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Nonregenerative Dual-Hop Cooperative Links with Selection Diversity

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The end-to-end performance of dual-hop cooperative diversity systems equipped with nonregenerative relays and a selection combining receiver at the destination terminal over independent and nonidentical Nakagami-*m* fading channels is studied. Closed-form expressions for the cumulative distribution function and the probability density function of the end-to-end signal-to-noise ratio (SNR) are presented, while analytical formulae are derived for the moments and the moment generating function. Using these statistical results, closed-form expressions for the outage probability are presented for both channel state information and fixed gain relays. Furthermore, for the case of fixed gain relay, the average end-to-end SNR, the amount of fading, and the average bit error rate can be numerically evaluated. The proposed mathematical analysis is complemented by numerical examples, including the effects on the overall performance of the SNRs unbalancing as well as the fading severity.

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1. INTRODUCTION

Relaying technology is a promising solution for the high throughput/data-rate coverage, as required in future cellular and ad hoc wireless and satellite communications systems. There are two main advantages of this relaying technology:

(1) very low transmit RF power requirements,
(2) use of spatial/multiuser diversity to combat fading.

Recently, the concept of cooperative diversity, where the mobile users relay signals for each other to emulate an antenna array and exploit the benefits of spatial diversity, has gained great interest [1–9]. More specifically, Emamian et al. [1] have studied the multiuser spatial diversity systems with channel-state-information- (CSI-) based relays, as was first proposed in [10] (and later in [6]), and have derived closed-form expressions for the outage probability over Rayleigh fading channels. CSI-based relays use the instantaneous CSI of the incoming signal to control the output gain and as a result limit the power of the retransmitted signal. In another contribution, Sendonaris et al. [3, 4] have proposed the user cooperation concept and considered practical issues related to its implementation. Moreover, Anghel and Kaveh have presented tight bounds for the outage and error probability of a distributed spatial diversity wireless system in the presence of Rayleigh fading [2]. Note that the relay considered in [2] is an ideal type of CSI-based relay, where the noise figure has been ignored from the relay gain. Also, efficient lower bounds for the average error probability in dual-hop cooperative diversity systems, especially in low average signal-to-noise ratio (SNR) region, have been presented in [5]. Moreover, Laneman et al. have proposed a variety of low-complexity cooperative protocols using the three-terminal case [6]. These protocols have applied on different relaying modes as amplify-and-forward (i.e., nonregenerative relays) and decode-and-forward (i.e., regenerative relays) and also the outage probability, using high-SNR approximations, has been analyzed. The work of Laneman et al. has been extended by Nabar et al. [7], where a new cooperative protocol is presented realizing maximum degrees of broadcasting and exhibits no receive collision. Furthermore, Zimmermann et al. [8] have presented an overview of cooperating relaying protocols and compared their performance with that of direct transmission and conventional relaying. Recently, an interesting work using selection diversity among cooperative users was presented by Bletsas et al. [9].

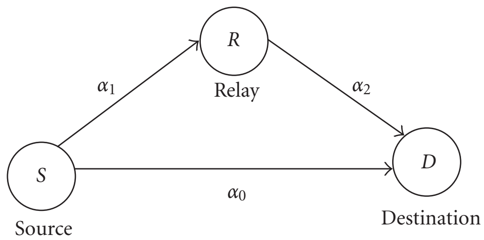

FIGURE 1: A typical wireless communication system with a cooperative diversity link.

In this work, a method of opportunistic relaying as an efficient cooperative diversity scheme has been proposed. This scheme selects the "best" relay between source and destination based on instantaneous channel measurements and neither topology information nor communication among the relays is needed.

Based upon the above, our paper presents for the first time a completely analytical approach in obtaining the end-to-end performance of dual-hop cooperative links over Nakagami-m fading channels. In doing so, we first present closed-form expressions for the probability density function (PDF) and cumulative distribution function (CDF) of the end-to-end SNR. Moreover, analytical expressions for the moments and moment generating function (MGF) can be easily obtained. These new results are then applied to study the end-to-end performance of dual-hop cooperative links such as outage probability, average end-to-end SNR, amount of fading (AoF), and average bit error rate (BER) when a selection combining (SC) receiver is assumed at the destination terminal. It will be shown that our general expressions for Nakagami-m fading reduce to previously published results for $m = 1$, that is, Rayleigh fading.

The remainder of this paper is organized as follows. Section 2 introduces the system and channel model under consideration. In Section 3, the performance analysis of the system is presented. In particular, closed-form expressions are derived for the outage probability for both types of relays. The analysis is complemented by presenting the moments and the average BER in closed form when fixed gain relay is considered. Numerical results are presented in Section 4 and concluding remarks are given in Section 5.

