total revenue from dairy operations to ensure that milk output is the dominant or the single output of the farm.

Output is measured by total pounds of milk sold. Herd size in each year is defined as the average number of milking cows in the herd during the year. Miscellaneous variable expenses incorporates several components such as feed purchased, custom work hired, utilities, gas and oil, fertilizer and lime, veterinary and medicine, machinery repair, and crop and seed supplies. The miscellaneous expenses category is taken as a measure of the farm's variable costs other than hired labor. No information is available on the quantities used for the variable inputs other than hired labor. The implicit farm-specific hourly wage rate is determined as the ratio of annual hired labor expenses to hired labor hours per year.

The total farm assets item involves land, buildings, machinery and equipment, livestock, and cash. Land, buildings, machinery and equipment and livestock are reported as stock variables where the reported values are book values. No market value of the farm's assets is available. These assets can be categorized according to their average useful life. Machinery and equipment can be classified as an intermediate-run asset with an average life ranging between 1 and 10 years, where land and buildings can be considered a long-run asset with an average useful life of more than 10 years.

Several quasi-fixed factors are present in the dairy production. The quasi-fixed factors are land, buildings, machinery and equipment, livestock and family labor. The depreciation rates used for buildings, machinery and equipment and livestock are 3%, 10% and 20%, respectively.[9]

Total debt consists of debt for farm operation. No information is provided on the allocation of the farm debts among different uses as well as on the possible different rates of interest associated with specific debts. The implicit farm-specific interest rate is determined as the ratio of total interest payments over total farm debts. The implicit farm-specific rate of interest is used as the rental cost price of capital and assumed to be the same for all quasi-fixed factors except family labor. The farm-specific wage rate is used as the rental cost price of family labor.

## 6. Empirical Implementation of the Nonparametric Tests

Three types of nonparametric tests are used to investigate the consistency of the data series with the behavioral hypothesis of dynamic cost minimization: the deterministic, goodness-of-fit and stochastic tests (Varian, 1984, 1990, 1985). The deterministic test consists in an exhaustive pairwise comparison of observations to determine whether the data satisfy fully the WADCM, indicating the percentage of observed violations (Varian, 1984). The goodness-of-fit test assesses the economic significance of the hypothesis violations (Varian, 1990). A measure of the magnitude of the violation of dynamic cost minimizing behavior is given as

$$100 \cdot \left(1 - \frac{w^{i'} x^j + W_k^{bi'} I^j}{w^{i'} x^i + W_k^{bi'} I^i}\right), \quad y^j \geq y^i, k^i \geq k^j.$$

Average values of the cost savings in percentages terms are calculated by summing the violation magnitudes and divided by the total number of possible violations. The deterministic and goodness-of-fit tests are conducted using quantity and observed input price data and both tests require information on the behavioral shadow value of capital.

The stochastic test for dynamic cost minimizing behavior investigates the statistical significance of the hypothesis violations, yielding a measure of what the standard error of the quantity data would have to be in order to reject the hypothesis (Varian, 1985). The stochastic test is performed by running problem (2) and the shadow value of capital is one of the variables to be determined within this problem.

The technological hypothesis of CRS and homotheticity are investigated using three types of nonparametric tests: deterministic test using quantity data, deterministic test using quantity and observed input price data and the stochastic test. The deterministic test for CRS and homotheticity using quantity data is conducted by running, respectively problems (9) and (17). The stochastic test for CRS and homotheticity is performed by running the quadratic programming problems in (10) and (18), respectively. In both cases, the shadow value of the quasi-fixed factors is one of the choice variables to be determined within each problem.

Condition (2) in theorem 4 and the programming problem in (16), respectively, are used to implement the deterministic test for CRS and homotheticity using quantity and observed input price data. Performing this test requires information on the behavioral shadow value of the quasi-fixed factors.

The behavioral shadow value of the quasi-fixed factors is estimated using the kernel estimation method and the negative of the marginal cost of adjustment evaluated at the observed level of the gross investment vector.[10] The kernel estimation procedure generates point estimates of the marginal cost of adjustment for each quasi-fixed factor and for each farm in each year.[11] The value of these estimates are nearly zero for all quasi-fixed factors and for all farms in all years, implying small changes in the initial stock of the quasi-fixed factors has no impact on the value function. The standard errors associated with these estimates are approximately zero implying there is little variability in these estimated values.[12]

The value of these estimates might be biased for several reasons. First, the reported values of the stock of the quasi-fixed factors as well as of the gross investment are book values rather than market values. Second, due to the absence of information on the quantity and price of the variable inputs other than hired labor, the component miscellaneous cost incorporates all other possible variable inputs and it is a significant component of the short-run variable cost. Finally, the kernel estimators are sensitive to the choice of the window-width.[13] The bias of the kernel estimators is an increasing function of the window-width where the standard error is a decreasing function of this parameter.

