*Econometrica*, Vol. 64, No. 5 (September, 1996), 1183–1193

# NONPARAMETRIC TESTS OF STOCHASTIC DOMINANCE IN INCOME DISTRIBUTIONS

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Tests for stochastic dominance, based upon extensions of the Goodness of Fit Test to the nonparametric comparison of income distributions, are proposed, implemented, and compared with indirect tests of second order stochastic dominance currently utilized in income distribution studies.

KEYWORDS: Stochastic dominance, poverty orderings, nonparametric tests.

## INTRODUCTION

THE COMPARISON OF INCOME, wealth, and earnings distributions has long been an integral part of income, wealth, and poverty studies and an important component in the public economist's toolkit. A popular means of comparison is the Lorenz curve and its related statistics, the Gini and Schutz coefficients (Lambert (1989)). Statistical comparison has been facilitated by the development of asymptotic distributions of Lorenz curve ordinates (Beach and Davidson (1983); Gastwirth and Gail (1985)) and joint confidence bands for such ordinates (Beach and Richmond (1985)). Concerns regarding Lorenz curves (Sen (1973)) lead to the development of the generalized Lorenz curve (Shorrocks (1983)) (the asymptotic distribution of its ordinates had in fact been developed implicitly in Beach and Davidson (1983)[2]). However the range of statements that can be made using generalized Lorenz curve comparisons remains limited.

Recent interest in poverty issues has centered on unambiguous ranking of income distributions in the sense that for any potential poverty line the ordering between two distributions remains unaltered. These orderings can be based upon a range of poverty measures, many of which are members of a general class proposed in Foster, Greer, and Thorbecke (1984). For specific members of this class an equivalence has been established between poverty orderings and stochastic dominance of various forms between the two income distributions being compared. Furthermore this equivalence extends to orderings of social welfare in the class of utilitarian social welfare functions (see Propositions 1 and 2 in Foster and Shorrocks (1988)[3]). Thus social preferences based upon mono-

[1] Many thanks are due to two anonymous referees for their helpful comments. Support from SSHRC on Research Grant Number 3-309-156-50 and the research assistance of Peter Ibbot is gratefully acknowledged.

[2] This was pointed out in Bishop, Chakraborti, and Thistle (1989).

[3] This paper also establishes that generalized Lorenz dominance implies second order stochastic dominance in the underlying distributions.

tonic utilitarian functions corresponds to first order stochastic dominance of the income distribution, preference for mean preserving progressive transfer corresponds to second order dominance, and preference for progressive transfers at lower income levels corresponds to third order dominance. The establishment of this rich relationship between poverty orderings, stochastic dominance in distributions, and social welfare functions, given appropriate statistical instruments, admits a wider range of assessments regarding such orderings than is possible using generalized Lorenz curves. Thus techniques that compare underlying wealth or income distributions may be more appropriate in this context.

This paper proposes nonparametric methods of comparing distributions directly, avoiding the use of generalized Lorenz curves. The advantage is that specific aspects of potential differences can be addressed in the context of the original distribution rather than some transformation therefrom. Analogs of the Pearson goodness of fit test (PATs) yield omnibus tests of distributional differences and linear transformations of the underlying standard normal variates provide simple tests (involving multiple comparison procedures) of all three forms of stochastic dominance of interest in the poverty literature.[4] PAT's can be shown to be likelihood ratio tests of distributional differences (Anderson (1995)); the tests involving multiple comparison procedures are akin to the Wald tests for inequalities discussed in Wolak (1989) in the context of the linear regression model. Section 1 introduces and discusses PAT's and Section 2 describes the tests for stochastic dominance. An application to Canadian household income data is provided in Section 3, and conclusions are drawn in Section 4.

