

Non-singular AdS-dS transitions in a landscape scenario

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Understanding transitions between different vacua of a multiverse allowing eternal inflation is an open problem whose resolution is important to gain insights on the global structure of the spacetime as well as the problem of measure. In the classical theory, transitions from the anti-deSitter to deSitter vacua are forbidden due to the big crunch singularity. In this article, we consider toy landscape potentials: a double well and a triple well potential allowing anti-deSitter and de-Sitter vacua, in the effective dynamics of loop quantum cosmology for the $k = -1$ FRW model. We show that due to the non-perturbative quantum gravity effects as understood in loop quantum cosmology, non-singular anti-deSitter to de-Sitter transitions are possible. In the future evolution, an anti-deSitter bubble universe does not encounter a big crunch singularity but undergoes a big bounce occurring at a scale determined by the underlying quantum geometry. These non-singular transitions provide a mechanism through which a probe or a ‘watcher’, used to define a local measure, can safely evolve through the bounce and geodesics can be smoothly extended from anti-deSitter to de-Sitter vacua.

I. INTRODUCTION

In the eternal inflation scenario based on new inflation [1, 2], the universe starts inflating in a false vacuum producing a vast volume of the spacetime. Different parts of this vast spacetime inflate in different false vacua, giving rise to a landscape of numerous vacua. At the scale of the entire vastness of the spacetime, inflation goes on forever, producing an infinite number of pocket universes [3]. This picture of a big universe leading to a fractal like structure of several pocket universes is often termed as a “multiverse”. The idea of eternal inflationary multiverse has recently gained a lot of attention due to the possibility that it may naturally arise from the string landscape which is composed of many metastable vacua. Transitions of the configuration of the compact manifold in the string theory, results in transitions between different vacua in the 4-dimensional effective field theory description. A metastable vacuum with a positive minima leads to a de-Sitter (dS) evolution, and one with a negative minimum results in an anti-deSitter (AdS) phase of evolution in the landscape. A bubble universe which undergoes an AdS evolution reaches a maximum size determined by the local minimum of the effective potential, recollapses and evolves to a big crunch singularity in a finite time. Since a big crunch singularity is a generic property of AdS pocket universes in general relativity (GR), a transition from an AdS to a dS bubble is forbidden. It is therefore important to understand mechanisms originating from quantum gravity which can allow the transitions between AdS to dS vacua in the landscape to be non-singular.

The issue of obtaining such non-singular transitions is also important for a related problem of defining appropriate measures for computing the probability of inflation to occur in an eternal inflationary multiverse (see Ref. [4] for a discussion of various measures used in this setting). A promising proposal in the eternal inflation multiverse is to introduce a watcher, a probe which is conjectured to go through infinite transitions in the landscape [5]. It is claimed that a local measure defined via such a watcher does not suffer from various ambiguities resulting from the choice of the cut-off surface. Since the watcher encounters a big crunch singularity at the end of

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an AdS phase of evolution where the geodesics break down, the watcher proposal faces a severe limitation in the classical theory. This problem is also shared by other approaches to define a local measure in the multiverse [6–9]. Unfortunately, current prescriptions to introduce a measure from a more fundamental theory in a landscape, such as using holographic ideas, also face difficulties associated with the big crunch singularity in the AdS evolution [10, 11].

Since near the singularity, the spacetime curvature becomes Planckian, the continuum spacetime description of general relativity (GR) breaks down. It is expected that modifications to the dynamical equations originating from a quantum theory of gravity would provide insights on the resolution of such singularities. Although a full quantum theory of gravity is yet to be found, loop quantum cosmology (LQC), a framework of homogeneous quantum cosmology based on loop quantum gravity (LQG), presents a promising avenue to tackle issues involving curvature singularities (see Ref. [12] for an extensive review). A key result in LQC for spatially flat isotropic spacetimes, is that non-perturbative quantum gravity effects originating from discrete quantum geometry in LQG cause a bounce of the scale factor when the energy density of the universe reaches a universal maxima [13–16]. The big bang and big crunch singularities of the classical theory are replaced by a big bounce. The mechanism of non-singular bounce in the isotropic spacetimes has been studied in great detail for several flat and curved models with massless scalar field and other matter sources [14–24]. Rigorous quantization of various anisotropic spacetimes has also been recently performed [25–32]. It is notable that the bounce in loop quantized spacetimes occurs without any fine tuning of the parameters or a specific choice of the energy conditions.