## 7. Empirical Results

The nonparametric test results for the WADCM are presented in Table 1. Violations of the WADCM are detected in all years. The deterministic test indicates that more than 60% of the observation comparisons violate the WADCM in each year. The average percentage error ranges between 79.9% and 564.6%, implying the observed departures from the behavioral

*Table 1.* Nonparametric tests for dynamic cost minimization.

| | Deterministic Test | Goodness-of-Fit Test | Stochastic Test |
|---|---|---|---|
| Year | Percentage of Violations | Average Percent Error | Critical Value of Standard Error (%)[a] |
| 1987 | 69.0 | 79.9 | 25.87 |
| 1988 | 70.5 | 564.6 | 27.04 |
| 1989 | 61.8 | 266.2 | 39.05 |
| 1990 | 65.3 | 301.3 | 29.84 |
| 1991 | 60.9 | 128.6 | 29.35 |
| 1992 | 68.0 | 129.8 | 37.23 |

[a]Critical value of the standard error of the input quantity data is calculated at the 1% significance level.

hypothesis of dynamic cost minimization are economically significant. The results of the deterministic and goodness-of-fit tests show a high percentage of violations of the WADCM with some violations being relatively high as indicated by the average percentage error. The stochastic test for WADCM identifies the lower bound for the standard error in the input quantity data due to measurement error which ranges between 25.87% and 39.05% across years. Using the rejection criterion of 10% measurement error, it can be inferred from the stochastic test that violations of WADCM are statistically significant.[14]

The test results for WADCM indicate inconsistency of the data series with the dynamic cost minimization hypothesis.[15] This inconsistency may be due to several reasons. First, the dynamic technology may not be well-behaved. Nonconvexities in the dynamic technology may be the sources of the observed violations. There is a growing body of empirical studies offering empirical evidence on the lumpiness of investments and production in a variety of industries (e.g., Ramey, 1991; Bresnahan and Ramey, 1994; Caballero, Engel and Haltiwanger, 1995).[16] Second, a significant percentage of the dairy operators may be economically inefficient in the use of variable or/and quasi-fixed factors. Failure of the WADCM can be caused by a poor efficiency performance of some dairy operators.

Third, excluded variables may be important in explaining the dairy operator's behavior. Factors such as the stock of human capital (e.g., number of years of education), the flow of human capital (e.g., new knowledge acquired by attending extension programs), risk and unexpected weather conditions may affect the variable input and investment decision making process, and, consequently affect the economic performance of dairy operators.

Fourth, the nonparametric test may be biased toward rejection since necessary as well as sufficient conditions are being checked. The nonparametric approach to the dynamic theory of production is paradoxically both a less and a higher structured approach relative to the parametric approach. On one hand, the nonparametric approach is a free-functional form approach permitting the analysis and measurement of the production structure without imposing explicitly or implicitly a functional form on the production technology. On the other hand, first- and second-order conditions are incorporated simultaneously in the production analysis. As a result, the weakness of the data is more easily and fully revealed in a nonparametric approach than in the parametric approach which embodies first-order conditions only.[17]

Fifth, the quality (accuracy) of the data may be another source for the rejection of the WADCM. Accuracy of the physical factor use data can be questioned. One can expect some input levels used such as energy consumption, custom work hired, crop and seed supplies, veterinary expenses to be reliably recorded and reported. However, reporting of quasi-fixed factor levels can be subject to considerable error. It is difficult to obtain an accurate reporting of the value of a farm's barn buildings, specialized durable equipment such as a milking parlor, and acreage. In addition, one may also suspect of the price data. The presence of unobservable variables (the shadow value of the quasi-fixed factors) leads to employment of two techniques that are of a different nature: mathematical programming and nonparametric regression methods.

The nonparametric tests for CRS (homotheticity) are performed as a joint hypothesis of dynamic cost minimization and CRS (homotheticity). Given the data series is not consistent with the behavioral hypothesis of dynamic cost minimization, rejection of the joint hypothesis is expected. Nevertheless, the deterministic and stochastic tests for CRS and homotheticity are conducted in order to check whether the results of the three nonparametric tests (deterministic test using quantity data, deterministic test using quantity and observed input price data and the stochastic test) are consistent. The results of the stochastic test and the deterministic test using quantity and observed input price data are expected to indicate rejection of each of the joint hypotheses, since similar tests conducted for the behavioral hypothesis of dynamic cost minimization indicate rejection of the WADCM. The deterministic test using quantity data is checking whether there exist input prices (including the shadow value of the quasi-fixed factors) cost-racionalizing the data series and simultaneously making it consistent with a particular technological hypothesis (CRS or homotheticity). If the deterministic test using quantity data fails to reject the joint hypothesis and the results of the other two tests indicate rejection of the hypothesis, a poor quality of the input price data may be the source of this rejection or the behavioral hypothesis underlying these tests is not valid (Hanoch and Rothschild, 1972).

Table 2 reports the nonparametric test results for CRS and dynamic cost minimization. The deterministic test using quantity data reveals a strictly positive $\gamma$, implying the data do not satisfy fully the joint hypothesis of CRS and dynamic cost minimization in all years.

*Table 2.* Nonparametric test for constant returns to scale and dynamic cost minimization.

| | Deterministic Test (Quantity Data) | Deterministic Test (Quantity and Price Data) | Stochastic Test |
|---|---|---|---|
| Year | $\gamma$ | Percentage of Violations | Critical Value of Standard Error (%)[a] |
| 1987 | 0.834 | 4.7 | 79.98 |
| 1988 | 0.695 | 5.1 | 77.51 |
| 1989 | 1.453 | 4.8 | 84.80 |
| 1990 | 6.243 | 5.6 | 75.09 |
| 1991 | 0.035 | 5.8 | 62.69 |
| 1992 | 0.544 | 6.9 | 48.64 |

[a]Critical value of the standard error of the input quantity data is calculated at the 1% significance level.

