Brazilian Journal of Probability and Statistics
2015, Vol. 29, No. 4, 866–877
DOI: 10.1214/14-BJPS250

Nonparametric estimation of the residual entropy function with censored dependent data

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Abstract. The residual entropy function introduced by Ebrahimi [*Sankhyā A* **58** (1996) 48–56], is viewed as a dynamic measure of uncertainty. This measure finds applications in modeling and analysis of life time data. In the present work, we propose nonparametric estimators for the residual entropy function based on censored data. Asymptotic properties of the estimator are established under suitable regularity conditions. Monte Carlo simulation studies are carried out to compare the performance of the estimators using the mean-squared error. The methods are illustrated using two real data sets.

1 Introduction

In the recent past, many researchers have taken keen interest in the measurement of uncertainty associated with a probability distribution. Of particular interest is the notion of entropy, introduced by Shannon (1948). If X is a nonnegative random variable admitting an absolutely continuous distribution function $F(x)$ with probability density function $f(x)$, the Shannon's entropy associated with the random variable X is defined as

$$H(X) = H(f) = -\int_0^\infty f(x)\log f(x)\,dx. \tag{1.1}$$

If X represents the life time of a unit, then $H(f)$ can be used as a potential measure for the associated uncertainty. However, if a unit has survived up to an age t, the information about the remaining age is of special importance in reliability and survival analysis. In this scenario, Ebrahimi and Pellerey (1995) followed by Ebrahimi (1996) have proposed the concept of residual entropy. For a nonnegative random variable X, representing the life time of a component, the residual entropy function is the Shannon's entropy associated with the random variable $X - t$ truncated at $t \geq 0$ and is defined as

$$\begin{aligned} H(f;t) &= -\int_t^\infty \frac{f(x)}{(1-F(t))}\log\biggl(\frac{f(x)}{1-F(t)}\biggr)\,dx \\ &= \log\bigl(1-F(t)\bigr) - \frac{1}{(1-F(t))}\int_t^\infty f(x)\log f(x)\,dx, \end{aligned} \tag{1.2}$$

Key words and phrases. Residual entropy function, kernel estimate, α-mixing, residual life.
Received July 2013; accepted May 2014.

where $f(x)$ and $F(x)$ denotes the density function and distribution function respectively. Belzunce et al. (2004) has established that if $H(f;t)$ is increasing in t then $H(f;t)$ determines the distribution uniquely. Given that an item has survived up to time t, $H(f;t)$ measures the uncertainty about its remaining life. For practical purposes, we need to develop some inference techniques about this measure. Ebrahimi (1996) proposed new ageing classes namely, IURL(DURL), using this measure. Ebrahimi (1997) proposed a test of exponentiality against IURL(DURL) alternatives. Belzunce et al. (2001) proposed kernel type estimation of the residual entropy function in the case of independent complete data sets. It is more realistic to assume some form of dependence among the data are observed. In this paper, we provide nonparametric kernel type estimation for $H(f;t)$ under right censored dependent data. We consider only situations where the data under study are dependent. In this situation, the underlying lifetimes are assumed to be α-mixing (see Rosenblatt (1956)) and whose definition is given below.

Definition 1. Let $\{X_i; i \geq 1\}$ denote a sequence of random variables. Given a positive integer n, set

$$\alpha(n) = \sup_{k\geq 1}\{|P(A\cap B) - P(A)P(B)|; A \in \mathfrak{F}_1^k, B \in \mathfrak{F}_{k+n}^{\infty}\}, \quad (1.3)$$

where $\mathfrak{F}_i^k$ denote the σ-field of events generated by $\{X_j; i \leq j \leq k\}$. The sequence is said to be α-mixing (strong mixing), if the mixing coefficient $\alpha(n) \to 0$ as $n \to \infty$. Among various mixing conditions, α-mixing is reasonably weak and has many practical applications. Many stochastic processes satisfy the α-mixing condition, see, for example, Doukhan (1994) and Carrasco et al. (2007). Fakoor (2010) examined the strong uniform consistency of kernel density estimators for censored dependent data. Cai (1998a) proposed hazard rate estimation for censored dependent data and Cai (1998b) established the asymptotic properties of Kaplan–Meier estimator for censored dependent data.