2. SYSTEM AND CHANNEL MODEL

A dual-hop relaying system operating over independent and nonidentical Nakagami-m fading channels is illustrated in Figure 1. The source terminal S communicates with the destination terminal D not only directly but also via the cooperative diversity link through terminal R, which acts as a non-regenerative gain relay. Each transmission period is divided into two signaling intervals: in the first signaling interval, terminal S communicates with the relay and the destination terminal, while in the second one, only the relay communicates with terminal D. The above transmission protocol was originally proposed in [10]. The destination terminal combines the received signals using a SC. Assuming that S is transmitting a signal with an average power normalized to unity, the instantaneous equivalent end-to-end SNR of the dual-hop path can be expressed as in [11, 12]

$$\gamma_{\text{end}} = \frac{(\alpha_1^2/N_{0,1})(\alpha_2^2/N_{0,2})}{(\alpha_2^2/N_{0,2}) + (1/g^2 N_{0,1})}. \tag{1}$$

In the above equation, α_i is the fading amplitude of the ith path, $i = 0, 1, 2$ (α_0 is the fading amplitude of the direct path), $N_{0,i}$ is the one-sided power spectral density of the additive white Gaussian noise at the input of R and D, respectively, and g is the gain of the relay. Since α_i is modeled as Nakagami-m random variable (RV), the instantaneous SNR, $\gamma_i = \alpha_i^2/N_{0,i}$, is a gamma distributed RV with PDF given by

$$f_{\gamma_i}(\gamma) = \frac{m_i^{m_i}}{\overline{\gamma}_i^{m_i}\Gamma(m_i)}\gamma^{m_i-1}\exp\left(-\frac{m_i\gamma}{\overline{\gamma}_i}\right), \tag{2}$$

where $\Gamma(\cdot)$ is the gamma function [13, equation (8.310/1)], $m_i \geq 1/2$ is a parameter describing the fading severity, and $\overline{\gamma}_i$ the average SNR of the ith path. Hence, its CDF can be written as

$$F_{\gamma_i}(\gamma) = 1 - \frac{\Gamma(m_i, (m_i/\overline{\gamma}_i)\gamma)}{\Gamma(m_i)}, \tag{3}$$

where $\Gamma(\cdot,\cdot)$ is the incomplete gamma function defined in [13, equation (8.350.2)].

When R has available CSI from the first hop and its gain aims to limit the output power of the relay, one kind of gain proposed by Laneman and Wornell [10] is given by

$$g_1^2 = \frac{1}{\alpha_1^2 + N_{0,1}}. \tag{4}$$

Therefore, the instantaneous equivalent end-to-end SNR of the dual-hop path can be expressed as in [1, 2, 11, 12]

$$\gamma_{\text{eq1}} = \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 + 1}. \tag{5}$$

When R introduces fixed gain to the received signal given by [14]

$$g_2^2 = \frac{1}{C N_{0,1}}, \tag{6}$$

where C is a positive constant, the instantaneous end-to-end SNR of the dual-hop path can be expressed as in [14]

$$\gamma_{\text{eq2}} = \frac{\gamma_1\gamma_2}{C + \gamma_2}. \tag{7}$$

3. PERFORMANCE ANALYSIS OF DUAL-HOP COOPERATIVE LINKS

3.1. CSI-based gain relay

3.1.1. Outage probability

When a SC receiver is employed in terminal D, the end-to-end outage probability at D, P_{out}, is defined as the probability that the equivalent output SNR, $\gamma_{\text{eq,csi}} = \max(\gamma_{\text{eq1}}, \gamma_0)$, falls below a given threshold, γ_{th}. Since it is assumed independent fading channels, the end-to-end outage probability can be written as

$$P_{out,csi} = \Pr[\gamma_{eq,csi} \leq \gamma_{th}] = F_{\gamma_0}(\gamma_{th}) F_{\gamma_{eq1}}(\gamma_{th}), \quad (8)$$

where $F_{\gamma_{eq1}}(\gamma_{th})$ and $F_{\gamma_0}(\gamma_{th})$ are the CDFs of the instantaneous end-to-end SNRs of the relay fading channel and direct path, evaluated at $\gamma = \gamma_{th}$, respectively. Since $F_{\gamma_0}(\gamma)$ is given by (3), only $F_{\gamma_{eq1}}(\gamma)$ has to be evaluated. The CDF, $F_{\gamma_{eq1}}(\gamma)$, using (5), can be expressed as

$$\begin{aligned} F_{\gamma_{eq1}}(\gamma) &= \int_0^\infty \Pr\left[\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1} \leq \gamma \mid \gamma_2\right] f_{\gamma_2}(\gamma_2)\, d\gamma_2 \\ &= \int_0^\gamma \Pr\left[\gamma_1 \geq \frac{\gamma(\gamma_2+1)}{\gamma_2-\gamma} \mid \gamma_2\right] f_{\gamma_2}(\gamma_2)\, d\gamma_2 \\ &\quad + \int_\gamma^\infty \Pr\left[\gamma_1 \leq \frac{\gamma(\gamma_2+1)}{\gamma_2-\gamma} \mid \gamma_2\right] f_{\gamma_2}(\gamma_2)\, d\gamma_2 \\ &= I_1 + I_2, \end{aligned} \quad (9)$$