## 1. COMPARING DISTRIBUTIONS

Pearson's goodness of fit test (Andrews (1988)) is commonly used in studying distributional assumptions. It is based upon partitioning the range of a random variable $Y$ and $k$ mutually exclusive and exhaustive categories. Then $x_i$, the number of observations on $Y$ falling in the $i$th category, is distributed multinomially with probabilities $p_i$, $i = 1, \ldots, k$, such that

$$\sum_{i=1}^{k} x_i = n, \quad \sum_{i=1}^{k} p_i = 1,$$

where the $p_i$ are given by the hypothesized distribution of $Y$. A multivariate central limit theorem (Kendall and Stewart (1979)) implies that the $k \times 1$

[4] One note of caution: Techniques explored here appear useful in studying stochastic dominance issues in analyzing asset returns over time; however critical to these tests is the assumption of independent sampling which is generally not tenable for most asset return applications. Klecan, McFadden, and McFadden (1991) deal with this issue and provide a Kolmogorov–Smirnoff test for stochastic dominance when $n$ observations on $k$ alternatives are available.

dimensioned empirical frequency vector $x$ (with typical element $x_i$) is asymptotically distributed $N(\mu, \Omega)$ where

$$n^{-1}\mu = \left\| \begin{matrix} p_1 \\ p_2 \\ \cdot \\ \cdot \\ p_k \end{matrix} \right\|; \quad n^{-1}\Omega = \left\| \begin{matrix} p_1(1-p_1) & -p_1p_2 & \cdots & -p_1p_k \\ -p_2p_1 & p_2(1-p_2) & \cdots & -p_2p_k \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ -p_kp_1 & -p_kp_2 & \cdots & p_k(1-p_k) \end{matrix} \right\|.$$

Given the usual caveat that cell sizes are chosen so that $np_i > 5$ for all $i = 1, \ldots, k$ (see (Rao (1973)), a test, distributed $\chi^2(k-1)$, of the null distribution is given by

$$(x-\mu)'\Omega^g(x-\mu)$$

where $\Omega^g$ is the generalized inverse of $\Omega$.[5]

Extending these ideas to the comparison of two distributions, let $x^A$ and $x^B$ be the empirical frequency vectors based upon samples of size $n^A$ and $n^B$ drawn respectively from populations $A$ and $B$ (partitioned in the same manner in each case). Under a null of common population distributions and the assumption of independence of the two samples, it can easily be shown that

(1) $\quad v = x^A/n^A - x^B/n^B$ is asymptotically distributed as $N(0, m\Omega)$

where $m = n^{-1}(n^A + n^B)/n^A n^B$ and $v'(m\Omega)^g v$ is asymptotically distributed as $\chi^2(k-1)$.

Equation (1) can either be predicated on a mutually exclusive and exhaustive partition of the rangespace of $Y$, so that $p$ has to be determined, or upon prior specification of $p$, leaving the corresponding intervals or fractiles of $Y$ to be determined. In the absence of a specified common null distribution either has to be estimated, by $p^* = (x^A + x^B)/(n^A + n^B)$ in the case of predetermined partition of the rangespace, or by the appropriate joint sample fractiles for some given $p$ in the latter case. Proof of consistent estimation in each case is readily available in the literature (see, for example, Rao (1973) or White (1984)). Denoting $n^{-1}\Omega^*$ as the estimate of $n^{-1}\Omega$, it may be shown that a PAT $v'(m\Omega^*)^g v = v'(m\Omega)^g v + O(1/n)$; hence

(2) $\quad v'(m\Omega^*)^g v$ is asymptotically distributed as $\chi^2(k-1)$.[6,7]

[5] Modifications of this test, based upon power enhancing linear combinations of the elements of $x - \mu$, that focus upon particular aspects of the null and allow for testing against specific alternatives are considered in Anderson (1994a).

[6] The degree to which this statistic behaves according to its asymptotic distribution in small samples is the subject of a small Monte Carlo study reported in Appendix 1 of Anderson (1994b). Essentially the results suggest that little of the size and power characteristics are lost when the underlying parameters remain unspecified.

[7] (2) is easily established as the likelihood ratio test for the difference in two distributions based upon the multinomial distribution of $x$ (see Anderson (1995)).