The organization of the paper is as follows. In Section 2, we present nonparametric estimators for $H(f;t)$ using censored samples. In Section 3, we examine asymptotic properties of the estimator. In Section 4, we evaluate the estimator to two real data sets and Section 5 a simulation study to illustrate the behavior of the estimators is undertaken.

2 Estimation

In this section, we propose nonparametric estimators for the residual entropy function for censored data sets. In reliability and life testing, due to time constraints or cost consideration the experimenter is forced to terminate the experiment after specific period of time or after a failure of a specified number of units. In this context, the underlying data will be censored. In the context of right censoring, only the lower bounds on life time will be available for some individuals and in the context of left censoring data will be recorded as the upper bound of life time for some individuals. Another common type of censoring is random censoring.

Let $\{X_i; 1 \le i \le n\}$ be a sequence of nonnegative random variables representing the life times for n components/devices. The random variables are not assumed to be mutually independent. However, they have a common unknown continuous marginal distribution function $F(x)$ with a probability density function $f(x) = F'(x)$. Let the random variable X_i be censored on the right by the random variable Y_i. In this random censorship model, the censoring times Y_i are assumed to be independently and identically distributed and they are also assumed to be independent of X_i. The censoring times $Y_1, Y_2, \dots, Y_n$ have the common distribution function $G(x)$. This scheme is very common in clinical trials. In such experiments, patients enter into the study at random time points, while the experiment itself is terminated at a pre specified time. Let $Z_i = X_i \wedge Y_i$ and $\delta_i = I(X_i \le Y_i)$, where $I(\cdot)$ denotes the indicator function of the event specified in parentheses. The actually observed Z_i's have a distribution function $L(x)$ satisfying

$$1 - L(t) = \big(1 - F(t)\big)\big(1 - G(t)\big), \qquad t \in R_+ = [0, \infty).$$

Let $L^*(t) = P(Z_1 \le t; \delta_1 = 1)$ be the corresponding sub-distribution function for the uncensored observations and $l^*(t) = f(t)(1 - G(t))$ be the corresponding sub-density. A reasonable estimator of f should behave like $\frac{l_n^*(t)}{(1-G(t))}$ where $l_n^*(t) = b_n^{-1} \int_{R^+} K(\frac{t-x}{b_n})\, dL_n^*(x)$ is the kernel estimator pertaining to $L_n^*(t) = \frac{1}{n}\sum_{i=1}^n I(Z_i \le t; \delta_i = 1)$.

A nonparametric density estimator for the density function $f(x)$ (see Cai (1998a)) under dependence condition is given by

$$f_n(x) = \frac{1}{b_n} \int_{R^+} \frac{K((x-u)/b_n)}{1 - G(u)}\, dL_n^*(u). \tag{2.1}$$

Under α-mixing dependence conditions, expressions for the bias and variance of $f_n(x)$ are given by

$$\mathrm{Bias}\big(f_n(x)\big) \backsimeq \frac{b_n^s c_{s^+}}{s!} f^{(s)}(x) \tag{2.2}$$

and

$$\mathrm{Var}\big(f_n(x)\big) \backsimeq \frac{1}{nb_n} \frac{f(x)}{(1 - G(x))} C_K, \tag{2.3}$$

where $c_{s^+} = \int_{R^+} u^s K(u)\, du$ and $C_K = \int_{-\infty}^{\infty} K^2(u)\, du$.

Let $N_n(t) = \sum_{i=1}^n I(Z_i \le t; \delta_i = 1)$ be the number of uncensored observations less than or equal to t and $Y_n(t) = \sum_{i=1}^n I(Z_i \ge t)$ be the number of censored or uncensored observations greater than or equal to t. Then, the Kaplan–Meier estimator is given by (see Kaplan and Meier (1958))

$$1 - F_n(t) = \prod_{s \le t}\left(1 - \frac{dN_n(s)}{Y_n(s)}\right), \tag{2.4}$$

where $dN_n(s) = N_n(s) - N_n(s-)$.