where

$$\begin{aligned} I_1 &= \int_0^\gamma f_{\gamma_2}(\gamma_2)\, d\gamma_2 = F_{\gamma_2}(\gamma) = 1 - \frac{\Gamma(m_2, (m_2/\overline{\gamma}_2)\gamma)}{\Gamma(m_2)}, \\ I_2 &= \int_\gamma^\infty F_{\gamma_1}\left[\gamma\left(\frac{\gamma_2+1}{\gamma_2-\gamma}\right)\right] f_{\gamma_2}(\gamma_2)\, d\gamma_2 \\ &= \int_\gamma^\infty \left\{1 - \frac{\Gamma(m_1, (m_1\gamma/\overline{\gamma}_1)((\gamma_2+1)/(\gamma_2-\gamma)))}{\Gamma(m_1)}\right\} f_{\gamma_2}(\gamma_2)\, d\gamma_2 \\ &= \frac{\Gamma(m_2, m_2\gamma/\overline{\gamma}_2)}{\Gamma(m_2)} - \frac{m_2^{m_2}}{\overline{\gamma}_2^{m_2}\Gamma(m_1)\Gamma(m_2)} I_3, \end{aligned} \quad (10)$$

$$\begin{aligned} I_3 &= \int_\gamma^\infty \gamma_2^{m_2-1} \exp\left(-\frac{m_2\gamma_2}{\overline{\gamma}_2}\right) \\ &\quad \times \Gamma\left(m_1, \frac{m_1\gamma}{\overline{\gamma}_1}\left(\frac{\gamma_2+1}{\gamma_2-\gamma}\right)\right) d\gamma_2. \end{aligned} \quad (11)$$

Since the integral I_3 is not a tabulated one, it can be solved in closed form (see the appendix) and by using (10) and after some straightforward mathematical manipulations, $F_{\gamma_{eq1}}(\gamma)$ can be expressed as

$$\begin{aligned} F_{\gamma_{eq1}}(\gamma) &= 1 - \frac{2m_2^{m_2}(m_1-1)!\exp[-(m_1\gamma/\overline{\gamma}_1 + m_2\gamma/\overline{\gamma}_2)]}{\overline{\gamma}_2^{m_2}\Gamma(m_1)\Gamma(m_2)} \\ &\quad \times \sum_{k=0}^{m_1-1}\sum_{\ell=0}^{k}\sum_{r=0}^{m_2-1} \left\{\frac{1}{k!}\binom{k}{l}\binom{m_2-1}{r}\left(\frac{m_2}{\overline{\gamma}_2}\right)^{(\ell-r-1)/2} \right. \\ &\quad \times \left(\frac{m_1}{\overline{\gamma}_1}\right)^{(2k-\ell+r+1)/2} \\ &\quad \times \gamma^{(2k+2m_2-\ell-r-1)/2}(\gamma+1)^{(\ell+r+1)/2} \\ &\quad \left. \times K_{\ell-r-1}\left(2\sqrt{\frac{m_1m_2\gamma(1+\gamma)}{\overline{\gamma}_1\overline{\gamma}_2}}\right)\right\}, \end{aligned} \quad (12)$$

where $K_\nu(\cdot)$ is the νth-order modified Bessel function of the second kind, defined in [13, equation (8.432)]. Note, that for the case of $m_i = 1$, that is, Rayleigh fading channels, (12) reduces to the previously published result presented in [1, equation (14)].

It is clear now that by using (3) and (12), the outage probability of cooperative dual-hop links with CSI-based relays when SC diversity is considered in the destination terminal, can be expressed in closed form as

$$\begin{aligned} &P_{out,csi} \\ &= \left[1 - \frac{\Gamma(m_0, (m_0/\overline{\gamma}_0)\gamma_{th})}{\Gamma(m_0)}\right] \\ &\quad \times \left\{1 - \frac{2m_2^{m_2}(m_1-1)!\exp[-(m_1\gamma_{th}/\overline{\gamma}_1 + m_2\gamma_{th}/\overline{\gamma}_2)]}{\overline{\gamma}_2^{m_2}\Gamma(m_1)\Gamma(m_2)} \right. \\ &\quad \times \sum_{k=0}^{m_1-1}\sum_{\ell=0}^{k}\sum_{r=0}^{m_2-1}\left[\frac{1}{k!}\binom{k}{l}\binom{m_2-1}{r}\left(\frac{m_2}{\overline{\gamma}_2}\right)^{(\ell-r-1)/2} \right. \\ &\quad \times \left(\frac{m_1}{\overline{\gamma}_1}\right)^{(2k-\ell+r+1)/2} \\ &\quad \times \gamma_{th}^{(2k+2m_2-\ell-r-1)/2}(\gamma_{th}+1)^{(\ell+r+1)/2} \\ &\quad \left.\left. \times K_{\ell-r-1}\left(2\sqrt{\frac{m_1m_2\gamma_{th}(1+\gamma_{th})}{\overline{\gamma}_1\overline{\gamma}_2}}\right)\right]\right\}. \end{aligned} \quad (13)$$

3.2. *Fixed gain relay*

3.2.1. *Outage probability*

For the case of fixed gain relay, the outage probability is given by

$$P_{out,fg} = F_{\gamma_0}(\gamma_{th}) F_{\gamma_{eq2}}(\gamma_{th}), \quad (14)$$

where $F_{\gamma_{eq2}}(\gamma_{th})$ is the CDF of the instantaneous end-to-end SNR for the dual-hop path.