Similar results are inferred from the deterministic test using quantity and observed input price data. The percentage of violations of the joint hypothesis ranges between 4.7% in 1987 and 6.9% in 1992, implying the data are not fully consistent with this hypothesis in all years. The lower bound of the standard error in the input quantity data identified by the stochastic test for CRS ranges between 48.64% in 1992 and 84.80% in 1989. Given the rejection criterion of 10% measurement error, the joint hypothesis is rejected in all years.[18]

The results of the deterministic test using quantity data are consistent with the results of the stochastic test and the deterministic test using quantity and observed input price data, reinforcing the rejection of the joint hypothesis of CRS and dynamic cost minimization. However, the nonparametric test for CRS and dynamic cost minimization is inconclusive as far as CRS is concerned. The technological hypothesis of CRS is tested in conjunction with the behavioral hypothesis of dynamic cost minimization and the WADCM is rejected for this data series. Thus, one does not know whether CRS is satisfied or not by the data series. Nevertheless, one may suspect the data are not consistent with the technological hypothesis of CRS, given the results of the deterministic test using only quantity data.

The nonparametric test results for homotheticity and dynamic cost minimization are presented in Table 3. The deterministic test using quantity and observed input price data generates a positive value for $\beta$ in all years, implying the data series is not fully consistent with the joint hypothesis. The lower bound for the standard error of the input quantity data ranges between 78.69% and 120.02% across years. Employing the rejection criterion of 10% measurement error, the observed departures from dynamic cost minimization and homotheticity are statistically significant in all years. However, the deterministic test for homotheticity using quantity data indicates the data are fully consistent with homotheticity in all years.[19]

The joint hypothesis of homotheticity and dynamic cost minimization is not rejected by the deterministic test using quantity data, implying that there exists input prices (including the shadow value of the quasi-fixed factors) cost-rationalizing the data series and making it consistent with a homothetic technology. However, the joint hypothesis is rejected by the stochastic test and the deterministic test using quantity and observed input price data.

*Table 3.* Nonparametric tests for homotheticity and dynamic cost minimization.

| | Deterministic Test (Quantity Data) | Deterministic Test (Quantity and Price Data) | Stochastic Test |
|---|---|---|---|
| Year | $\alpha$ | $\beta$ | Critical Value of Standard Error (%)[a] |
| 1987 | 0 | 1.54 | 79.85 |
| 1988 | 0 | 5.89 | 78.69 |
| 1989 | 0 | 3.82 | 80.71 |
| 1990 | 0 | 0.47 | 120.02 |
| 1991 | 0 | 1.59 | 86.43 |
| 1992 | 0 | 2.51 | 89.42 |

[a] Critical value of the standard error of the input quantity data is calculated at the 1% significance level.

A poor quality of the input price data may be the source of the rejection of the joint hypothesis by the two nonparametric tests and, as argued before, it may be also one of the reasons to reject the behavioral hypothesis of dynamic cost minimization.

## 8. Concluding Comments

The theoretical formulation proposed allows the recovery of information concerning the dynamic production structure in a nonparametric fashion. Building on the foundation of dynamic production analysis in the context of intertemporal cost minimization, the characterization of the dynamic production structure is developed leading to nonparametric tests for constant returns to scale and homotheticity.

A nonparametric dynamic dual cost approach to production analysis requires consistency of the data series with intertemporal cost minimizing behavior. Nonparametric tests to analyze the structure of a dynamic technology and to check for consistency from a dynamic cost minimizing perspective are developed.

The empirical implementation of these nonparametric tests is presented for a panel data set of Pennsylvania dairy operators during the time period 1986–1992. The empirical results indicate the weak axiom of dynamic cost minimization is violated by this data series. The results of the nonparametric tests indicate also rejection of the joint hypothesis of CRS and dynamic cost minimization. However, the results of the nonparametric tests conducted for homotheticity and dynamic cost minimization are inconclusive. The joint hypothesis of homotheticity and dynamic cost minimizing behavior is rejected by the stochastic test and the deterministic test using quantity and observed input price data. The deterministic test using quantity data fails to reject that hypothesis. These inconsistencies revealed between the results of the deterministic test using only quantity data and the other types of tests rise some concerns about the quality of the input price data.

This theoretical framework can provide analysts with a useful pre-test methodology before resorting to more precise parametric methodologies. Implementation of nonparametric dynamic analysis is best suited to panel data.

Extensions and modifications of the theoretical framework proposed in this study are necessary. There are several factors relevant to the production analysis that are not explicitly considered in this framework. Learning and technical change are not considered in the nonparametric approach to the dynamic theory of production. Learning can play a significant role both in the decision-making process and as a source of intertemporal shifts in the production technology and the production structure can change with technical change. Relaxing the convexity property of the dynamic technology is another important factor to be considered, requiring the development of another type of nonparametric approach to the dynamic theory of production.

Nonparametric dynamic production analysis has the potential to be the foundation for efficiency measurement where one can distinguish between the efficiency of variable inputs and quasi-fixed factors (such as land, durable equipment). The efficient management of production operations is addressed at how the variable factors of production (such as hired labor, fuel, materials) are utilized in comparison to the challenges of managing the assets of the operation.