Evolution in LQC is governed by a quantum difference equation on the quantum geometry. For states which lead to a macroscopic universe peaked on a classical trajectory at late times, it is possible to obtain an effective continuum spacetime description [33–35]. The effective dynamics obtained from the effective Hamiltonian in LQC has been shown to be in excellent agreement with the underlying quantum evolution for states which remain sharply peaked and bounce at scale factor larger than the Planck length [35]. Based on the effective description of LQC, the strong singularities have been shown to be resolved in isotropic [36, 37] and Bianchi-I spacetime [38], and bounds on energy density, shear scalar and expansion scalar in different models haven computed [39, 40]. Given these results, LQC provides an excellent approach to address the difficulties, arising due to big crunch singularity, to understand the transitions from the AdS to dS phase in the effective 4-dimensional spacetime description of the multiverse.

Several investigations in the landscape paradigm imply that a pocket universe is born out of a tunneling event from an other vacuum in the neighborhood. The metric of the spacetime inside the bubble is given by a $k = -1$ FRW universe using which several physical and observational consequences of bubbles have been studied in detail [41–48], (see Refs. [49, 50] for recent reviews). Our aim in this this work is to use the insights from the new physics of non-perturbative quantum gravity as understood in LQC to understand transitions in multiple vacua of the landscape. We consider a $k = -1$ universe with a scalar field with self interacting potentials motivated by the landscape scenario in the multiverse in the effective spacetime description of LQC. We will study the way loop quantum effects resolve the big crunch singularity while making a non-singular transition from an AdS to a dS vacuum. It is important to emphasize some of the caveats in our analysis. We assume the validity of effective equations in LQC for the $k = -1$ model. So far the quantization of $k = -1$ model in LQC is based on considering holonomies of the extrinsic curvature, unlike the Ashtekar-Barbero connection for spatially flat and positively curved spacetimes. It is possible that a quantization based on holonomies of connection may lead to some modifications to the effective equations, though these are not expected to change the qualitative aspects of dynamics. Further, strictly speaking, effective equations are valid only for scale factors greater than the Planck length [35]. Thus, we focus our analysis on those cases where the bounce occurs at scale factors much greater than the Planck length. On the same lines, given the limitations of the effective equations

at scale factors comparable and smaller than the Planck length, any questions pertaining to such a regime can not be addressed in our approach. Finally, we assume the existence of the self-interacting potentials motivated by the landscape scenario in LQC. Whether or not such scenarios can consistently arise in LQC at the level of the quantum theory is an open question, which goes beyond the scope of this work.¹

The main result of this paper is based on the resolution of curvature singularities in LQC. Let us consider a pocket universe described by a negatively curved open FRW spacetime which is in an AdS vacuum at some point in its evolution. In the classical description, such a pocket universe would develop big crunch singularity in a finite proper time. In a striking contrast, we show that in LQC, the big crunch singularity is resolved due to the quantum geometric corrections which become important in the Planck regime. As conjectured in the Ref. [5], such a resolution of big crunch singularity would provide a way to define a measure with respect to a ‘watcher’ who travels on the worldline of the multiverse. This in turn opens promising ways to address the measure problem in eternal inflation. We note that the goal of this work is neither to address the measure problem in the landscape scenario in LQC, nor the way quantum gravity effects may influence the probabilities for transitions.² Our goal in this paper is to demonstrate that non-singular AdS-dS transitions can be achieved using loop quantum gravitational effects. Specifically, we show that depending on the initial conditions, the pocket universe may transit to a dS vacuum in the future evolution or undergo another AdS phase before such a transition. This alters the global structure of multiverse, providing the watcher a safe passage from AdS to dS vacuum.

It is to be emphasized that a non-singular evolution of the scalar field from a negative part of the potential to a dS vacuum is a very non-trivial result. In the classical theory, such a transition can not be achieved due to singularity theorems and there is no well defined mechanism to obtain this, unless one considers violation of the weak energy condition. This problem has also been stressed in various earlier works (see for eg. [5]), and it has been hoped that quantum gravity effects may resolve the big crunch singularity, and lead to a non-singular AdS-dS transitions. Though in LQC singularities have been resolved in various models, one must exercise care while expecting results in the present setting. Here it is important to note that even though quantum gravity effects in LQC may resolve the singularity, it is not at all obvious that an AdS-dS transition would necessarily occur. An example on these lines occurs in the case of ekpyrotic model in isotropic LQC, where though the singularity is resolved, the scalar field *does not* make a transition from the negative to the positive part of the ekpyrotic potential [52], unless one considers the presence of anisotropies [53].³ A closer example is the case of landscape potentials in $k = 0$ isotropic cosmology (see Ref. [56] for an earlier work on these lines, however these results do not extend to the singularities encountered in the bubble universes as considered here.⁴). As we show in the Appendix A, though in the $k = 0$ model, the big crunch singularity is resolved, there are no AdS-dS transitions. As in Ref. [56], if the initial conditions are chosen in such a way that the energy density is positive

¹ Our work will be on the lines of similar treatment of Ekpyrotic/Cyclic model in LQC, where singularity resolution was achieved assuming the existence of an effective 4-dimensional potential motivated from string theory [51–53].