A simple nonparametric estimator for $H(f;t)$ based on the censored data is

$$H^*(f;t) = \frac{-1}{n}\sum_{i=1}^{n}\log\left(\frac{f_n(Z_i)}{1 - F_n(t)}\right)I_{(Z_i>t)}, \tag{2.5}$$

where $f_n(Z_i) = \frac{1}{(n-1)}\sum_{j\neq i}^{n}\frac{1}{b_n}K(\frac{Z_i - Z_j}{b_n})$ is the kernel estimator obtained from the sample without Z_i and $1 - F_n(t)$ is the Kaplan–Meier estimator given in (2.4).

A kernel estimator for $H(f;t)$ under censoring is

$$\begin{aligned} H_n(f;t) &= -\int_t^{\infty}\frac{f_n(x)}{(1 - F_n(t))}\log\left(\frac{f_n(x)}{(1 - F_n(t))}\right)dx \\ &= \log\bigl(1 - F_n(t)\bigr) - \frac{1}{(1 - F_n(t))}\int_t^{\infty} f_n(x)\log f_n(x)\,dx. \end{aligned} \tag{2.6}$$

The assumptions used in this paper are listed below.

1. Suppose that $\{X_i; 1 \le i \le n\}$ is a sequence of stationary α-mixing random variables with continuous distribution function $F(x)$.
2. Suppose that the censoring time variables $\{Y_i; 1 \le i \le n\}$ are independent and identically distributed with a continuous distribution function $G(y)$ and are independent of $\{X_i; 1 \le i \le n\}$.
3. $\alpha(n) = O(n^{-\nu})$ for some $\nu > 3$.
4. K is a continuously differentiable probability density function vanishing outside some finite interval, $\infty < s_1 < 0 < s_2 < \infty$.
5. The bandwidth $b_n \to 0$ and $nb_n \to \infty$ as $n \to \infty$.
6. For each $j \ge 2$, the joint probability density function of X_1 and X_j, $f_{1,j}(\cdot,\cdot)$ exists and $|f_{1,j}(u,v) - f(u)f(v)| \le C$ for all $j \ge 2$ and all u, $v \in \Re$ and some constant C.
7. There exists a sequence of real numbers $\{m_n\}$ such that $1 \le m_n \le n$, $m_n \to \infty$, $m_n b_n \to 0$ and $b_n^{-\gamma}\sum_{j\ge m_n}\alpha^{\gamma}(j) \to 0$ for some $\gamma \in (0,1)$.
8. Let $\{c_n\}$ and $\{d_n\}$ be sub-sequences of $\{n\}$ tending to infinity and such that $c_n + d_n \le n$ and let μ_n be the largest positive integer for which $\mu_n(c_n + d_n) \le n$. Then

(a) $\frac{d_n\mu_n}{n} \to 0$, $\mu_n\alpha(d_n) \to 0$ and $\frac{c_n}{\sqrt{nb_n}} \to 0$.

(b) $b_n^{-\gamma}\sum_{j\ge c_n}\alpha^{\gamma}(j)$ is bounded, where γ is as in (7).

3 Asymptotic properties

In this section, we look into the strong convergence and asymptotic normality of the estimator given in (2.6).

In order to simplify the notation, define

$$\begin{aligned} a_n(t) &= \int_t^\infty f_n(x) \log f_n(x)\, dx, \\ a(t) &= \int_t^\infty f(x) \log f(x)\, dx, \\ m_n(t) &= \log\bigl(1 - F_n(t)\bigr) \quad \text{and} \quad m(t) = \log\bigl(1 - F(t)\bigr). \end{aligned} \tag{3.1}$$

Using (3.1) on (1.2) and (2.6), we get

$$H(f;t) = m(t) - \frac{a(t)}{(1 - F(t))} \tag{3.2}$$

and

$$H_n(f;t) = m_n(t) - \frac{a_n(t)}{(1 - F_n(t))}. \tag{3.3}$$

In the following theorem, we prove the almost sure convergence of the estimator $H_n(f;t)$. In addition to the assumptions given in Section 2, we have the following assumptions.

(i) Let for the distribution functions $F(\cdot)$ and $G(\cdot)$, the possibly infinite times τ_F and τ_G is given by $\tau_F = \inf\{y : F(y) = 1\}$ and $\tau_G = \inf\{y : G(y) = 1\}$.

