

Nonparametric System Identification

Presenting a thorough overview of the theoretical foundations of nonparametric systems identification for nonlinear block-oriented systems, Włodzimierz Greblicki and Mirosław Pawlak show that nonparametric regression can be successfully applied to system identification, and they highlight what you can achieve in doing so.

Starting with the basic ideas behind nonparametric methods, various algorithms for nonlinear block-oriented systems of cascade and parallel forms are discussed in detail. Emphasis is placed on the most popular systems, Hammerstein and Wiener, which have applications in engineering, biology, and financial modeling.

Algorithms using trigonometric, Legendre, Laguerre, and Hermite series are investigated, and the kernel algorithm, its semirecursive versions, and fully recursive modifications are covered. The theories of modern nonparametric regression, approximation, and orthogonal expansions are also provided, as are new approaches to system identification. The authors show how to identify nonlinear subsystems so that their characteristics can be obtained even when little information exists, which is of particular significance for practical application. Detailed information about all the tools used is provided in the appendices.

This book is aimed at researchers and practitioners in systems theory, signal processing, and communications. It will also appeal to researchers in fields such as mechanics, economics, and biology, where experimental data are used to obtain models of systems.

Włodzimierz Greblicki is a professor at the Institute of Computer Engineering, Control, and Robotics at the Wrocław University of Technology, Poland.

Mirosław Pawlak is a professor in the Department of Electrical and Computer Engineering at the University of Manitoba, Canada. He was awarded his Ph.D. from the Wrocław University of Technology, Poland.

Both authors have published extensively over the years in the area of nonparametric theory and applications.



Nonparametric System Identification

WŁODZIMIERZ GREBLICKI

Wrocław University of Technology

MIROSŁAW PAWLAK

University of Manitoba, Canada





CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press

32 Avenue of the Americas, New York, NY 10013-2473, USA

www.cambridge.org

Information on this title: www.cambridge.org/9780521868044

© Cambridge University Press 2008

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2008

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication Data

Greblicki, Włodzimierz.

Nonparametric system identification / W. Greblicki, M. Pawlak.

p. cm.

Includes bibliographical references and index.

ISBN 978-0-521-86804-4 (hardcover)

- 1. System identification. 2. Nonparametric signal detection. 3. Signal processing Mathematics.
- 4. Mathematical optimization. I. Pawlak, M. (Mirosław), 1954– II. Title.

QA402.G7315 2008

003'.1-dc22 2008014570

ISBN 978-0-521-86804-4 hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party Internet Web sites referred to in this publication and does not guarantee that any content on such Web sites is, or will remain, accurate or appropriate.



Contents

| | Prefa | ice | page ix |
|---|-----------------------------------|---------------------------------------|---------|
| 1 | Intro | duction | 1 |
| 2 | Discrete-time Hammerstein systems | | 3 |
| | 2.1 | The system | 3 |
| | 2.2 | Nonlinear subsystem | 4 |
| | 2.3 | Dynamic subsystem identification | 8 |
| | 2.4 | Bibliographic notes | 9 |
| 3 | Kerne | el algorithms | 11 |
| | 3.1 | Motivation | 11 |
| | 3.2 | Consistency | 13 |
| | 3.3 | Applicable kernels | 14 |
| | 3.4 | Convergence rate | 16 |
| | 3.5 | The mean-squared error | 21 |
| | 3.6 | Simulation example | 21 |
| | 3.7 | Lemmas and proofs | 24 |
| | 3.8 | Bibliographic notes | 29 |
| 4 | Semirecursive kernel algorithms | | 30 |
| | 4.1 | Introduction | 30 |
| | 4.2 | Consistency and convergence rate | 31 |
| | 4.3 | Simulation example | 34 |
| | 4.4 | Proofs and lemmas | 35 |
| | 4.5 | Bibliographic notes | 43 |
| 5 | Recursive kernel algorithms | | 44 |
| | 5.1 | Introduction | 44 |
| | 5.2 | Relation to stochastic approximation | 44 |
| | 5.3 | Consistency and convergence rate | 46 |
| | 5.4 | Simulation example | 49 |
| | 5.5 | Auxiliary results, lemmas, and proofs | 51 |
| | 5.6 | Bibliographic notes | 58 |