Using (7), $F_{\gamma_{eq2}}(\gamma_{th})$ can be written as

$$\begin{aligned} &F_{\gamma_{eq2}}(\gamma) \\ &= \int_0^\infty \Pr\left[\frac{\gamma_1\gamma_2}{\gamma_2+C} \leq \gamma \mid \gamma_2\right] f_{\gamma_2}(\gamma_2)\, d\gamma_2 \\ &= \int_0^\infty \Pr\left[\gamma_1 \leq \frac{\gamma(\gamma_2+C)}{\gamma_2} \mid \gamma_2\right] f_{\gamma_2}(\gamma_2)\, d\gamma_2 \\ &= \int_0^\infty \left\{1 - \frac{\Gamma(m_1, (m_1\gamma/\overline{\gamma}_1)((\gamma_2+C)/\gamma_2))}{\Gamma(m_1)}\right\} f_{\gamma_2}(\gamma_2)\, d\gamma_2 \\ &= 1 - \frac{m_2^{m_2}}{\overline{\gamma}_2^{m_2}\Gamma(m_1)\Gamma(m_2)} \mathcal{I}_1 \end{aligned} \quad (15)$$

with

$$\begin{aligned} \mathcal{I}_1 &= \int_0^\infty \gamma_2^{m_2-1}\exp\left(-\frac{m_2\gamma_2}{\overline{\gamma}_2}\right) \\ &\quad \times \Gamma\left(m_1, \frac{m_1\gamma}{\overline{\gamma}_1}\left(\frac{\gamma_2+C}{\gamma_2}\right)\right) d\gamma_2. \end{aligned} \quad (16)$$

Since the integral $\mathcal{I}_1$ is not a tabulated one, it can be solved in closed form by setting $y = 0$ in the integral I_3 (see the appendix) as

$$\begin{aligned}\mathcal{I}_1 = &\sum_{k=0}^{m_1-1}\sum_{\ell=0}^{k}\frac{2(m_1-1)!}{k!}\exp\left(-\frac{m_1\gamma}{\overline{\gamma}_1}\right)\binom{k}{\ell}C^{k-\ell}\\ &\times\left(\frac{m_1\gamma}{\overline{\gamma}_1}\right)^k\left(\frac{m_2}{\overline{\gamma}_2}\right)^{(k-\ell-m_2)/2}\left(\frac{\overline{\gamma}_1}{Cm_1\gamma}\right)^{(k-\ell-m_2)/2}\\ &\times K_{k-m_2-\ell}\left(2\sqrt{\frac{m_1m_2C\gamma}{\overline{\gamma}_1\overline{\gamma}_2}}\right).\end{aligned} \tag{17}$$

Using (15) and (17), $F_{\gamma_{eq2}}(\gamma)$ can be written as

$$\begin{aligned}F_{\gamma_{eq2}}(\gamma) = 1 - &\sum_{k=0}^{m_1-1}\sum_{\ell=0}^{k}\Xi[k,\ell]\exp\left(-\frac{m_1\gamma}{\overline{\gamma}_1}\right)\gamma^{(k+\ell+m_2)/2}\\ &\times K_{k-m_2-\ell}(2\sqrt{\tau\gamma}),\end{aligned} \tag{18}$$

where

$$\begin{aligned}\Xi[k,\ell] &= 2\binom{k}{\ell}\frac{(m_1-1)!C^{(k+m_2-\ell)/2}}{\Gamma(m_1)\Gamma(m_2)k!}\\ &\quad\times\left(\frac{m_1}{\overline{\gamma}_1}\right)^{(k+m_2+\ell)/2}\left(\frac{m_2}{\overline{\gamma}_2}\right)^{(k+m_2-\ell)/2},\\ \tau &= \frac{m_1m_2C}{\overline{\gamma}_1\overline{\gamma}_2}.\end{aligned} \tag{19}$$

Note that for the case of Rayleigh fading channels, (18) reduces to the previously published result presented in [14, equation (9)].

Therefore, using (3) and (18), the outage probability of cooperative dual-hop links with fixed gain relays when SC diversity is considered in the destination terminal can be expressed in closed form as

$$\begin{aligned}P_{\text{out},fg} = &\left[1-\frac{\Gamma(m_0,(m_0/\overline{\gamma}_0)\gamma_{\text{th}})}{\Gamma(m_0)}\right]\\ &\times\left\{1-\sum_{k=0}^{m_1-1}\sum_{\ell=0}^{k}\left[\Xi[k,\ell]\exp\left(-\frac{m_1\gamma_{\text{th}}}{\overline{\gamma}_1}\right)\gamma_{\text{th}}^{(k+\ell+m_2)/2}\right.\right.\\ &\left.\left.\times K_{k-m_2-\ell}(2\sqrt{\tau\gamma_{\text{th}}})\right]\right\}.\end{aligned} \tag{20}$$

3.2.2. Moments of the end-to-end SNR

The first- and the second-order moments of the end-to-end SNR are statistical parameters which can be efficiently used to evaluate important performance system measures, such as average output SNR and variance. The higher-order moments are also useful in signal processing algorithms for signal detection, classification, and estimation and they play a fundamental role in understanding the performance of wideband communication systems in the presence of fading [15].