## 2. TESTS OF STOCHASTIC DOMINANCE

Possibilities of exploring linear combinations of the vector $v$ directly are explored here without the null distribution of $Y$ being formally specified[8] but, as above, maintaining a null of a common underlying distribution. A special case of these is the test for differences in population proportions available in standard texts (for example, Rao (1973)) which is clearly a test of percentile dominance. Combining the two data sets to establish the overall percentile $Y^*$, partition each sample into $y < Y^*$ and $y > Y^*$ and define $x_A$ and $x_B$ accordingly (in this case $k = 2$) letting $i^{*\prime} = [1 \;\; 0]$; $i^{*\prime} v$ is then $N(0, i^{*\prime} m \Omega^* i^*)$ under the null of nondominance and provides a means of examining the hypothesis.

More generally let $Y$ be the rangespace of incomes from two income distributions $A$ and $B$ with cumulative distribution functions $F_A(y)$ and $F_B(y)$ respectively. Stochastic dominance of $B$ with respect to $A$ is equivalent to and requires tests of the following:

First Order Stochastic Dominance:

$$F_A(y) \leq F_B(y), \quad F_A(y_i) \neq F_B(y_1) \quad \text{for some } i, \quad \forall y \in Y.$$

Second Order Stochastic Dominance:

$$\int_{-\infty}^{y_i} [F_B(z) - F_A(z)]\, dz \geq 0, \quad F_B(y_i) \neq F_A(y_i) \quad \text{for some } y_i, \quad \forall y \in Y.$$

Third Order Stochastic Dominance:

$$\int_{-\infty}^{y_i} \int_{-\infty}^{w} [F_B(z) - F_A(z)]\, dz\, dw \geq 0, \quad F_B(y_i) \neq F_A(y_i) \quad \text{for some } y_i, \quad \forall y \in Y.$$

In the context of a common partitioning of the rangespace of the two distributions into $k$ mutually exclusive and exhaustive intervals with respective relative frequency vectors $p_A$ and $p_B$, let $d_j$ be the $j$th interval length; then, were $F_a$ and $F_B$ known, probabilities of falling in the $j$th category would be given by

$$p_j = F(y^j) - F(y^{j-1}) \quad \text{where} \quad y^h = \sum_{i=1}^{h} d_i, \qquad F(y^0) = 0.$$

[8] Indeed it appears to be very difficult to choose an appropriate distribution; see Harrison (1982).

Modifying the trapezoidal rule for approximating integrals (Goodman (1967)) to permit nonequal intervals suggests that

$$F(y^j) = \sum_{i=1}^{j} p_i,$$

$$C(y^j) = \int_0^{y^j} F(z)\,dz \simeq 0.5\left[F(y^j)\,d_j + \sum_{i=1}^{j-1}(d_i + d_{i+1})F(y^i)\right],$$

$$\int_0^{y^j} C(z)\,dz \simeq 0.5\left[C(y^j)\,d_j + \sum_{i=1}^{j-1}(d_i + d_{i+1})C(y^i)\right].$$

By defining two matrices as follows:

$$I_f = \begin{Vmatrix} 1 & 0 & 0 & \cdot & \cdot & 0 \\ 1 & 1 & 0 & \cdot & \cdot & 0 \\ 1 & 1 & 1 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & 1 & 1 & \cdot & \cdot & 1 \end{Vmatrix},$$

$$I_F = 0.5\begin{Vmatrix} d_1 & 0 & 0 & \cdot & \cdot & 0 \\ d_1 + d_2 & d_2 & 0 & \cdot & \cdot & 0 \\ d_1 + d_2 & d_2 + d_3 & d_3 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ d_1 + d_2 & d_2 + d_3 & d_3 + d_4 & \cdot & \cdot & d_k \end{Vmatrix},$$

discrete analogues can be used to contemplate the three forms of dominance by focusing on a null of no dominance, viz:

First Order Stochastic Dominance:

$H_0$: $I_f(p^A - p^B) = 0$ against $H_1$: $I_f(p^A - p^B) \leq 0$;
note $H_2$: $I_f(p^A - p^B) \nleq \wedge \ngeq 0 \Rightarrow$ indeterminate;