**Appendix**

*Proof of Theorem 1.* (1) $\Rightarrow$ (2) Suppose that $\{V(y:k)\}$ is a family of nested and "reverse nested" input requirement sets that cost-rationalizes the data set $S^c$. If $y^j \geq y^i$ and $k^j \leq k^i$, by the nestedness and "reverse nestedness" properties of $V(y:k)$, it must be the case that $(x^j, I^j) \in V(y^i : k^i)$. Since $\{V(y^i : k^i)\}$ cost-rationalizes the data set $S^c$, then, by definition 1, the following statement is true

$$w^{i'}x^j + W_k^{i'}I^j \geq w^{i'}x^i + W_k^{i'}I^i; \quad \forall i = 1, \ldots, n, \text{ and } j \text{ such that } y^j \geq y^i, \text{ and } k^j \leq k^i.$$

(2) $\Rightarrow$ (3) Assume condition (2) holds at all data points of $S^c$. Let $V(y:k)$ be the convex monotonic hull of $(x^i, I^i)$ such that $y^i \geq y$ and $k^i \leq k$; i.e.,

$$V(y:k) = com\{z^i + e^i : y^i \geq y; k^i \leq k; e^i \geq 0\} \tag{A.1}$$

where $z^i = (x^i, I^i)$, $e^i = (e_x^i, -e_I^i)$ and *com* denotes convex monotonic hull. If $\nexists (y^i, k^i)$ such that $y^i \geq y$ and $k^i \leq k$, then let $V(y:k) = \emptyset$. By construction, the set $V(y:k)$ defined in (A.1) is closed, convex in $(x, I)$, nested in $y$, "reverse nested" in $k$, positive monotonic in $x$ and negative monotonic in $I$ for each $y$, given $k$.

It is left to be shown that $\{V(y:k)\}$, where $V(y:k)$ is defined in (A.1), cost-rationalizes the data set $S^c$. More specifically, one needs to show that

$$\forall (x, I) \in V(y^i : k^i), w^{i'}x^i + W_k^{i'}I^i \leq w^{i'}x + W_k^{i'}I; \tag{A.2}$$

for all $i$, $i = 1, \ldots, n$. Using (A.1), $V(y^i : k^i)$ can be written as

$$V(y^i : k^i) = com\{z^j + e^j : y^j \geq y^i; k^j \leq k^i; e^j \geq 0\}$$

for all $i$. Given that $V(y^i : k^i)$ is a convex polytype (i.e., the intersection of a finite number of closed half-spaces), one only needs to check (A.2) at the vertices of this set. The vertices of $V(y^i : k^i)$ is a subset of the following set $\{(x^j, I^j) : y^j \geq y^i; k^j \leq k^i\}$. However, by condition (2) in the theorem one can state that $w^{i'}x^i + W_k^{i'}I^i \leq w^{i'}x^j + W_k^{i'}I^j$; for all $i = 1, \ldots, n$, and $j$ such that $y^j \geq y^i$ and $k^j \leq k^i$. (3) $\Rightarrow$ (1) (3) is a special case of (1). ■

*Proof of Theorem 2.* (1) Let $V_I(y:k)$ be the convex monotonic hull of $(x^i, I^i)$ as defined in (A.1). By theorem 1, $V_I(y:k)$ cost-rationalizes the data set $S^c$.

(2) Since $V(y:k)$ is nested in $y$ and "reverse nested" in $k$, $y^i \geq y$ and $k \geq k^i$ imply that $(x^i, I^i) \in V(y:k)$. Given that $V(y:k)$ is convex in $(x, I)$, positive monotonic in $x$ and negative monotonic in $I$, one can establish that $V(y:k) \supset V_I(y:k)$.

(3) If $V'(y:k)$ is a input requirement set that cost-rationalizes the data, then it must be the case that $V'(y:k) \supset \{(x^i, I^i) : y^i \geq y; k \geq k^i\}$. But $V_I(y:k)$ is the smallest closed, convex input requirement set, positive monotonic in $x$ and negative monotonic in $I$, that contains all these points. ■

*Proof of Theorem 3.* (1) Let $V_O(y:k) = \{(x, I) : w^{i'}x + W_k^{i'}I \geq w^{i'}x^i + W_k^{i'}I^i; y \geq y^i; k \leq k^i\}$.

$(x^j, I^j) \in V_O(y^j : k^j)$ for all $j$. If $(x^j, I^j) \notin V_O(y^j : k^j)$, then there is some $y^i \leq y^j$ and $k^i \geq k^j$ such that $w^{i'}x^j + W_k^{i'}I^j < w^{i'}x^i + W_k^{i'}I^i$. But this contradicts the WADCM. In addition, let $(x, I) \in V_O(y^i : k^i)$. Then, by construction, $w^{i'}x + W_k^{i'}I \geq w^{i'}x^i + W_k^{i'}I^i$; $y \geq y^i$, $k \leq k^i$. Hence, $V_O(y^i : k^i)$ cost-rationalizes the data set $S^c$.

(2) Let $\{V(y : k)\}$ be any family of input requirement sets that cost-rationalizes the observed data. Assume $(x, I) \in V(y^j : k^j)$, but $(x, I) \notin V_O(y^j : k^j)$. Then, $w^{i'}x + W_k^{i'}I < w^{i'}x^i + W_k^{i'}I^i$ for some $y^j \geq y^i$ and $k^j \leq k^i$. Since $y^j \geq y^i$, $k^j \leq k^i$ and $(x, I) \in V(y^j : k^j)$, the nestedness and "reverse nestedness" properties of $V(y : k)$ imply that $(x, I) \in V(y^i : k^i)$. Given that $V(y : k)$ cost-rationalizes the data set $S^c$, then $w^{i'}x + W_k^{i'}I \geq w^{i'}x^i + W_k^{i'}I^i$. One reaches a contradiction thus establishing the desired result.