The nth-order moment of the end-to-end SNR at terminal D is given by

$$\mu_n = E[\gamma_{\text{eq},fg}^n] = \int_0^\infty \gamma^n f_{\gamma_{\text{eq},fg}}(\gamma)d\gamma, \tag{21}$$

where $E[\cdot]$ denotes the statistical average operator, $\gamma_{\text{eq},fg} = \max(\gamma_{\text{eq2}},\gamma_0)$ and $f_{\gamma_{\text{eq},fg}}$ is the PDF of the SC output. This PDF can be found by taking the derivative of (14) with respect to γ_{th}, yielding

$$\begin{aligned}f_{\gamma_{\text{eq},fg}}(\gamma) = &\frac{m_0^{m_0}}{\overline{\gamma}_0^{m_0}\Gamma(m_0)}\exp\left(-\frac{m_0\gamma}{\overline{\gamma}_0}\right)\gamma^{m_0-1}F_{\gamma_{eq2}}(\gamma)\\ &+F_{\gamma_0}(\gamma)f_{\gamma_{eq2}}(\gamma).\end{aligned} \tag{22}$$

The PDF of the dual-hop path, $f_{\gamma_{eq2}}(\gamma)$, can be evaluated by taking the derivative of (18) with respect to γ, yielding

$$\begin{aligned}f_{\gamma_{eq2}}(\gamma) = &\sum_{k=0}^{m_1-1}\sum_{\ell=0}^{k}\Xi[k,\ell]\exp\left(-\frac{m_1\gamma}{\overline{\gamma}_1}\right)\\ &\times\left[\frac{m_1}{\overline{\gamma}_1}\gamma^{(k+\ell+m_2)/2}K_{k-\ell-m_2}(2\sqrt{\tau\gamma})\right.\\ &+\frac{\sqrt{\tau}}{2}\gamma^{(k+\ell+m_2-1)/2}K_{k-\ell-m_2-1}(2\sqrt{\tau\gamma})\\ &+\frac{\sqrt{\tau}}{2}\gamma^{(k+\ell+m_2-1)/2}K_{k+1-\ell-m_2}(2\sqrt{\tau\gamma})\\ &\left.-\frac{1}{2}(k+\ell+m_2)\gamma^{(k+\ell+m_2-2)/2}K_{k-\ell-m_2}(2\sqrt{\tau\gamma})\right].\end{aligned} \tag{23}$$

By substituting (23) into (21), the resulting integral can be expressed as

$$\begin{aligned}\mu_n = &\frac{m_0^{m_0}}{\overline{\gamma}_0^{m_0}\Gamma(m_0)}\left\{\int_0^\infty \gamma^n\exp\left(-\frac{m_0\gamma}{\overline{\gamma}_0}\right)\gamma^{m_0-1}d\gamma\right.\\ &-\sum_{k=0}^{m_1-1}\sum_{\ell=0}^{k}\int_0^\infty\left[\gamma^n\Xi[k,\ell]\exp\left(-\frac{m_1\gamma}{\overline{\gamma}_1}\right)\gamma^{(k+\ell+m_2)/2}\right.\\ &\left.\left.\cdot K_{k-m_2-\ell}(2\sqrt{\tau\gamma})\right]d\gamma\right\}\\ &+\int_0^\infty\gamma^n f_{\gamma_{eq2}}(\gamma)d\gamma\\ &-\int_0^\infty\left[\frac{\Gamma(m_0,(m_0/\overline{\gamma}_0)\gamma)}{\Gamma(m_0)}\right]\gamma^n f_{\gamma_{eq2}}(\gamma)d\gamma.\end{aligned} \tag{24}$$

The first integral can be solved in closed form using [13, equation (3.381.4)]. The second and the third integrals, after a simple transformation, are in the form of $\int_0^\infty x^r\exp(-\beta x^2)K_v(\zeta x)dx$, which can be evaluated using [13, equation (6.631.3)] yielding a complicated closed-form mathematical expression involving the Whittaker function. This expression can be simplified further by applying the identities [13, equations (9.220.3), (9.220.4)] of the well-known confluent hypergeometric function, ${}_1F_1(x,y;z)$, [13, equations (9.210.1) and (9.220.4)]. Hence, the result is a finite nested sum of ${}_1F_1(x, y; z)$. The last integral is not a tabulated one since it involves the incomplete gamma function. By applying [13, equation (8.354.2)], incomplete gamma function can be written as an infinite series sum (i.e., $\Gamma(\alpha, x) = \Gamma(\alpha) - \sum_{n=0}^{\infty}(-1)^n x^{\alpha+n}/n!(\alpha+n)$). It is obvious, after the previous expansion, that the last integral has similar form as the previous ones yielding to an infinite nested sums of ${}_1F_1(x, y; z)$. The analytical expression for the moments[1] is very useful since it can be used to directly obtain the average end-to-end SNR (i.e., μ_1) or the amount of fading (AoF) defined as

$$\text{AoF} = \frac{\mu_2}{\mu_1^2} - 1 \tag{25}$$

under various fading channel conditions.