² For a discussion on the latter issue in chaotic inflationary scenario in LQC, see Ref. [54, 55].

³ One should be careful in a comparison of the landscape potential considered here with the analysis of the ekpyrotic potential such as in Ref. [52]. Since both the potentials have a negative part, one may be tempted to conclude that in the ekpyrotic potential one also has an AdS phase. However, as has been demonstrated in Ref. [52], the approach to the singularity in ekpyrotic scenario is kinetic dominated, and thus there is no AdS phase near the big bang/crunch. Therefore, even though the landscape potentials (as considered in this work), and the ekpyrotic potential have some common features, there are sharp differences in the dynamical evolution and the equation of state on approach to the singularity.

⁴ Similar transitions from AdS to dS vacua has been discussed in the setting of cyclic inflation by considering quadratic corrections to the Friedmann equation in Ref. [57, 58].

in the negative vacua of the potential (which is possible by an appropriate choice of the kinetic energy of the scalar field), transitions from the negative to positive minima are obtained. However, we found that in such cases, the presence of the field in the negative minima does not correspond to an AdS phase of the evolution (see the Appendix A). For the spatially flat model, the form of the effective Friedmann equation is such that a negative energy density, which is required to capture the AdS like phase, leads to an imaginary Hubble rate.⁵ Thus, the analysis of landscape motivated potentials in the spatially flat model is unable to capture the relevant physics of the AdS-dS transitions.

This paper is organized as follows. In the next section we briefly review the loop quantization of an open FRW spacetime. We introduce the loop variables, the SU(2) Ashtekar-Barbero connection A_a^i and triads E_a^i , discuss their relation with the usual metric variables and derive the dynamical equations both in the classical theory and the effective spacetime description of LQC. In Sec. III, we consider two different types of landscape potentials: a double and a triple well potential, and solve the equations of motion in these scenarios. We discuss the evolution of scale factor, scalar field and also provide phase diagrams of the dynamical trajectories of the scalar field. In this section, we show non-singular transition from AdS to dS vacua with help of numerical simulations. We summarize and discuss our results in section A.

II. LOOP QUANTUM COSMOLOGY OF $k = -1$ FRW SPACETIME

In this section we begin by an introduction of the FRW spacetime with negative spatial curvature in terms of loop variables – the Ashtekar-Barbero connection A_a^i and triads E_a^i . We discuss the relationship of these variables with usual metric variables, write the classical Hamiltonian constraint and obtain the classical Friedmann and Raychaudhuri equations using the Hamilton's equations. Then, we discuss the effective description of loop quantum cosmology for $k = -1$ spacetime. The dynamical equations hence obtained are then used to derive modified Friedmann equations for $k = -1$ FRW spacetime. We will also describe the important features of the modified Friedmann equations which lead to vital differences between the classical and effective LQC trajectories in the deep Planck regime.

A. Classical theory

We consider a homogeneous and isotropic open FRW spacetime with negative spatial curvature, i.e. $k = -1$ with the following metric

$$dS^2 = -N^2 dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (2.1)$$

where $N(t)$ is the lapse function, $a(t)$ is the scale factor, r is the radial co-ordinate of the spatial metric and $d\Omega^2$ is the metric on the surface of a 2-sphere. Thus the spatial topology of the metric is $\Sigma = \mathbb{R} \times \mathbb{S}^2$. In order to define symplectic structure on the spatial manifold we introduce a fiducial cell \mathcal{V} whose volume is given as \mathcal{V}_o . The edges of the fiducial cell are chosen to lie along the fiducial triads \hat{e}_a^i . The fiducial metric is \hat{q}_{ab} taken to be compatible with the fiducial co-triads. Utilizing the underlying symmetry of $k = -1$ FRW spacetime the Ashtekar variables can be written in terms of the symmetry reduced connection and triads as follows

$$A_a^i = c \mathcal{V}_o^{1/3} \hat{\omega}, \quad \text{and} \quad E_a^i = p \mathcal{V}_o^{-2/3} \sqrt{q} \hat{e}_a^i, \quad (2.2)$$