| VI | Contents |
|----|----------|
| | |
| | |

| 6 | Ortho | gonal series algorithms | 59 |
|----|--------------------------------------|-------------------------------------|-----|
| U | 6.1 | Introduction | 59 |
| | 6.2 | Fourier series estimate | 61 |
| | 6.3 | Legendre series estimate | 64 |
| | 6.4 | Laguerre series estimate | 66 |
| | 6.5 | Hermite series estimate | 68 |
| | 6.6 | Wavelet estimate | 69 |
| | 6.7 | Local and global errors | 70 |
| | 6.8 | Simulation example | 71 |
| | 6.9 | Lemmas and proofs | 72 |
| | 6.10 | Bibliographic notes | 78 |
| 7 | Algorithms with ordered observations | | 80 |
| | 7.1 | Introduction | 80 |
| | 7.2 | Kernel estimates | 81 |
| | 7.3 | Orthogonal series estimates | 85 |
| | 7.4 | Lemmas and proofs | 89 |
| | 7.5 | Bibliographic notes | 99 |
| 8 | Continuous-time Hammerstein systems | | 101 |
| | 8.1 | Identification problem | 101 |
| | 8.2 | Kernel algorithm | 103 |
| | 8.3 | Orthogonal series algorithms | 106 |
| | 8.4 | Lemmas and proofs | 108 |
| | 8.5 | Bibliographic notes | 112 |
| 9 | | ete-time Wiener systems | 113 |
| | 9.1 | The system | 113 |
| | 9.2 | Nonlinear subsystem | 114 |
| | 9.3 | Dynamic subsystem identification | 119 |
| | 9.4 | Lemmas | 121 |
| | 9.5 | Bibliographic notes | 122 |
| 10 | | el and orthogonal series algorithms | 123 |
| | 10.1 | Kernel algorithms | 123 |
| | 10.2 | Orthogonal series algorithms | 126 |
| | 10.3 | Simulation example | 129 |
| | 10.4 | Lemmas and proofs | 130 |
| | 10.5 | Bibliographic notes | 142 |
| 11 | Continuous-time Wiener system | | 143 |
| | 11.1 | Identification problem | 143 |
| | 11.2 | Nonlinear subsystem | 144 |
| | 11.3 | Dynamic subsystem | 146 |
| | 11.4 | Lemmas | 146 |
| | 11.5 | Bibliographic notes | 148 |



| | | Contents | vii |
|----|----------------------------------|---|-----|
| | | | |
| 12 | Other | block-oriented nonlinear systems | 149 |
| | 12.1 | Series-parallel, block-oriented systems | 149 |
| | 12.2 | Block-oriented systems with nonlinear dynamics | 173 |
| | 12.3 | Concluding remarks | 218 |
| | 12.4 | Bibliographical notes | 220 |
| 13 | Multiv | ariate nonlinear block-oriented systems | 222 |
| | 13.1 | Multivariate nonparametric regression | 222 |
| | 13.2 | Additive modeling and regression analysis | 228 |
| | 13.3 | Multivariate systems | 242 |
| | 13.4 | Concluding remarks | 248 |
| | 13.5 | Bibliographic notes | 248 |
| 14 | - | arametric identification | 250 |
| | 14.1 | Introduction | 250 |
| | 14.2 | Semiparametric models | 252 |
| | 14.3 | Statistical inference for semiparametric models | 255 |
| | 14.4 | Statistical inference for semiparametric Wiener models | 264 |
| | 14.5 | Statistical inference for semiparametric Hammerstein models | 286 |
| | 14.6 | Statistical inference for semiparametric parallel models | 287 |
| | 14.7 | Direct estimators for semiparametric systems | 290 |
| | 14.8 | Concluding remarks | 309 |
| | 14.9 | Auxiliary results, lemmas, and proofs | 310 |
| | 14.10 | Bibliographical notes | 316 |
| A | Convolution and kernel functions | | |
| | A.1 | Introduction | 319 |
| | A.2 | Convergence | 320 |
| | A.3 | Applications to probability | 328 |
| | A.4 | Lemmas | 329 |
| В | - | gonal functions | 331 |
| | B.1 | Introduction | 331 |
| | B.2 | Fourier series | 333 |
| | B.3 | Legendre series | 340 |
| | B.4 | Laguerre series | 345 |
| | B.5 | Hermite series | 351 |
| | B.6 | Wavelets | 355 |
| C | Probability and statistics | | 359 |
| | C.1 | White noise | 359 |
| | C.2 | Convergence of random variables | 361 |
| | C.3 | Stochastic approximation | 364 |
| | C.4 | Order statistics | 365 |
| | References | | 371 |
| | Index | | 387 |



To my wife, Helena, and my children, Jerzy, Maria, and Magdalena – WG To my parents and family and those whom I love – MP



Preface

The aim of this book is to show that the nonparametric regression can be applied successfully to nonlinear system identification. It gathers what has been done in the area so far and presents main ideas, results, and some new recent developments.