It is worth to derive a closed-form expression for the third integral, which represents the moments of the dual-hop relay fading channel using a fixed gain relay over Nakagami-m environment, given by

$$\begin{aligned}
\mu_n &= \sum_{k=0}^{m_1-1}\sum_{\ell=0}^{k}\Xi[k,\ell] \\
&\times\Bigg\{\xi_1[k,\ell,n]\Bigg[\frac{{}_1F_1(k+n,k-m_2-\ell;\overline{\gamma}_1\tau/m_1)}{\Gamma(1-k+m_2+\ell)^{-1}} \\
&\qquad - \frac{{}_1F_1(k+n,1+k-m_2-\ell;\overline{\gamma}_1\tau/m_1)}{\big((k+m_2+\ell)\Gamma(-k+m_2+\ell)\big)^{-1}}\Bigg] \\
&+\xi_2[k,\ell,n]\Bigg[\frac{{}_1F_1(1+k+n,1+k-m_2-\ell;\overline{\gamma}_1\tau/m_1)}{\Gamma(-k+m_2+\ell)^{-1}} \\
&\qquad + \frac{\overline{\gamma}_1\tau}{2m_1}\frac{{}_1F_1(1+k+n,2+k-m_2-\ell;\overline{\gamma}_1\tau/m_1)}{\Gamma(-1-k+m_2+\ell)^{-1}}\Bigg] \\
&+\xi_3[k,\ell,n]\Bigg[\frac{{}_1F_1(m_2+n+\ell,-k+m_2+\ell;\overline{\gamma}_1\tau/m_1)}{\Gamma(1+k-m_2-\ell)^{-1}} \\
&\qquad - \frac{{}_1F_1(m_2+n+\ell,1-k+m_2+\ell;\overline{\gamma}_1\tau/m_1)}{\big((k+m_2+\ell)\Gamma(k-m_2-\ell)\big)^{-1}}\Bigg] \\
&+\xi_4[k,\ell,n] \\
&\times\Bigg[\frac{{}_1F_1(1+m_2+n+\ell,1-k+m_2+\ell;\overline{\gamma}_1\tau/m_1)}{\Gamma(k-m_2-\ell)^{-1}} \\
&\qquad + \frac{\overline{\gamma}_1\tau}{2m_1}\frac{{}_1F_1(1+m_2+n+\ell,2-k+m_2+\ell;\overline{\gamma}_1\tau/m_1)}{\Gamma(-1+k-m_2-\ell)^{-1}}\Bigg]\Bigg\},
\end{aligned} \tag{26a}$$

[1] Although the analytical expression for the moments of the cooperative dual-hop link with fixed gain relay has been derived, it is not presented here since it is very complicated and does not provide much insight to the performance analysis. However, since ${}_1F_1(x, y, ; z)$ is a built-in standard function to the most well-known mathematical software packages, the results can be easily extracted.

where

$$\begin{aligned}
\xi_1[k,\ell,n] &= \frac{1}{2}\left(\frac{m_1}{\overline{\gamma}_1}\right)^{-k-n}\tau^{(k-m_2-\ell)/2}\Gamma(k+n), \\
\xi_2[k,\ell,n] &= \left(\frac{m_1}{\overline{\gamma}_1}\right)^{-k-n}\tau^{(k-m_2-\ell)/2}\Gamma(1+k+n), \\
\xi_3[k,\ell,n] &= \frac{1}{2}\left(\frac{m_1}{\overline{\gamma}_1}\right)^{-m_2-n-\ell}\tau^{(-k+m_2+\ell)/2}\Gamma(m_2+n+\ell), \\
\xi_4[k,\ell,n] &= \left(\frac{m_1}{\overline{\gamma}_1}\right)^{-m_2-n-\ell}\tau^{(-k+m_2+\ell)/2}\Gamma(1+m_2+n+\ell).
\end{aligned} \tag{26b}$$

3.2.3. Average BER

The MGF of the end-to-end SNR at terminal D is defined as

$$\mathcal{M}_{\gamma_{\text{eq},fg}}(s) \triangleq E[e^{s\gamma_{\text{eq2}}}] = \int_0^{\infty} e^{s\gamma} f_{\gamma_{\text{eq},fg}}(\gamma)d\gamma. \tag{27}$$

Following the same procedure as for the moments, the MGF for the dual-hop cooperative links can be derived in terms of nested sums of ${}_1F_1(x, y, ; z)$. Again, the analytical result of $\mathcal{M}_{\gamma_{\text{eq},fg}}(s)$ is not presented here for the same reason as for the moments. However, with the aid of $\mathcal{M}_{\gamma_{\text{eq},fg}}(s)$, the performance of a great variety of modulation schemes can be easily obtained [16]. Examples of such modulation schemes include M-ary quadrature amplitude modulation (M-QAM), M-ary phase-shift keying (M-PSK), and noncoherent binary schemes such as binary frequency-shift keying (BFSK) and differential phase-shift keying (DPSK). For example, the average BER of binary DPSK can be given by $\overline{P}_b(E) = 0.5\mathcal{M}_{\gamma_{\text{eq},fg}}(-1)$.