(3) Let $(x, I) \in V'(y : k)$, but $(x, I) \notin V_O(y : k)$. Then, by construction, $w^{i'}x + W_k^{i'}I < w^{i'}x^i + W_k^{i'}I^i$ for $y \geq y^i$ and $k \leq k^i$. Thus, $V'(y : k)$ cannot cost-rationalize the data set $S^c$. ■

*Proof of Theorem 4.* (1) ⇒ (2) Let $F(x, I, k)$ be a homogeneous function of degree 1 in $(x, I)$ that cost-rationalizes the data set $S^c$. Then, by definition 2, the following statement is true

$$y^i = F(x^i, I^i, k^i),\ F(x, I, k) \geq y^i,\ k \leq k^i \Rightarrow w^{i'}x^i + W_k^{i'}I^i \leq w^{i'}x + W_k^{i'}I; \tag{A.3}$$

$i = 1, \ldots, n$.

By (A.3), one can state that $y^i = F(x^i, I^i, k^i)$, for all $i$, and by the linear homogeneity property of this function, the following statements are true

$$1 = F\left(\frac{x^i}{y^i}, \frac{I^i}{y^i}, k^i\right), \forall i \quad \text{and} \quad y^j = F\left(x^i\left(\frac{y^j}{y^i}\right), I^i\left(\frac{y^j}{y^i}\right), k^i\right), \forall i \neq j;$$

where $y^j = F(x^j, I^j, k^j)$. Since $F(x, I, k)$ cost-rationalizes the data set $S^c$, then

$$w^{j'}x^j + W_k^{j'}I^j \leq w^{j'}x^i(y^j/y^i) + W_k^{j'}I^i(y^j/y^i), \quad \text{for } k^i \leq k^j;$$

or, equivalently

$$\frac{w^{j'}x^i + W_k^{j'}I^i}{w^{j'}x^j + W_k^{j'}I^j} \geq \frac{y^i}{y^j}, \quad k^i \leq k^j.$$

(2) ⇒ (3) Assume condition (2) holds; i.e.,

$$\frac{w^{j'}x + W_k^{j'}I}{w^{j'}x^j + W_k^{j'}I^j} \geq \frac{y}{y^j}, \quad k \leq k^j,\ j = 1, \ldots, n.$$

Then,

$$y \leq y^j \frac{w^{j'}x + W_k^{j'}I}{w^{j'}x^j + W_k^{j'}I^j}, \quad k \leq k^j.$$

Let

$$F(x, I, k) = \min_j \left\{ y^j \frac{w^{j'}x + W_k^{j'}I}{w^{j'}x^j + W_k^{j'}I^j}, k \leq k^j \right\}.$$

By construction, $F(x, I, k)$ is continuous, linearly homogeneous, concave in $(x, I, k)$, positive monotonic in $(x, k)$, and negative monotonic in $I$.

One needs to prove that $F(x, I, k)$ cost-rationalizes the data set $S^c$; or equivalently that condition (A.3) holds at all data points. The first step is to prove $y^i = F(x^i, I^i, k^i)$ for all $i = 1, \dots, n$. By construction,

$$F(x^i, I^i, k^i) = \min_j \left\{ y^j \frac{w^{j'} x^i W_k^{j'} I^i}{w^{j'} x^j + W_k^{j'} I^j}; k^i \le k^j \right\}$$

and, thus, the following statement is true

$$\begin{aligned} F(x^i, I^i, k^i) &= y^m \frac{w^{m'} x^i + W_k^{m'} I^i}{w^{m'} x^m + W_k^{m'} I^m} \\ &\le y^i \frac{w^{i'} x^i W_k^{i'} I^i}{w^{i'} x^i + W_k^{i'} I^i} = y^i. \end{aligned}$$

Therefore, $F(x^i, I^i, k^i) \le y^i$. From condition (2) in the theorem, one can also state that

$$y^m \frac{w^{m'} x^i + W_k^{m'} I^i}{w^{m'} x^m + W_k^{m'} I^m} \ge y^i$$

or equivalently $F(x^i, I^i, k^i) \ge y^i$. $F(x^i, I^i, k^i) \le y^i$ and $F(x^i, I^i, k^i) \ge y^i$ implies that $F(x^i, I^i, k^i) = y^i$, for all $i = 1, \dots, n$. To show that the second part of condition (A.3) is satisfied at all data points, suppose $y^i = F(x^i, I^i, k^i) \le F(x, I, k)$, $k \le k^i$. Then, by construction of the production function,

$$y^i \le F(x, I, k) = \min_j \left\{ y^j \frac{w^{j'} x + W_k^{j'} I}{w^{j'} x^j + W_k^{j'} I^j}, k \le k^i \right\}$$

which is equivalent to stating that

$$y^i \le F(x, I, k) \le y^i \frac{w^{i'} x + W_k^{i'} I}{w^{i'} x^i + W_k^{i'} I^i}, \quad k \le k^i.$$

Hence,

$$y^i \le y^i \frac{w^{i'} x + W_k^{i'} I}{w^{i'} x^i + W_k^{i'} I^i}, \quad k \le k^i,$$

or equivalently

$$\frac{w^{i'} x + W_k^{i'} I}{w^{i'} x^i + W_k^{i'} I^i} \ge 1; \quad k \le k^i.$$

(3) $\Rightarrow$ (1) (3) is a special case of (1). ■

*Proof of Theorem 5.* (1) $\Rightarrow$ (2) Let $F(x, I, k) = M(h(x, I, k))$ be a homothetic production function that cost-rationalizes the data set $S^c$. Since $M(\cdot)$ is a monotonic function, $M^{-1}(\cdot)$ is well-defined. Let $h^i = M^{-1}(y^i) = h(x^i, I^i, k^i)$, for all $i$, $i = 1, \dots, n$. Then, the data set

$\overline{S^c} = \{(h^i, x^i, I^i, k^i, w^i, c^i, W_k^i); i = 1, \ldots, n\}$ is consistent with cost minimization and a homogeneous production function of degree 1; i.e.,

$$h^i = h(x^i, I^i, k^i), h(x, I, k) \geq h^i, k \leq k^i \Rightarrow w^{i'} x^i + W_k^{i'} I^i \leq w^{i'} x + W_k^{i'} I. \qquad \text{(A.4)}$$

By the homogeneity property of $h(x, I, k)$, the following statements are true

$$1 = h\left(\frac{x^i}{(h^i)}, \frac{I^i}{(h^i)}, k^i\right); \quad \text{and} \quad h^j = h\left(x^i\left(\frac{h^j}{h^i}\right), I^i\left(\frac{h^j}{h^i}\right), k^i\right);$$

where $h^j = h(x^j, I^j, k^j)$.