Then for the marginal distribution function of $L(\cdot)$ of Z, it holds $\tau_L = \tau_F \wedge \tau_G$ (see Stute and Wang (1993)).

(ii) For $0 < \tau < \infty$, $L(\tau) < 1$ (see Cai (1998b)).

(iii) By combining (i) and (ii), we can write for any $0 < \tau < \tau_L$ such that $L(\tau) < 1$.

Theorem 3.1. *Let $H_n(f;t)$ be a nonparametric estimator of $H(f;t)$ satisfying the assumptions in Section 2 and assumption* (iii) *given above. Then*

$$\lim_{n\to\infty} \sup_{0 \le t \le \tau} \bigl|H_n(f;t) - H(f;t)\bigr| = 0 \qquad a.s. \tag{3.4}$$

Proof. Using (3.2) and (3.3), we get

$$H_n(f;t) - H(f;t) = \bigl(m_n(t) - m(t)\bigr) - \left(\frac{a_n(t)}{1 - F_n(t)} - \frac{a(t)}{1 - F(t)}\right). \tag{3.5}$$

We have

$$\bigl|m_n(t) - m(t)\bigr| \backsimeq \frac{|F_n(t) - F(t)|}{(1 - F(t))} \tag{3.6}$$

and

$$\left|a_n(t) - a(t)\right| \backsimeq \int_t^\infty (1 + \log f(x)) \left|f_n(x) - f(x)\right| dx. \tag{3.7}$$

Since $\sup_{0\le t\le\tau} |F_n(t) - F(t)| \to 0$ a.s. (see Cai (1998b)) and simplifies we get,

$$\begin{aligned} &\left|\frac{a_n(t)}{(1 - F_n(t))} - \frac{a(t)}{(1 - F(t))}\right| \\ &\quad \backsimeq \frac{(1 - F(t))|a_n(t) - a(t)| + a(t)|F_n(t) - F(t)|}{(1 - F(t))^2}. \end{aligned} \tag{3.8}$$

Using (3.6), (3.7) and (3.8) in (3.5), we get

$$\begin{aligned} &\left|H_n(f;t) - H(f;t)\right| \\ &\quad \backsimeq \frac{|F_n(t) - F(t)|}{(1 - F(t))} + \frac{1}{(1 - F(t))} \int_t^\infty (1 + \log f(x)) \left|f_n(x) - f(x)\right| dx \\ &\qquad + \frac{a(t)}{(1 - F(t))^2} \left|F_n(t) - F(t)\right|. \end{aligned}$$

By using the almost sure convergence of $f_n(x)$ and $F_n(x)$ given in Cai (1998a) and Cai (1998b) respectively, we get

$$\lim_{n\to\infty} \sup_{0\le t\le\tau} \left|H_n(f;t) - H(f;t)\right| = 0 \qquad \text{a.s.} \qquad \square$$

In the following theorem, we focus attention on the asymptotic normality of the estimator $H_n(f;t)$.

Theorem 3.2. *Let $H_n(f;t)$ be a nonparametric estimator of $H(f;t)$ satisfying the assumptions in Section 2. Then*

$$\sqrt{nb_n} \frac{(H_n(f;t) - H(f;t))}{\sigma_H} \tag{3.9}$$

has a standard normal distribution as $n \to \infty$ with

$$\begin{aligned} \sigma_H^2 \backsimeq{}& \frac{C_K}{(1 - F(t))^2} \int_t^\infty \frac{f(x)}{(1 - G(x))} (1 + \log f(x))^2 \, dx \\ &+ \left(\frac{a^2(t)}{(1 - F(t)^2)} + 1\right) \frac{b_n \sigma^2}{(1 - F(t))}, \end{aligned} \tag{3.10}$$

where $\sigma^2(t) = \mathrm{Var}(\xi(Z_1, \delta_1, t)) + 2\sum_{j=2}^\infty \mathrm{Cov}(\xi(Z_1, \delta_1, t), \xi(Z_j, \delta_j, t))$ (see, Cai (1998b)), $\{\xi(Z_j, \delta_j, t)\}_i$ is a sequence of stationary α-mixing bounded random variables and $C_K = \int_{-\infty}^\infty K^2(u)\, du$.