Second Order Stochastic Dominance:

$H_0$: $I_F I_f(p^A - p^B) = 0$ against $H_1$: $I_F I_f(p^A - p^B) \leq 0$;
note $H_2$: $I_F I_f(p^A - p^B) \nleq \wedge \ngeq 0 \Rightarrow$ indeterminate;

Third Order Stochastic Dominance:

$$H_0\colon I_F I_F I_f(p^A - p^B) = 0 \quad \text{against} \quad H_1\colon I_F I_F I_f(p^A - p^B) \leq 0;$$
$$\text{note} \quad H_2\colon I_F I_F I_f(p^A - p^B) \nleq \wedge \ngeq 0 \Rightarrow \text{indeterminate};$$

where in each case under the alternative strict inequality must hold for at least one element of the vector.

These hypotheses can be examined in the context of $v_f = I_f v$, $v_F = I_F I_f v$, and $v_C = I_F I_F I_f v$, which, for suitably specified partitions, have well defined asymptotically normal distributions;[9,10] however they do involve multiple comparison procedures. Fortunately these have been worked out (Richmond (1982)) and employed in the context of Lorenz curve ordinate confidence regions (Beach and Richmond (1985)) which require the use of the studentized maximum modulus distribution (Stoline and Ury (1979)).[11] Following the convenient convention in Bishop, Chakraborti, and Thistle (1989), the hypothesis of dominance of distribution $A$ over distribution $B$ requires that no element of the appropriate vector $v$ be significantly greater than 0 whilst at least one element is significantly less. Since the test is perfectly symmetric, dominance of $B$ over $A$ requires that no element of $v$ be significantly less than 0 whilst at least one is significantly greater.[12] Note that as first order stochastic dominance implies second order dominance which in turn implies third order dominance (Foster and Shorrocks (1988)), the respective tests are correspondingly stronger.

In the following example the pooled sample was split into deciles; thus $p$ was given determining the $d_j$'s in the matrix $I_F$ as the difference between the $j$th and $j-1$th deciles of the pooled sample. The relative frequencies for the individual samples are then calculated for each decile and the vector $v$ with typical element $v_j = x_{Aj}/n_A - x_{Bj}/n_B$ is calculated. Inferences are based upon the vectors $I_f v$, $I_F I_f v$, and $I_F I_F I_f v$ which, from (1), are each normally distributed asymptotically. Dividing each element by its standard deviation permits multiple comparisons using the studentized maximum modulus distribution.

It should be noted that partitioning into decile groups is arbitrary, chosen in this case for coherence with usual practice in the existing income comparison literature. With regard to power considerations the arguments in the standard Pearson test case are appropriate here. In that case, if there is a specific

[9] Interestingly these premultiplying matrices behave like aggregators, placing most weight on the poverty end of the income scale, which accords with the concerns expressed in Atkinson (1983) that in income studies such a region should be the main focus of attention.

[10] In practice the pooled sample is employed to determine the fractiles (employed in calculating $I_F$) in the case of predetermined $p$, or the vector $p$ (employed in calculating $\Omega$) in the case of a predetermined partition of $Y$.

[11] Appropriately normed quadratic forms of $v_f$ and $v_F$ would yield identical statistics to (2); however this is not the case for multiple comparison procedures involving these vectors.

[12] Note that implementation of a similar test based upon generalized Lorenz curve ordinates for Lorenz curve (second order stochastic) dominance would require the reversal of this decision rule.

alternative distribution in mind, power is to be gained by location of partition points at intersections of the null and alternative distribution functions (see Anderson (1994a)). In the problem confronted here identical arguments indicate that power will be gained by locating partition points at fractiles where it is thought that the two distributions may intersect; of course the number of intervals will be governed by the number of potential intersection points. Absent this information, the standard tenet for gaining power in Pearson tests of equalizing cell probabilities under the null whilst maintaining an expected cell frequency of at least 5 is a good one to follow (see Cochran (1952)).