4. NUMERICAL EXAMPLES AND SIMULATIONS

In this section, numerical examples will be presented, which also verify the accuracy of our mathematical analysis. Figure 2 depicts the outage probability as a function of the average SNR of the direct link, $\overline{\gamma}_0$, for both types of relays and for different outage threshold values. Comparing the performance between CSI-based relay with the equivalent, in terms of average introduced gain, fixed gain relay (i.e., $g_2^2 = E[1/(\alpha_1^2 + N_{0,1})]$), it can be seen that for medium-to-large SNR values the CSI-based relay outperforms those with fixed gain. However, for the low-SNR region, dual-hop systems employed with fixed gain relays outperform those with variable gain. This happens due to the fact that the maximum value of the CSI-based gain relays, g_1, is $1/N_0$ when $\alpha_1 \to 0$, which is very possible in the low average SNR regime. Considering the higher complexity nature of CSI-based relays, our results show that fixed gain relays may serve as an efficient solution for relayed transmissions. Similar observations have been made for the Rayleigh fading channel in [14]. Furthermore, it is observed that as γ_{th} increases, the SNR range where fixed gain outperforms CSI-based relays also increases.

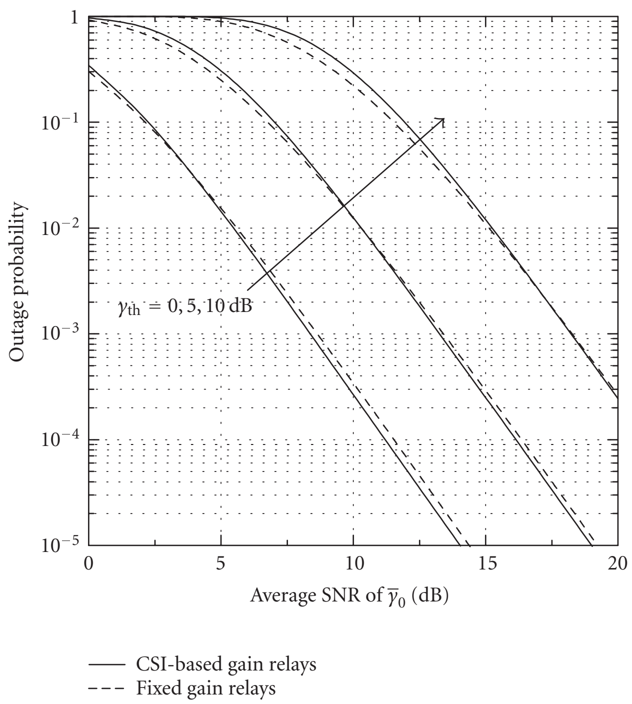

Figure 2: Outage probability of dual-hop cooperative links with CSI-based relay and SC diversity receiver versus average SNR $\overline{\gamma}_0$, for different relay types ($m_1 = m_2 = 2$, $m_0 = 1.5$, $\overline{\gamma}_1 = 2\overline{\gamma}_0$, and $\overline{\gamma}_2 = 5\overline{\gamma}_0$).

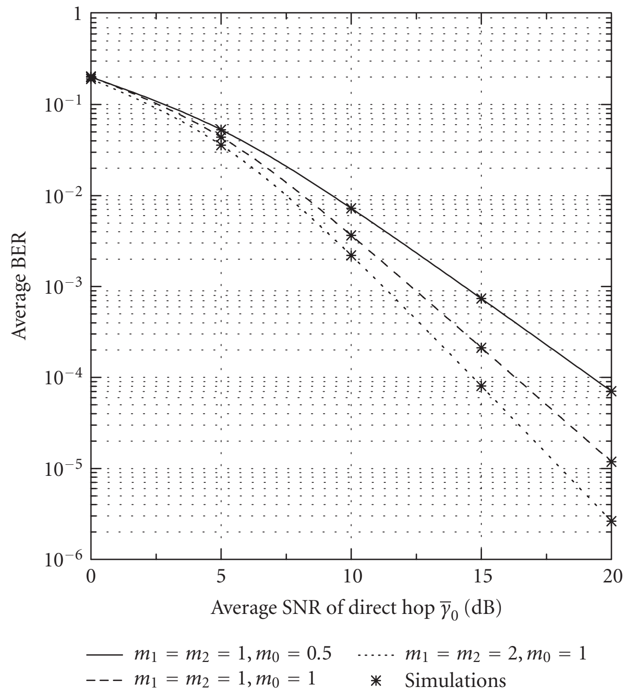

Figure 4: Average BER for DPSK for dual-hop cooperative link with fixed gain relay and SC diversity receiver versus $\overline{\gamma}_0$ ($\overline{\gamma}_1 = 1.5\overline{\gamma}_0$, $\overline{\gamma}_2 = 2\overline{\gamma}_0$, and $C = 2$).