Proof. We have,

$$
\begin{aligned}
&\sqrt{nb_n}\big(H_n(f;t)-H(f;t)\big)\\
&\quad=\sqrt{nb_n}\bigg(\big(m_n(t)-m(t)\big)-\bigg(\frac{a_n(t)}{(1-F_n(t))}-\frac{a(t)}{(1-F(t))}\bigg)\bigg)\\
&\quad\backsimeq\sqrt{nb_n}\bigg(\frac{-(F_n(t)-F(t))}{(1-F(t))}-\frac{(a_n(t)-a(t))}{(1-F_n(t))}\bigg)\\
&\qquad-\sqrt{nb_n}\bigg(\frac{a(t)(F_n(t)-F(t))}{(1-F_n(t))(1-F(t))}\bigg).
\end{aligned}
\tag{3.11}
$$

Since $\sup_{0\le t\le\tau}|F_n(t)-F(t)|\to 0$ a.s. (see Cai (1998b)), (3.11) is asymptotically equal to

$$
\begin{aligned}
&\sqrt{nb_n}\big(H_n(f;t)-H(f;t)\big)\\
&\quad\backsimeq\sqrt{nb_n}\bigg(-\frac{1}{(1-F(t))}-\frac{a(t)}{(1-F(t))^2}\bigg)\big(F_n(t)-F(t)\big)\\
&\qquad-\sqrt{nb_n}\bigg(\frac{a_n(t)-a(t)}{(1-F(t))}\bigg)\\
&\quad\backsimeq\sqrt{nb_n}\bigg(-\frac{1}{(1-F(t))}-\frac{a(t)}{(1-F(t))^2}\bigg)\big(F_n(t)-F(t)\big)\\
&\qquad-\sqrt{nb_n}\frac{1}{(1-F(t))}\int_t^\infty\big(f_n(u)-f(u)\big)\big(1+\log f(u)\big)\,du.
\end{aligned}
$$

By using the assymptotic normality of $f_n(x)$ and $F_n(x)$ given in Cai (1998a) and Cai (1998b) respectively, it is immediate that

$$
(nb_n)^{1/2}\bigg\{\frac{(H_n(f;t)-H(f;t))}{\sigma_H}\bigg\}
$$

is asymptotically normal with mean zero and variance given in (3.10). □

4 Numerical illustration

Example 1. To illustrate the usefulness of the proposed kernel estimator $H_n(f;t)$ discussed in Section 2 with real situations, we consider the failure times (measured in millions of operations) of 40 randomly selected mechanical switches given in Nair (1984) and Nair (1993) and is reproduced in Table 1. Three of the test positions became available much later than the others, so the three switches tested at these positions were still operating at the termination of the test. The corresponding censored observations are indicated by the code +. This data set is a typical example of a competing-risk problem where a system fails due to one or more competing causes and one observes only the time to failure of the system and the corresponding failure mode (component). Here the censoring mechanisms are dependent, in fact, since the components are subject to the same stress and operating environment, it is likely that the failure times of the components are positively dependent. The Gaussian kernel $K(z) = \frac{1}{\sqrt{2\pi}} \exp(\frac{-z^2}{2})$ is used as the kernel function for the estimation. Figure 1 shows the plot of $H_n(f;t)$ calculated using Gaussian kernel. From Figure 1, it is easy to see that for the data set considered $H_n(f;t)$ is decreasing.

Example 2. Here we consider the data of time to tumor appearance in a litter matched tumorigenesis experiment reported by Mantel and Ciminera (1979). Ying and Wei (1994) used this data for estimating survival function under dependent censoring. Animal studies in which it is desired to control for genetic factors often use a litter-matched design. In such a design, a random sample of litters is chosen. One or more animals in each litter are, perhaps, treated with a suspected carcinogen while the remaining animals in the litter are untreated. The data consists of fifty litters of three female rats each. A suspected carcinogen was administered to one rat in each litter, the other two rats in each litter served as controls. The experiment lasted 104 weeks. The data set consists of the week at which each rat developed a tumor or was lost to follow-up by tumor less death and is given in Table 2. In Table 2, + indicates the weeks of death prior to any tumor. Here also we use the Gaussian kernel for the estimation. Figure 2 shows the plot of $H_n(f;t)$ for this