The study of nonparametric regression estimation began with works published by Cencov, Watson, and Nadaraya in the 1960s. The history of nonparametric regression in system identification began about ten years later. Such methods have been applied to the identification of composite systems consisting of nonlinear memoryless systems and linear dynamic ones. Therefore, the approach is strictly connected with so-called block-oriented methods developed since Narendra and Gallman's work published in 1966. Hammerstein and Wiener structures are most popular and have received the greatest attention in numerous applications. Fundamental for nonparametric methods is the observation that the unknown characteristic of the nonlinear subsystem or its inverse can be represented as regression functions.

In terms of the a priori information, standard identification methods and algorithms work when it is parametric, that is, when our knowledge about the system is rather large; for example, when we know that the nonlinear subsystem has a polynomial characteristic. In this book, the information is much smaller, nonparametric. The mentioned characteristic can be, for example, any integrable or bounded or, even, any Borel function.

It can thus be said that this book associates block-oriented system identification with nonparametric regression estimation and shows how to identify nonlinear subsystems, that is, to recover their characteristics when the a priori information is small. Because of this, the approach should be of interest not only to researchers but also to people interested in applications.

Chapters 2–7 are devoted to discrete-time Hammerstein systems. Chapter 2 presents basic discussion of the Hammerstein system and its relationship with the concept of the nonparametric regression. The nonparametric kernel algorithm is presented in Chapter 3, its semirecursive versions are examined in Chapter 4, and Chapter 5 deals with fully recursive modifications derived from the idea of stochastic approximation. Next, Chapter 6 is concerned with the nonparametric orthogonal series method. Algorithms using trigonometric, Legendre, Laguerre, and Hermite series are investigated. Some space is devoted to estimation methods based on wavelets. Nonparametric algorithms based on ordered observations are presented and examined in Chapter 7. Chapter 8 discusses the nonparametric algorithms when applied to continuous-time Hammerstein systems.



x Preface

The Wiener system is identified in Chapters 9–11. Chapter 9 presents the motivation for nonparametric algorithms that are studied in the next two chapters devoted to the discrete and continuous-time Wiener systems, respectively. Chapter 12 is concerned with the generalization of our theory to other block-oriented nonlinear systems. This includes, among others, parallel models, cascade-parallel models, sandwich models, and generalized Hammerstein systems possessing local memory. In Chapter 13, the multivariate versions of block-oriented systems are examined. The common problem of multivariate systems, that is, the curse of dimensionality, is cured by using low-dimensional approximations. With respect to this issue, models of the additive form are introduced and examined. In Chapter 14, we develop identification algorithms for a semiparametric class of block-oriented systems. Such systems are characterized by a mixture of finite dimensional parameters and nonparametric functions being typically a set of univariate functions.

The reader is encouraged to look into the appendices, in which fundamental information about tools used in the book is presented in detail. Appendix A is strictly related to kernel algorithms, and Appendix B is tied with the orthogonal series nonparametric curve estimates. Appendix C recalls some facts from probability theory and presents results from the theory of order statistics used extensively in Chapter 7.

Over the years, our work has benefited greatly from the advice and support of a number of friends and colleagues with interest in ideas of nonparametric estimation, pattern recognition, and nonlinear system modeling. There are too many names to list here, but special mention is due to Adam Krzyżak, as well as Danuta Rutkowska, Leszek Rutkowski, Alexander Georgiev, Simon Liao, Pradeepa Yahampath, and Yongqing Xin – our past Ph.D. students, now professors at universities in Canada, the United States, and Poland. Cooperation with them has been a great pleasure and given us a lot of satisfaction. We are deeply indebted to Zygmunt Hasiewicz, Ewaryst Rafajłowicz, Uli Stadtmüller, Ewa Rafajłowicz, Hajo Holzmann, and Andrzej Kozek, who have contributed greatly to our research in the area of nonlinear system identification, pattern recognition, and nonparametric inference.

Last, but by no means least, we would like to thank Mount-first Ng for helping us with a number of typesetting problems. Ed Shwedyk and January Gnitecki have provided support for correcting English grammar.

We also thank Anna Littlewood, from Cambridge University Press, for being a very supportive and patient editor. Research presented in this monograph was partially supported by research grants from Wrocław University of Technology, Wrocław, Poland, and NSERC of Canada.

Wrocław, Winnipeg February 2008 Włodzimierz Greblicki, Mirosław Pawlak