⁵ In $k = 0$ model, the Friedmann equation in LQC is: $H^2 = \frac{8\pi G}{3} \rho \left(1 - \frac{\rho}{\rho_{\text{crit}}} \right)$, where $\rho_{\text{crit}} = 0.41 \rho_{\text{Pl}}$.

where \hat{e}_i^a are the densitized triads and $\hat{\omega}_a^i$ are the fiducial co-triads compatible with the fiducial metric \hat{q}_{ab} . The symmetry reduced connection, c and the triad p form a canonical pair which satisfies the following Poisson bracket

$$\{c, p\} = \frac{8\pi G\gamma}{3}, \quad (2.3)$$

where $\gamma \approx 0.2375$ is the Barbero-Immirzi parameter whose value is fixed via black hole entropy computation in LQG. The triad, p (whose orientation is chosen to be positive in this analysis) and the classical connection, c are related to the scale factor, a and its time derivative as follows

$$p = a^2, \quad c = \gamma\dot{a} + k, \quad (2.4)$$

where the ‘dot’ represents the time derivative with respect to the proper time. It is important to note that the relation between the connection, c and the time derivative of the scale factor, as given above, holds true only in the classical theory. This relation is modified in the effective description of LQC. On the other hand, the triad is given by the same relation both in the classical and the effective description of LQC.

The classical Hamiltonian constraint, for FRW spacetime with the lapse function chosen to be $N = 1$, can be written in terms of the symmetry reduced connection and triad as follows

$$\mathcal{H}_{\text{cl}} = -\frac{3}{8\pi G\gamma^2}\sqrt{p}\left[(c-k)^2 + k\gamma^2\right] + \mathcal{H}_{\text{matt}}. \quad (2.5)$$

From the vanishing of the Hamiltonian constraint, $\mathcal{H}_{\text{cl}} = 0$ we obtain

$$\left(\frac{c-k}{\gamma}\right)^2 = \frac{8\pi G}{3}\rho p - k \quad (2.6)$$

where ρ is the energy density related to the matter part of the Hamiltonian as $\mathcal{H}_{\text{matt}} = \rho p^{3/2}$. Substituting the expressions of p and c in terms of metric variables from eq. (2.4) to the above equation, one can obtain the classical Friedmann equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}. \quad (2.7)$$

The dynamical equations of the triad and the connection are given via the Hamilton’s equation of motion

$$\dot{p} = \{p, \mathcal{H}_{\text{cl}}\}, \quad \dot{c} = \{c, \mathcal{H}_{\text{cl}}\}. \quad (2.8)$$

One can now use the equations of motion of the connection and the triad to obtain the Raychaudhuri equation

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P), \quad (2.9)$$

where P denotes the pressure of the matter field. Using the classical Friedmann and Raychaudhuri equation, we can obtain classical trajectories for arbitrary matter. For the case of the positive cosmological constant, the future evolution asymptotes to dS phase. Where as for the negative cosmological constant, the future evolution is asymptotically AdS which ends in a big crunch singularity.

B. Effective dynamics

Quantization of a cosmological spacetimes in LQC is a symmetry reduced quantization based on the techniques of LQG. The quantization procedure involves considering the field strength operator of the holonomies of the Ashtekar-Barbero connection, whose action on the states in the geometrical representation leads to a quantum difference equation with uniform spacing in the volume. The quantum difference equation is a direct consequence of the underlying quantum geometry, in particular of the minimum non-zero eigenvalue of the area operator in LQG. The evolution given by the quantum difference equation turns out to be non-singular. Due to certain technical difficulties, for the $k = -1$ model, one considers field strength operator not constructed from the holonomies of the connection, but of the extrinsic curvature, which results in a similar non-singular quantum difference equation. Interestingly, for states which evolve to a macroscopic universe at late times, it is possible to derive an effective Hamiltonian constraint using geometrical formulation of quantum mechanics [33–35]. Using Hamilton's equation of motion, effective dynamical equations are then derived from the effective Hamiltonian. These dynamical equations govern the evolution of a cosmological model under the effective description of LQC, and in turn give rise to modified Friedmann and Raychaudhuri equations. Various extensive numerical simulations show that the effective dynamical trajectory stands in very good agreement with the full quantum evolution for sharply peaked semi-classical states (see Ref. [59] for a review).