## 3. AN APPLICATION

Roughly 8000 household units were randomly sampled from the Canadian Family Income Survey in each of the years 1973, 1977, 1981, 1985, and 1989 and pre and post tax unit income data were collected and deflated by the consumer price index (base year 1988) for the corresponding year. Units with zero and negative reported incomes were eliminated from the sample (this always corresponded to less than 0.6% of the sample in each case).[13] Basic statistical detail for each sample is reported in Table I.

Of significance is the fall in both pre and post tax mean incomes between 1981 and 1985; the respective standard normal test statistics for these are 2.80 and 3.643. The situation does appear to have recovered by 1989 when the difference between 1989 and 1981 means in pre and post tax incomes yield respective standard normal test statistics of 2.41 and 1.20. Note also that during the observation periods the standard deviation of the pretax income distribution was steadily increasing as is the case, with the exception of 1989, for post tax incomes.

Tests were performed for the three forms of stochastic dominance together with the Beach–Davidson test for generalized Lorenz dominance of each sample

TABLE I

| | Year | | | | |
|---|---|---|---|---|---|
| | 1973 | 1977 | 1981 | 1985 | 1989 |
| Sample size | 8599 | 7841 | 8146 | 8145 | 8552 |
| CPI Income Deflator | 0.380 | 0.534 | 0.763 | 0.963 | 1.130 |
| Mean Income Pre (Post) Tax | 29079 | 32299 | 34511 | 33468 | 35431 |
| | (24968) | (27906) | (29456) | (28388) | (29115) |
| Median Income Pre (Post) Tax | 25989 | 29177 | 30712 | 28912 | 30822 |
| | (22876) | (25981) | (26763) | (25212) | (26204) |
| Std Deviation Pre (Post) Tax | 19583 | 21747 | 23585 | 24700 | 25795 |
| | (15391) | (17143) | (18433) | (18982) | (18228) |

[13] It should be acknowledged that whilst the sampling from the income survey was random, the observations making up the original survey were not; as in all other studies this difficulty is simply assumed away.

survey over its predecessor. For the stochastic dominance tests partitioning was based upon the population decile groups of the combined sample. For the generalized Lorenz tests the usual practice of partitioning into income decile groups of the combined sample was followed. The results of all the tests are always unambiguous with stochastic dominance of all orders and generalized Lorenz dominance being established in every case. Excluding 1985 the comparisons were very similar, with each distribution dominating its chronological predecessor in all categories both in pre and post tax situations; however 1985 was dominated by 1981. Thus, for brevity, a selection of the results, namely the 1973–1977 (a typical case), 1981–1985 (the exceptional case), and a 1981–1989 comparison, are reported in Table II (details of all tests are available from the author).

TABLE II

Income Comparison Years 1973–1977

| Before Tax Stochastic Dominance Order | | | | After Tax Stochastic Dominance Order | | | |
|---|---|---|---|---|---|---|---|
| 1st | 2nd | 3rd | GL | 1st | 2nd | 3rd | GL |
| 0.44 | 0.44 | 0.44 | −0.49 | 0.63 | 0.63 | 0.63 | −0.69 |
| 1.24 | 0.88 | 0.78 | −0.26 | 2.07 | 1.41 | 1.22 | −0.82 |
| 3.08 | 1.83 | 1.40 | −1.34 | 4.43 | 2.76 | 2.12 | −2.13 |
| 5.74 | 3.31 | 2.36 | −2.60 | 7.99 | 4.72 | 3.40 | −3.66 |
| 9.09 | 5.26 | 3.64 | −4.16 | 10.33 | 6.85 | 4.92 | −5.32 |
| 10.42 | 7.18 | 5.09 | −5.81 | 11.85 | 8.73 | 6.48 | −6.96 |
| 10.70 | 8.67 | 6.49 | −7.26 | 12.83 | 10.35 | 7.95 | −8.49 |
| 10.68 | 9.80 | 7.73 | −8.61 | 12.33 | 11.62 | 9.28 | −10.00 |
| 9.82 | 10.69 | 8.81 | −9.84 | 10.93 | 12.54 | 10.45 | −11.33 |
| 0.00 | 11.48 | 10.61 | −9.94 | 0.00 | 13.29 | 12.40 | −11.53 |
| | PAT ($\chi^2(9)$) | 171.37 | | | PAT ($\chi^2(9)$) | 218.03 | |
| | BDT ($\chi^2(9)$) | 35.07 | | | BDT ($\chi^2(9)$) | 28.34 | |
| | GBDT ($\chi^2(10)$) | 175.45 | | | GBDT ($\xi^2(10)$) | 216.75 | |