Table 1 *Failure times (in millions of operations) for a mechanical-switch life test*

1.151	1.170	1.248	1.331	1.381	1.499	1.508	1.534	1.577
1.584	1.667	1.695	1.710	1.955	1.965	2.012	2.051	2.076
2.109	2.116	2.119	2.135	2.197	2.199	2.227	2.250	2.254
2.261	2.349	2.369	2.547	2.548	2.738	2.79	2.883+	2.883+
2.910	3.015	3.017	3.793+					

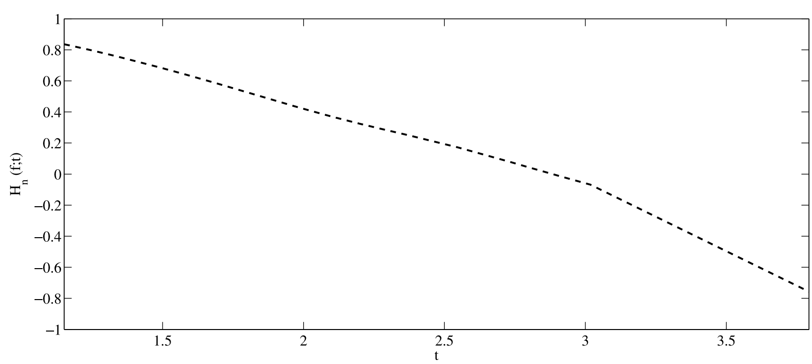

Figure 1 *Plot of $H_n(f;t)$ for the failure times (measured in millions of operations) of* 40 *randomly selected mechanical switches.*

Table 2 *Time (in weeks) to tumor appearance in a litter-matched tumorigenesis experiment*

Drug treated	Control 1	Control 2	Drug treated	Control 1	Control 2
101.0+	49	104.0+	89.0+	104.0+	104.0+
104.0+	102.0+	104.0+	78.0+	104.0+	104.0+
104.0+	104.0+	104.0+	104.0+	81	64
77.0+	97.0+	79.0+	86	55	94.0+
89.0+	104.0+	104.0+	34	104.0+	54
88	96	104.0+	76.0+	87.0+	74.0+
104	94.0+	77	103	73	84
96	104.0+	104.0+	102	104.0+	80.0+
82.0+	77.0+	104.0+	80	104.0+	73.0+
70	104.0+	77.0+	45	79.0+	104.0+
89	91.0+	90.0+	94	104.0+	104.0+
91.0+	70.0+	92.0+	104.0+	104.0+	104.0+
39	45.0+	50	104.0+	101	94.0+
103	69.0+	91.0+	76.0+	84	78
93.0+	104.0+	103.0+	80	81	76.0+
85.0+	72.0+	104.0+	72	95.0+	104.0+
104.0+	63.0+	104.0+	73	104.0+	66
104.0+	104.0+	74.0+	92	104.0+	102
81.0+	104.0+	69.0+	104.0+	98.0+	73.0+
67	104.0+	68	55.0+	104.0+	104.0+
104.0+	104.0+	104.0+	49.0+	83.0+	77.0+
104.0+	104.0+	104.0+	89	104.0+	104.0+
104.0+	83.0+	40	88.0+	79.0+	99.0+
87.0+	104.0+	104.0+	103	91.0+	104.0+
104.0+	104.0+	104.0+	104.0+	104.0+	79