The effective Hamiltonian for $k = -1$ FRW spacetime in terms of the symmetry reduced triad (p) and connection (c), with lapse ($N = 1$), given as [20, 21, 37]

$$\mathcal{H}_{\text{eff}} = -\frac{3p^{3/2}}{8\pi G\gamma^2\lambda^2} \left(\sin(\bar{\mu}(c-k))^2 - k\chi \right) + \frac{P_\phi^2}{2p^{3/2}} + V(\phi)p^{3/2}, \quad (2.10)$$

where $\bar{\mu} = \lambda/\sqrt{|p|}$ is the length of the edge along which holonomy is computed, $\lambda^2 = 4\sqrt{3}\pi\gamma l_{\text{Pl}}^2$ is the minimum area gap determined by quantum geometry and $\chi = -\gamma^2\bar{\mu}^2$ for $k = -1$. P_ϕ is the conjugate momentum of the scalar field and $V(\phi)$ is the self interacting potential. It is useful to note that the above effective Hamiltonian is derived in the approximation when $p > 2.24l_{\text{Pl}}^2$ [37].⁶ The dynamical equations for the connection and triad are given via the Hamilton's equations of motion:

$$\dot{p} = \{p, \mathcal{H}_{\text{eff}}\}, \quad \dot{c} = \{c, \mathcal{H}_{\text{eff}}\}, \quad (2.11)$$

which gives the following evolution equations for c , p , P_ϕ and ϕ :

$$\begin{aligned} \dot{c} = & -\frac{8\pi\gamma}{3} \left(\frac{3p^{3/2}}{8\pi\gamma^2\lambda^2} \left(\frac{\gamma^2\lambda^2}{p^2} - \bar{\mu}(1+c) \cos(\bar{\mu}(1+c)) \sin(\bar{\mu}(1+c)) \right) \right. \\ & + \frac{9\sqrt{p}}{16\pi\gamma^2\lambda^2} \left(-\frac{\gamma^2\lambda^2}{p} + \sin(\bar{\mu}(1+c))^2 \right) \\ & \left. + \frac{3P_\phi^2}{4p^{5/2}} - \frac{3}{2}\sqrt{p}V(\phi) \right) \end{aligned} \quad (2.12)$$

⁶ In conventions of Ref. [37], this constraint corresponds to $v > 1$ for $\gamma \approx 0.2375$.

$$\dot{p} = \frac{2}{\gamma\lambda} \cos(\bar{\mu}(1+c)) \sin(\bar{\mu}(1+c)), \quad (2.13)$$

$$\dot{\phi} = \frac{P_\phi}{p^{3/2}}, \quad (2.14)$$

$$\dot{P}_\phi = -p^{3/2} \frac{\partial V(\phi)}{\partial \phi}. \quad (2.15)$$

Combining the dynamical equations for $\dot{\phi}$ and \dot{P}_ϕ it is straightforward to obtain the Klein-Gordon equation, which is equivalent to the conservation equation $\dot{\rho} = -3H(\rho + P)$ (where $H = \dot{p}/4p = \dot{a}/a$ is the Hubble rate):

$$\ddot{\phi} + 3H\dot{\phi} = -V_{,\phi}. \quad (2.16)$$

The energy density and the pressure of the scalar field are given in terms of the velocity of scalar field and potential as follows:

$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi), \quad \text{and} \quad P = \frac{\dot{\phi}^2}{2} - V(\phi). \quad (2.17)$$

The Klein-Gordon equation given via eq. (2.16) can be written in terms of the energy density and the pressure as follows:

$$\dot{\rho} = -3H(\rho + P). \quad (2.18)$$

Using the dynamical equations hence obtained and the relation between the triad and scale factor, we obtain the following modified Friedmann and Raychaudhuri equation for $k = -1$ in the effective description of LQC [37]:

$$H^2 = \left(\frac{8\pi G}{3} \rho + \frac{\bar{\mu}^2}{\lambda^2} \right) \left(1 - \frac{\rho}{\rho_{\text{crit}}} - \gamma^2 \bar{\mu}^2 \right) \quad (2.19)$$

$$= \frac{8\pi G}{3} (\rho + \rho_1) \frac{1}{\rho_{\text{crit}}} (\rho_2 - \rho) \quad (2.20)$$

where $\rho_1 = \frac{3\bar{\mu}^2}{8\pi G\lambda^2}$ and $\rho_2 = \rho_{\text{crit}} (1 - \gamma^2 \bar{\mu}^2)$. Note that both ρ_1 and ρ_2 are non-negative.⁷