Income Comparison Years 1981–1985

| Before Tax Stochastic Dominance Order | | | | After Tax Stochastic Dominance Order | | | |
|---|---|---|---|---|---|---|---|
| 1st | 2nd | 3rd | GL | 1st | 2nd | 3rd | GL |
| −0.47 | −0.47 | −0.47 | 0.40 | −0.60 | −0.60 | −0.60 | 0.35 |
| −2.43 | −1.47 | −1.20 | 2.06 | −2.61 | −1.63 | −1.36 | 2.15 |
| −3.85 | −2.68 | −2.08 | 2.50 | −4.07 | −2.86 | −2.24 | 2.65 |
| −3.97 | −3.50 | −2.87 | 3.35 | −4.80 | −3.85 | −3.09 | 3.62 |
| −4.40 | −4.03 | −3.47 | 3.87 | −4.22 | −4.38 | −3.75 | 4.16 |
| −2.93 | −4.16 | −3.85 | 4.04 | −3.71 | −4.52 | −4.16 | 4.40 |
| −1.85 | −3.90 | −3.98 | 3.96 | −3.03 | −4.48 | −4.37 | 4.39 |
| −1.25 | −3.56 | −3.92 | 3.79 | −2.25 | −4.33 | −4.43 | 4.38 |
| −1.28 | −3.31 | −3.77 | 3.58 | −2.25 | −4.20 | −4.42 | 4.39 |
| 0.00 | −2.54 | −3.06 | 2.74 | 0.00 | −3.69 | −4.12 | 3.63 |
| | PAT ($\chi^2(9)$) | 26.51 | | | PAT ($\chi^2(9)$) | 27.60 | |
| | BDT ($\chi^2(9)$) | 26.57 | | | BDT ($\chi^2(9)$) | 29.70 | |
| | GBDT ($\chi^2(10)$) | 31.42 | | | GBDT ($\chi^2(10)$) | 42.41 | |

TABLE II—*Continued*

| Income Comparison Years 1973–1977 | | | | | | | |
|---|---|---|---|---|---|---|---|
| Before Tax Stochastic Dominance Order | | | | After Tax Stochastic Dominance Order | | | |
| 1st | 2nd | 3rd | GL | 1st | 2nd | 3rd | GL |
| 3.67 | 3.67 | 3.67 | −5.72 | 3.29 | 3.29 | 3.29 | −4.81 |
| 2.25 | 3.33 | 3.48 | −3.55 | 1.12 | 2.55 | 2.80 | −2.84 |
| 0.41 | 2.30 | 2.88 | −2.50 | −0.98 | 1.18 | 1.95 | −1.83 |
| 0.42 | 1.51 | 2.21 | −1.83 | −1.28 | 0.12 | 1.05 | −0.83 |
| 0.22 | 1.11 | 1.70 | −1.37 | −1.48 | −0.48 | 0.33 | −0.18 |
| 1.29 | 1.05 | 1.41 | −1.26 | −1.88 | −0.92 | −0.18 | 0.29 |
| 2.51 | 1.37 | 1.35 | −1.52 | −1.09 | −1.15 | −0.56 | 0.53 |
| 4.09 | 1.96 | 1.51 | −1.99 | −0.31 | −1.12 | −0.80 | 0.62 |
| 2.36 | 2.44 | 1.81 | −2.36 | −0.39 | −1.04 | −0.91 | 0.77 |
| 0.00 | 2.70 | 2.65 | −2.40 | 0.00 | −0.81 | −0.93 | 1.20 |
| | PAT ($\chi^2(9)$) | 39.13 | | | PAT ($\chi^2(9)$) | 24.77 | |
| | BDT ($\chi^2(9)$) | 41.08 | | | BDT ($\chi^2(9)$) | 39.18 | |
| | GBDT ($\chi^2(10)$) | 56.64 | | | GBDT ($\chi^2(10)$) | 39.43 | |