By the cost minimization assumption, it must be the case that

$$w^{j'} x^j + W_k^{j'} I^j \leq w^{j'} x^i \left(\frac{h^j}{h^i}\right) + W_k^{j'} I^i \left(\frac{h^j}{h^i}\right),$$

where $k^i \leq k^j$. Rearranging the previous expression yields

$$\frac{w^{j'} x^i + W_k^{j'} I^i}{w^{j'} x^j + W_k^{j'} I^j} \geq \left(\frac{h^i}{h^j}\right); \quad k^i \leq k^j.$$

(2) $\Rightarrow$ (3) Assume condition (2) holds at all data points. Then, the following statement is true for all $j$, $j = 1, \ldots, n$,

$$\frac{w^{j'} x + W_k^{j'} I}{w^{j'} x^j + W_k^{j'} I^j} \geq \left(\frac{h}{h^j}\right); \quad k \leq k^j,$$

or, equivalently

$$h \leq h^j \left(\frac{w^{j'} x + W_k^{j'} I}{w^{j'} x^j + W_k^{j'} I^j}\right); \quad k \leq k^j.$$

Let

$$h(x, I, k) = \min_j \left\{ h^j \left(\frac{w^{j'} x + W_k^{j'} I}{w^{j'} x^j + W_k^{j'} I^j}\right); k \leq k^j \right\}.$$

By construction, $h(x, I, k)$ is a continuous, concave and homogeneous function of degree 1 in $(x, I)$, positive monotonic in $(x, k)$, and negative monotonic in $I$. Thus, it is left to be shown that $h(x, I, k)$ cost-rationalizes the data set $\bar{S}^c$

$$h(x^i, I^i, k^i) = \min_j \left\{ h^j \left(\frac{w^{j'} x^i + W_k^{j'} I^i}{w^{j'} x^j + W_k^{j'} I^j}\right); k^i \leq k^j \right\}.$$

$$h(x^i, I^i, k^i) = h^m \left(\frac{w^{m'} x^i + W_k^{m'} I^i}{w^{m'} x^m + W_k^{m'} I^m}\right) \leq h^i \left(\frac{w^{i'} x^i + W_k^{i'} I^i}{w^{i'} x^i + W_k^{i'} I^i}\right) = h^i. \qquad \text{(A.5)}$$

From condition (2) in the theorem, the following statement is true

$$h^m \left(\frac{w^{m'} x^i + W_k^{i'} I^i}{w^{m'} x^m + W_k^{m'} I^m}\right) \geq h^i. \qquad \text{(A.6)}$$

(A.5) and (A.6) lead to $h(x^i, I^i, k^i) \leq h^i, h(x^i, I^i, k^i) \geq h^i \Rightarrow h(x^i, I^i, k^i) = h^i$ for all $i, i = 1, \ldots, n$.

To prove the second part of (A.4), using $h^i \leq h(x, I, k)$ leads to

$$h^i = h(x^i, I^i, k^i) \leq h(x, I, k) = \min_j \left\{ h^j \left( \frac{w^{j'} x + W_k^{j'} I}{w^{j'} x^j + W_k^{j'} I^j} \right) \right\} \leq h^i \left( \frac{w^{i'} x + W_k^{i'} I}{w^{i'} x^i + W_k^{i'} I^i} \right).$$

Hence,

$$h^i \leq h^i \left( \frac{w^{i'} x + W_k^{i'} I}{w^{i'} x^i + W_k^{i'} I^i} \right) \quad \text{or, equivalently} \quad \left( \frac{w^{i'} x + W_k^{i'} I}{w^{i'} x^i + W_k^{i'} I^i} \right) \geq 1;\, k \leq k^i.$$

Rearranging the previous expression, yields $w^{i'} x + W_k^{i'} I \geq w^{i'} x^i + W_k^{i'} I^i;\, k \leq k^i$ where $h^i \leq h(x, I, k)$.

Since $h^i = h(x^i, I^i, k^i)$ and $h^i$ is increasing in $y^i$ for all $i$, one can construct a monotonic transform mapping $h(x, I, k)$ into output. The resulting production function is continuous, quasi-concave, homothetic, positive monotonic in $(x, k)$ and negative monotonic in $I$. Given $h(x, I, k)$ cost-rationalizes the data set $\bar{S}^c$, a monotonic transform of this function cost-rationalizes $S^c$.