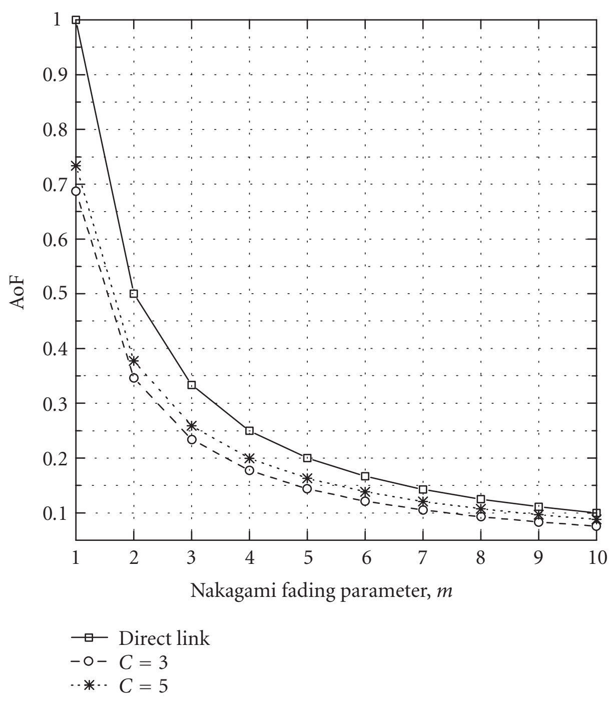

Figure 3: AoF for a dual-hop cooperative link with fixed gain relay and SC diversity receiver versus fading severity ($m_1 = m_2 = m_0 = m$, $\overline{\gamma}_1 = 1.5\overline{\gamma}_0$, $\overline{\gamma}_2 = 3\overline{\gamma}_0$, and $\overline{\gamma}_0 = 0$ dB).

In Figure 3 the AoF of a dual-hop cooperative link with fixed gain relay over integer-order Nakagami-*m* fading parameters is depicted. It is clear from Figure 3 that the overall AoF is reduced when a relay is used. Also, it is interesting to note that AoF deteriorates for low values of *m* even for high amplification gains. This can be explained by the fact that for strong fading conditions the fixed gain relay does not amplify only the received signal but also the noise of the end-to-end link. Moreover, as *m* increases, the overall difference between AoFs of a direct link and a dual-hop cooperative transmission decreases. Finally, in Figure 4 the average BER for DPSK is showed versus $\overline{\gamma}_0$. As it was expected, an increase in fading parameter upgrades the error performance of the dual-hop cooperative link.

5. CONCLUSION

In this paper, the performance of dual-hop cooperative diversity systems with nonregenerative relays and a SC receiver at the destination terminal over Nakagami-*m* fading channels was studied. Specifically, closed-form expressions for the outage probability for both CSI-based and fixed gain relays were presented. Regarding the outage probability, the analysis showed that the fixed gain relays may serve as an efficient solution for relayed transmissions in contrast to the high complexity nature of CSI-based relays. Moreover, analytical expressions for the AoF and the average BER when a fixed gain relay is considered were derived. These expressions were presented as infinite nested sums of the confluent hypergeometric function, which can be easily evaluated numerically.

APPENDIX

EVALUATION OF THE INTEGRAL I_3

The integral I_3 in (11), can be written in its general form as

$$I_3 = \int_y^\infty x^{a-1}\exp(-bx)\Gamma\left(c, d\left(\frac{x+e}{x-y}\right)\right)dx, \tag{A.1}$$

where a, c are positive integers and b, d, e real numbers. Then, the incomplete gamma function of the integrand in (A.1), using [13, equations (8.352/2), (1.111)], can be expressed as

$$\begin{aligned}\Gamma\left(c, d\left(\frac{x+e}{x-y}\right)\right) &= (c-1)!\exp(-d)\\ &\quad\times\exp\left[-d\left(\frac{y+e}{x-y}\right)\right]\\ &\quad\times\sum_{k=0}^{c-1}\sum_{\ell=0}^{k}\frac{d^k}{k!}\binom{k}{l}\left(\frac{y+e}{x-y}\right)^{\ell},\end{aligned} \tag{A.2}$$

where $\binom{\cdot}{\cdot}$ denotes the binomial coefficient. Using (A.1) and (A.2) after a simple transformation of the integration variable, I_3 can be expressed as

$$\begin{aligned}I_3 &= (c-1)!\exp\left[-(d+by)\right]\\ &\quad\times\sum_{k=0}^{c-1}\sum_{\ell=0}^{k}\sum_{r=0}^{a-1}\frac{d^k}{k!}\binom{k}{l}\binom{a-1}{r}(y+e)^{\ell}y^{a-r-1}\\ &\quad\times\int_0^\infty x^{r-\ell}\exp(-bx)\exp\left[-d\left(\frac{y+e}{x}\right)\right]dx.\end{aligned} \tag{A.3}$$

The inner integral of (A.3) can be solved by using [13, equation (3.471.9)], resulting in

$$\begin{aligned}I_3 &= (c-1)!\exp\left[-(d+by)\right]\\ &\quad\times\sum_{k=0}^{c-1}\sum_{\ell=0}^{k}\sum_{r=0}^{a-1}\frac{d^k}{k!}\binom{k}{l}\binom{a-1}{r}(y+e)^{(r+\ell+1)/2}y^{a-r-1}\\ &\quad\times 2\left(\frac{d}{b}\right)^{(r-\ell+1)/2}K_{\ell-r-1}\left(2\sqrt{bd(y+e)}\right).\end{aligned} \tag{A.4}$$

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