[a] The columns correspond to the ten decile variates which under the null are each distributed as the studentized maximum modulus distribution. It is easy to see that the 10th decile under first order dominance is degenerate because of the way the singular covariance matrix is transformed in this particular case. Otherwise with 10 multiple comparisons and infinite degrees of freedom the 1% critical value of this distribution is 3.29 (Stoline and Ury (1979, Table 1)). Note that the generalized Lorenz test requires a sign reversal for dominance in the same direction as the other tests.

[b] PAT, BDT, and GBDT correspond respectively to the Pearson analogue, the standard Beach–Davidson Lorenz curve, and the Beach–Davidson generalized Lorenz curve tests for differences in distribution. These are not tests of dominance per se but merely provide evidence of the existence of differences in distribution. The 1% critical value for these tests are respectively 21.67 for the $\chi^2(9)$ distribution and 23.21 for the $\chi^2(10)$ distribution. A Monte Carlo comparison of these tests is available from the author on request. Essentially there is very little to choose between PAT and GBDT in this context, both of which dominate BDT.

Unlike the standard Lorenz comparison, these approaches allow for the changes in overall income levels to be accommodated and the suggestion is that this factor has dominated, with the Canadian recession in 1982–1984 having a considerable impact in the patterns of dominance. This accords with the evidence in Table I, suggesting that over the 1981–1985 period incomes diminished on average whilst income variance increased. Of interest in this regard in Table II is the comparison between 1989 and 1981 which, in terms of the statistical results, is the most marginal, with post tax income distributions showing close to no stochastic dominance and the pretax distribution indicating only marginally significant dominance of the 1989 distribution over its 1981 counterpart. Observe that the generalized Lorenz test does indicate dominance in both these cases which is the only example of near conflict between the tests in this work. It would appear that the severe distributional effects of the recession had barely been overcome by the late 1980's which reflects the observations made in Table I.

## CONCLUSIONS

Recently a rich collection of criteria (essentially poverty measures) for ordering income distributions has been developed which has well established links to orderings of social welfare. Some of these criteria can be linked to the issue of whether or not one distribution stochastically dominates another to a given order. Unfortunately current statistical techniques for comparison via the generalized Lorenz curve only admit examining second order stochastic dominance. Here new nonparametric tests of the three major forms of stochastic dominance in income distributions have been proposed together with an omnibus test for the difference in two distributions. The dominance tests have been compared with the currently employed test for second order stochastic dominance. In being simple to deploy and facilitating examination of a wider range of dominance forms, these tests add to the economist's toolkit. It transpires that the distribution of the statistic for second order dominance has a simpler form than its generalized Lorenz curve equivalent and is found to have comparable size and power characteristics in the context of tests for distributional differences.

All the tests were implemented on Canadian household income data for the years 1973, 1977, 1981, 1985, and 1989. The results indicate that, with the exception of the interval immediately prior to 1985, within the class of utilitarian welfare functions, welfare gains were made over each observation period. Whether or not losses made prior to 1985 were recovered by 1989 is less clear. It is conjectured that the 1982–1984 recession was found to have a significant welfare impact in distributional terms which had not been substantially overcome by the time of the 1989 survey.

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*Manuscript received April, 1994; final revision received August, 1995.*

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## [Footnotes]

[2] **Asymptotically Distribution-Free Statistical Inference for Generalized Lorenz Curves**
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[8] **Earnings by Size: A Tale of Two Distributions**
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