(3) $\Rightarrow$ (1) (3) is a special case of (1). ■

## Notes

1. All the variables are time dependent. The time index, $t$, is dropped for the sake of clearer exposition.
2. By definition, the shadow value of capital, $W_k^i$, measures the impact on the value function due to a small change in the initial capital stock. Therefore, the shadow value of capital is an endogenous price of capital and influenced by the market input prices $(w^i, c^i)$, the production target and the initial capital stocks. Consequently, $W_k^i$ is not an element of $S^c$. The empirical implementation of the nonparametric tests for cost minimizing behavior, later in the paper, is accompanied by a discussion on the procedures adopted to generate the shadow value of capital.
3. Proof of this theorem and other theorems that follow are found in the appendix. More extensive discussion is found in Silva (1996).
4. The DPE in definition 1 can be rewritten as

$$rW(w^i, c^i, y^i, k^i) = \min_I \left\{ C(w^i, I, y^i, k^i) + c^{i'} k^i + W_k^{i'} (I - \delta k^i) \right\}$$

   where $C(.)$ is the short-run variable cost function. If the short-run variable cost function satisfies the usual regularity conditions, a sufficient condition for the intertemporal cost minimization problem is $C_I(w^i, I^*, y^i, k^i) = -W_k^i$, implying the marginal cost of adjustment must be equal to minus the shadow value of capital at each time period. The optimality condition implies perfect cost efficiency. Optimal and observed gross investment differ necessarily for a cost inefficient firm. In this case, the shadow value of capital, called the behavioral shadow value, can be approximated as the negative of the marginal cost of adjustment evaluated at the observed level of gross investment as $C_I(w^i, I^i, y^i, k^i) = -W_k^{bi}$. This condition and the kernel estimation method can be used to estimate the behavioral shadow value of capital, $W_k^{bi}$, for each observation.
5. As noted by Banker and Maindiratta (1988, p. 1321), the construction of the inner bound proposed by Varian (1984) in this way is equivalent to the Data Envelopment Analysis (DEA) construction of the technological possibilities of the firm and this equivalence provides an interesting link between DEA and nonparametric production analysis in economics.
6. See the discussion on the behavioral shadow value of capital in footnote 4.
7. See the discussion on the behavioral shadow value of capital in footnote 4.

8. The values of $h^i$ can be obtained from the nonparametric test in (16) when the data series is consistent with a well-behaved homothetic technology.
9. The annual gross investment in land and family labor are zero for many farms during the period 1987–1992 with 90–95 percent and 72–90 percent of the farms reporting zero gross investment in land and family labor, respectively. Gross investment in buildings is strictly negative for approximately 75 percent of the farms during the time period 1987–1989 where all the farms have a nonnegative gross investment during the years of 1990–1992. Gross investment in machinery and equipment is strictly negative for 42–48 percent of the farms in the time period 1987–1989 and becomes nonnegative for 95–100 percent of the farms in the last three years. Gross investment in livestock is strictly positive for 80–100 percent of the farms during 1987–1990 with 60 percent and 65 percent of the farms realizing a strictly positive gross investment in livestock in 1991 and 1992, respectively.
10. See footnote 4 for a discussion on the behavioral shadow value of capital. The kernel estimation method is a nonparametric regression method that does not impose a functional form for the regression equation. Hence, no parametric representation for the short-run variable cost function is assumed. The linear homogeneity property of the short-run variable cost function in the variable input prices is the only condition explicitly imposed on this function. The farm-specific wage rate is used as the numeraire.
11. The estimates of the negative of the marginal cost of adjustment evaluated at the observed gross investment vector are not reported due to space restrictions. In total, there are 1800 estimates. The results are available from the authors upon request.
12. Point estimates of the quasi-fixed shadow values close to zero do not necessarily imply the absence of adjustment costs and, thus, instantaneous adjustment. Instantaneous adjustment arises when the shadow value of the quasi-fixed factors is constant.
13. For each exogenous variable $z_j$, the window-width is defined as

$$h_j = s_j(n)^{-\frac{1}{4+m+2o}}; \quad s_j^2 = \sum_{i=1}^{n} \frac{\left(z_j^i - \overline{z_j}\right)^2}{n}$$

$j = 1, \ldots, m+2o$. $h_j$ is the estimator of the window-width minimizing the integrated mean square error (Ullah, 1988a and 1988b; Bierens, 1987).
14. No measurement error information is available for the specific data series used in this study. National income data are usually measured with a standard error higher than 10% (Morgenstern, 1963). The Department of Commerce reported that standard errors of state-level quantity data for the two-digit SIC category, Food and Kindred Products, are on average 8% (Lim and Shumway, 1992). Based on the evidence of measurement error in other data series, the 10% measurement error is adopted as the rejection criterion.
15. The nonparametric tests for WADCM are also conducted by splitting dairy operators among the following herd size classes: $hs_1 = [40, 60]$, $hs_2 = [61, 80]$ and $hs_3 = [81, 100]$. The idea underlying this division is that behavioral differences (e.g., differences in managerial practices) may be present across dairy operators with different herd sizes. The empirical results are, in general, similar to the ones presented in Table 1. However, they reveal some differences in the pattern of violations among the three herd size classes. The deterministic and the goodness-of-fit tests indicate that herd size class 3 has the highest percentage of violations but the least degree of seriousness as indicated by the relative magnitude of the average percentage error. The stochastic test results indicate observed departures from WADCM are not statistically significant for herd size class 1 in 1991. The results are available from the authors upon request.
16. One possible way to investigate the axioms proposed in this study allowing for lumpy investments and lumpy production is using a nonparametric approach called the Free-Disposal-Hull (FDH). FDH is a nonparametric approach relaxing the convexity assumption of the production possibilities set. FDH is suggested by Deprins, Simar and Tulkens (1984).
17. The parametric approach embodies first- and second-order conditions when the set of first-order conditions from a primal specification, or the system of equations from a dual specification plus the concavity (convexity) of the cost (profit) function are imposed in the estimation.
18. The test results for each of herd size class (see footnote 14) are, in general, similar to the results presented in Table 2. The results from the stochastic test and the deterministic test using quantity and observed input price data are consistent and indicate rejection of the joint hypothesis. However, the deterministic test using quantity data indicates, in general, failure of the joint hypothesis for all herd size classes in all years, except for herd size class 3 in three years. In 1987, 1988 and 1990, herd size class 3 is fully consistent with the technological

hypothesis of CRS and dynamic cost minimization. These results are inconsistent with the results generated by the stochastic test and the deterministic test using quantity and observed input price data. The results are available from the authors upon request.