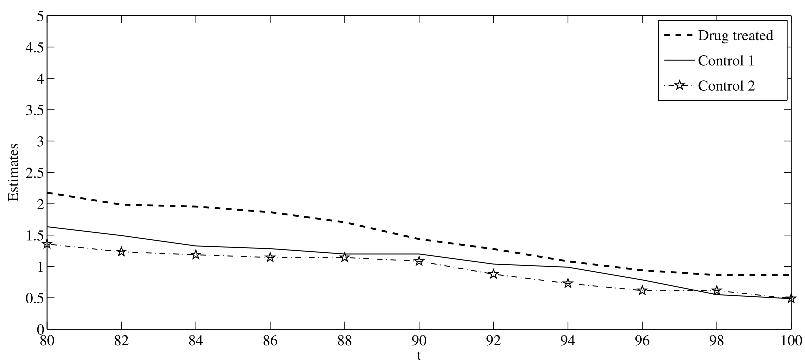

Figure 2 *Plots of $H_n(f;t)$ for the time (in weeks) to tumor appearance in a litter-matched tumorigenesis experiment.*

data calculated using Gaussian kernel. From Figure 2, we can say that $H_n(f;t)$ for rats treated with a suspected carcinogen is high when it is compared with the others which are untreated.

5 Simulation studies

A Monte Carlo simulation study is carried out to compare the kernel estimators $H_n(f;t)$ and $H^*(f;t)$ in terms of the mean-squared error (MSE). For the simulation under right censored sample, we generated $\{X_i\}$ from AR(1) with correlation coefficient $\rho = 0.2$ from an exponential distribution with parameter $\lambda = 5$. Observations are censored using uniform distribution $U(0, 1)$. For simulation we used four different censoring levels, that is, 15%, 25%, 50% and 80%, four different sample sizes, that is, 35, 50, 80 and 100 and three different kernels, that is, Gaussian kernel, Exponential kernel and Gompertz kernel. The mean-squared error of $H_n(f;t)$ and $H^*(f;t)$ are computed and are given in Table 3. The optimal b_n that minimizes the mean-squared error estimate for $H_n(f;t)$ and $H^*(f;t)$ is 0.3. From the Table 3, we can say that sample size and censoring level jointly affect the performance of estimators. When the censoring level is high (>80%), MSE of the estimators become high. MSE of $H_n(f;t)$ is small when it is compared with $H^*(f;t)$ and it decreases as the sample size increases. In the case of $H_n(f;t)$,

Table 3 *Mean-squared error of $H_n(f;t)$ and $H^*(f;t)$ using different censoring levels, sample sizes, kernels*

Censoring level	Sample size	MSE's of $H_n(f;t)$			MSE's of $H^*(f;t)$		
		Gaussian	Gompertz	Exponential	Gaussian	Gompertz	Exponential
0.15	$n = 100$	0.0264	0.0079	0.0116	0.3255	0.3257	0.2922
	$n = 80$	0.0341	0.0057	0.0342	0.2720	0.2678	0.2304
	$n = 50$	0.0355	0.0220	0.0363	0.3010	0.2998	0.2632
	$n = 35$	0.0499	0.1087	0.0510	0.3160	0.3122	0.2828
0.25	$n = 100$	0.0146	0.0305	0.0359	0.3254	0.3307	0.3038
	$n = 80$	0.0336	0.0105	0.0350	0.2670	0.2605	0.2278
	$n = 50$	0.0379	0.0553	0.0484	0.3090	0.3077	0.2745
	$n = 35$	0.0578	0.1070	0.0619	0.3177	0.3122	0.2828
0.5	$n = 100$	0.0339	0.0112	0.0295	0.3000	0.2995	0.2584
	$n = 80$	0.0421	0.0134	0.0639	0.2389	0.2291	0.1846
	$n = 50$	0.0464	0.0722	0.0407	0.3086	0.3059	0.3195
	$n = 35$	0.0589	0.1667	0.0956	0.3270	0.3250	0.2970
0.8	$n = 100$	0.0211	0.0667	0.0436	0.2818	0.2749	0.2641
	$n = 80$	0.0524	0.1096	0.0758	0.2970	0.2891	0.2470
	$n = 50$	0.0630	0.3085	0.1841	0.3276	0.3273	0.3030
	$n = 35$	0.1217	0.6176	0.3460	0.3323	0.3298	0.3298

when the censoring level is 15% the MSE of the estimator using Gompertz kernel is small when it is compared with the other 2 kernels. The MSE of the estimator using Gaussian kernel is small, when the censoring level is 80%.

Acknowledgments

The authors would like to express their gratitude to the referee and chief editor for their valuable suggestions which have considerably improved the earlier version of the paper.

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