$$\dot{H} = \left(-4\pi G(\rho + P) - \frac{\xi + \gamma^2 \bar{\mu}^2}{\gamma^2 \lambda^2} \right) \left(1 - 2 \left(\frac{\rho}{\rho_{\text{crit}}} + \gamma^2 \bar{\mu}^2 \right) \right) \quad (2.21)$$

where $\xi = \sin^2(\bar{\mu}) - \bar{\mu} \sin(\bar{\mu}) \cos(\bar{\mu})$ and $\rho_{\text{crit}} = \frac{3}{8\pi G\gamma^2 \lambda^2} = 0.41 \rho_{\text{Pl}}$ is the upper bound on the energy density for isotropic models. The modified form of the Friedmann and Raychaudhuri equations lead to resolution of various singularities which have been studied in detail in the Ref. [37]. The primary reason for the resolution of singularities lies in the quantum geometric effects which lead to the existence of ρ_{crit} . Note that if quantum geometric effects are absent, i.e. if the limit $\lambda \rightarrow 0$ is taken, we recover classical Friedmann and Raychaudhuri equations. These equations are also recovered when the spacetime curvature is negligible compared to the Planck curvature.

From eq. (2.19) it is clear that for a physical solution, which requires $H^2 \geq 0$, the energy density satisfies the following inequality:

$$-\rho_1 \leq \rho \leq \rho_2. \quad (2.22)$$

⁷ For ρ_2 to be negative, one requires $p < 0.2916l_{\text{Pl}}^2$. However, the effective Hamiltonian (2.10) is valid only for $p > 2.24l_{\text{Pl}}^2$.

This inequality clearly indicates that for the $k = -1$ FRW spacetime in LQC, the negative energy density is allowed as long as it remains greater than $-\rho_1$. Moreover, since ρ can take positive values as well, the energy density can change sign and become positive during the evolution. Note that for the energy density to dynamically change sign from negative to positive, the time derivative of the energy density should be positive, when $\rho = 0$ (at the point of sign change). It is straightforward to see that this is dynamically possible. If $\rho = 0$, then eq. (2.19) implies that $H^2 \neq 0$. Also, since $\rho = 0$ occurs for $\dot{\phi}^2/2 = -V(\phi)$, at the sign change of energy density, pressure is $P = \dot{\phi}^2 > 0$. Eq. (2.18), therefore implies that in the evolution when $\rho = 0$, $\dot{\rho} = -3H\dot{\phi}^2$. Thus, $\dot{\rho} > 0$ as the universe approaches a classical big crunch in the contracting branch after the recollapse. Hence, $k = -1$ FRW model not only allows negative energy density but also admits the change of sign of the energy density during the dynamical evolution. One can similarly carry out the above analysis for the $k = 0$ case (see Appendix), and one finds that such a change of sign of energy density is dynamically forbidden in the spatially flat model. *Therefore, unlike the flat model, the open FRW models can have transitions from AdS (negative ρ) to dS (positive ρ) vacuum.*

III. LANDSCAPE POTENTIALS AND ‘AdS to dS’ TRANSITIONS

In this section we consider two types of landscape potentials: (i) a double well landscape potential with one AdS and one dS vacuum, and (ii) a triple well landscape potential with one AdS and two dS vacua. We will consider these landscape potentials as self-interacting potentials of a scalar field and study the evolution in the effective description of LQC.

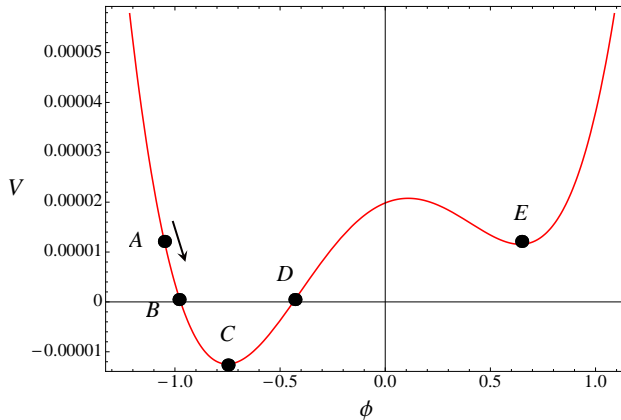


FIG. 1: A plot of the asymmetric double well potential motivated by the landscape scenario. Point ‘A’ refers to the position of the scalar field for one class of the initial conditions considered for evolution. Points ‘B’ and ‘D’ denote the zeros of the potential, ‘C’ marks the AdS vacuum and ‘E’ corresponds to the dS vacuum where the value of the potential is positive and locally minimum.