19. The nonparametric tests for homotheticity and dynamic cost minimization considering the three herd size classes are, in general, similar to the results presented in Table 3. The deterministic test using quantity data indicates all three herd size classes satisfy fully the joint hypothesis in all years. The deterministic test using quantity and observed input price data indicates that herd size class 2 is fully consistent with the joint hypothesis in 1987 and 1992; and herd size class 3 is fully consistent with the hypothesis in 1989, 1990 and 1992. The stochastic test results indicate the observed violations for herd size class 1 in 1991 are not statistically significant. The results are available from the authors upon request.

## References


Afriat, S. (1972). "Efficiency Estimation of Production Functions." *International Economic Review* 13(3), 568–598.

Banker, R. D. and A. Maindiratta. (1988). "Nonparametric Analysis of Technical and Allocative Efficiencies in Production." *Econometrica* 56(6), 1315–1332.

Bierens, H. (1987). Kernel Estimation of Regression Functions. In T. F. Bewley (ed.), *Advances in Econometrics-Fifth World Congress,* Cambridge: Cambridge University Press, 1.

Blackorby, C. and W. Schworm. (1982). "Aggregate Investment and Consistent Intertemporal Technologies." *Review of Economic Studies* 49, 595–614.

Blackorby, C. and W. Schworm. (1983). "Aggregating Heterogeneous Capital Goods in Adjustment-Cost Technologies." *Scandinavian Journal of Economics* 85, 207–222.

Bresnahan, T. F. and V. Ramey. (1994). "Output Fluctuations at the Plant Level." *Quarterly Journal of Economics* 593–624.

Caballero, R. J., E. M. R. A. Engel and J. C. Haltiwanger. (1995). "Plant-Level Adjustment and Aggregate Investment Dynamics." *Brookings Papers on Economic Activity* 2, 1–54.

Diewert, W. E. and C. Parkan. (1983). Linear Programming Tests of Regulatory Conditions for Production Functions. In W. Eichhorn, R. Henn, K. Neumann and R. W. Shephard (eds.), *Quantitative Studies on Production and Prices.* Physica Verlag, West Germany, 131–158.

Deprins, D., L. Simar and H. Tulkens. (1984). "Measuring Labor Efficiency in Post Offices." In M. Marchand, P. Pestieau and H. Tulkens (eds.), *The Performance of Public Enterprises: Concepts and Measurement.* Amsterdam: North-Holland, 243–267.

Eisner, R. and R. H. Strotz. (1963). Determinants of Business Investment. *Impacts of Monetary Policy*. Englewood Cliffs: Prentice Hall.

Epstein, L. G. (1983). "Aggregating Quasi-Fixed Factors." *Scandinavian Journal of Economics* 85, 191–205.

Farrell, M. J. (1957). "The Measurement of Productive Efficiency." *Journal of the Royal Statistical Society,* Series A General 120(3), 253–281.

Hanoch, G. and M. Rothschild. (1972). "Testing the Assumptions of Production Theory: A Nonparametric Approach." *Journal of Political Economy* 80(2), 256–275.

Lim, H. and C. R. Shumway. (1992). "Profit Maximization, Returns to Scale, and Measurement Error." *The Review of Economics and Statistics* 74, 430–438.

Lucas, R. E. (1967). "Adjustment Costs and the Theory of Supply." *The Journal of Political Economy* 75, 321–334.

McLaren, K. R. and R. J. Cooper. (1980). "Intertemporal Duality: Application to the Theory of the Firm." *Econometrica* 48, 1755–1762.

Morgenstern, O. (1963). *On the Accuracy of Economic Observations,* 2nd ed. Princenton: Princenton University Press.

Ramey, V. (1991). "Nonconvex Costs and the Behavior of Inventories." *Journal of Political Economy* 91, 306–334.

Silva, E. M. S. (1996). "A Unified Approach to Nonparametric Dynamic Production Analysis and Efficiency Measurement." Doctoral Dissertation, The Pennsylvania State University.

Treadway, A. B. (1969). "On Rational Entrepreneurial Behavior and the Demand for Investment." *Review of Economic Studies* 36, 227–240.

Treadway, A. B. (1970). "Adjustment Costs and Variable Inputs in the Theory of the Competitive Firm." *Journal of Economic Theory* 2, 329–347.

Ullah, A. (1988a). "Nonparametric Estimation of Econometric Functionals." *Canadian Journal of Economics* 21, 625–658.

Ullah, A. (1988b). "Nonparametric Estimation and Hypothesis Testing in Econometric Models." *Empirical Economics* 13, 223–249.

Varian, H. R. (1984). "The Nonparametric Approach to Production Analysis." *Econometrica* 52(3), 579–597.

Varian, H. R. (1985). "Nonparametric Analysis of Optimizing Behavior with Measurement Error." *Journal of Econometrics* 30(1/2), 445–458.

Varian, H. R. (1990). "Goodness-of-Fit in Optimizing Models." *Journal of Econometrics* 46, 125–140.