A. Double well potential

Let us consider a landscape potential as shown in Fig. 1. In order to model such a potential, we consider the following polynomial expression

$$V(\phi) = V_o \left(\left(\frac{\phi^2}{\alpha^2} - \delta \right)^2 + \frac{\beta}{\alpha} \phi \right), \quad (3.1)$$

where the parameters α , β and δ determine the shape of the potential; α controls the horizontal distance between the two local minima of the potential, β governs the vertical distance between the two local minima and δ governs the position of the $V|_{\phi=0}$ for given values of α and β .

The point ‘A’ in Fig. 1 corresponds to a point in the positive part of the potential where one set of initial conditions are given in our analysis. The arrow in Fig. 1 denotes that the field is considered to be initially rolling down, and enters the AdS well in future evolution. We also consider the initial conditions on the field such that the field begins the evolution at the bottom of the AdS well denoted by ‘C’. The points ‘B’ and ‘D’ denote the two zeros of the potential and ‘E’ marks the position of the positive local minimum. The vacuum ‘C’ of the potential corresponds to negative potential and would give rise to negative energy density. If the field spends some time in the minimum of the potential, then it mimics an AdS like evolution with almost constant negative energy density. This vacuum is referred to as the AdS vacuum. The second vacuum, denoted by ‘D’ has a positive potential which would mimic a positive cosmological constant (dS) vacuum. If the field is stuck in the positive side of the potential with a small field momentum, the bubble universe undergoes a de-Sitter (dS) like evolution. The value of the “effective cosmological constant” in this model can be controlled by changing the value of the magnitude of the potential V_o . The dynamics of the field in this potential has interesting phenomenological features due to AdS and dS like phases. In the classical theory, the evolution in AdS phase ends with a big-crunch singularity. Therefore, once the field comes into the negative potential regime, the spacetime has no escape but to collapse into big-crunch. In contrast, in LQC due to underlying quantum geometric effects, the big-crunch type singularity is avoided and evolution continues to an expanding branch.

The matter Hamiltonian for the scalar field with the potential eq. (3.1), is given as

$$\mathcal{H}_{\text{matt}} = \frac{P_\phi^2}{2p^{3/2}} + p^{3/2} V_o \left(\left(\frac{\phi^2}{\alpha^2} - \delta \right)^2 + \frac{\beta}{\alpha} \phi \right). \quad (3.2)$$

In the following, we solve the effective dynamical equations for the above matter Hamiltonian. The set of equations (2.12-2.15) form a well posed initial value problem. That is, by providing initial conditions on (p, c, ϕ, P_ϕ) at a given point of time $t = t_o$, the state of universe at a later time t can be computed. Since, the initial conditions must satisfy the effective Hamiltonian constraint, we provide the initial values of three of the variables and calculate the initial value of the remaining one from the vanishing of the effective Hamiltonian constraint, $\mathcal{H}_{\text{eff}} \approx 0$. Typically we provide $p(0)$, $\phi(0)$, and $P_\phi(0)$ and calculate $c(0)$ from the effective Hamiltonian constraint. We investigate the numerical evolution by providing the initial conditions on the scalar field at two different places in the landscape potential. In the first, the field is considered to roll down from a positive part of the potential (corresponding to point ‘A’), and in the second, the field starts to evolve from the bottom of the AdS well (corresponding to point ‘C’). The form of the dynamical equations are too complicated to be solved analytically, therefore we will solve them numerically.

1. Evolution in double well potential: initial conditions at ‘A’

In the evolution with a double well potential, we want to understand the way transition from the AdS to dS vacuum takes place without collapsing into the big-crunch singularity. In this subsection, we give the initial conditions in the expanding phase of the bubble universe when the field is high up in the potential so that the field is not in the AdS vacuum in the beginning, i.e. the field is near the point A shown in the Fig. 1. The evolution of scalar field and the scale factor is shown in Fig. 2. The solid curve in this figure depicts the LQC evolution while the dashed curve shows the classical trajectory. We have chosen initial conditions when the spacetime curvature is very small compared to the Planck scale. Thus, initially there is little difference between the

classical and LQC dynamics. As the evolution takes place, the field rolls down the potential and crosses point ‘B’. During this process, the field loses its kinetic energy due to Hubble friction as the scale factor expands. As the field enters the negative part of the potential near the AdS vacuum, it has a very small kinetic energy, so it spends some time at the bottom of the potential, corresponding to point ‘C’. During this phase, as shown in Fig. 2(a), the scalar field slowly oscillates in the AdS well around the bottom of the potential ‘C’. There, the negative potential energy dominates the kinetic energy and the total energy density of the field remains almost constant, as shown in Fig. 3(a), where the potential is almost constant negative as long as the field remains close to the bottom of the potential. The equation of state of the scalar field is shown in the Fig. 3(b). As long as the value of Hubble friction is small enough, the scalar field dwells around the AdS vacuum for a finite duration. Due to the negative energy density, the scale factor reaches a maximum size and starts to contract, making the Hubble rate negative. After the re-collapse takes place (shown in Fig. 2(b)), the kinetic energy of the field increases due to anti-friction originating from the negative Hubble rate. As a result, the field starts to roll up with a high kinetic energy. In the further evolution, as soon as the energy density of the field approaches Planckian value, the quantum geometric effects come into dominance, and the departures between classical theory and LQC become significant. The universe in LQC undergoes a non-singular bounce, as shown in Fig. 2(b). This behavior stands in a sharp contrast to the classical theory where a big crunch singularity is reached. As shown in Fig. 2, the scale factor in the classical theory (dashed curve), goes to big crunch, whereas the LQC trajectory bounces. The kinetic energy of the scalar field in the classical theory, diverges due to diverging Hubble rate. Due to this, the value of the scalar field diverges as well, and the scalar field does not settle into the dS vacuum.

In LQC, following the bounce, the Hubble rate again becomes positive, and its value increases rapidly. If the kinetic energy is high enough, the scalar field shoots into the positive well of the potential, as shown in the Fig. 2. In this phase of the evolution, the scale factor expands due to the positive Hubble rate and the field slows down, this time getting trapped into dS vacuum. In the dS vacuum state, the scale factor undergoes exponential expansion and due to high Hubble friction the field remains in the dS vacuum, undergoing exponential inflation during the rest of the evolution.

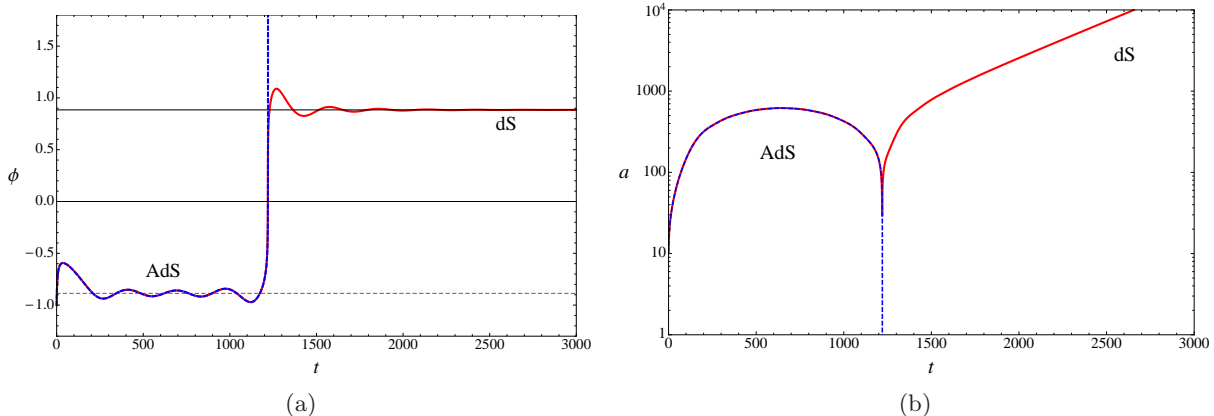


FIG. 2: These figures show the evolution starting from the left of the AdS vacuum of the landscape shown in Fig. 1. Figures (a) and (b) respectively show the evolution of the scalar field and the scale factor. The dashed curves show the classical trajectory, while the solid curves correspond to the LQC trajectories. It is clear from these figures that in LQC, instead of big-crunch singularity, there is a non-singular bounce. Following the bounce universe transits to a dS phase as the field evolves to the dS vacuum. The initial conditions for these plots are: $a(0) = 10.5$, $\phi(0) = -1.4$, $\dot{\phi}(0) = 0.14$ (in Planck units). The parameters of the potential are taken to be: $\alpha = 0.28$, $\beta = 0.16$, $\delta = 10$ and $V_o = 10^{-6}$.

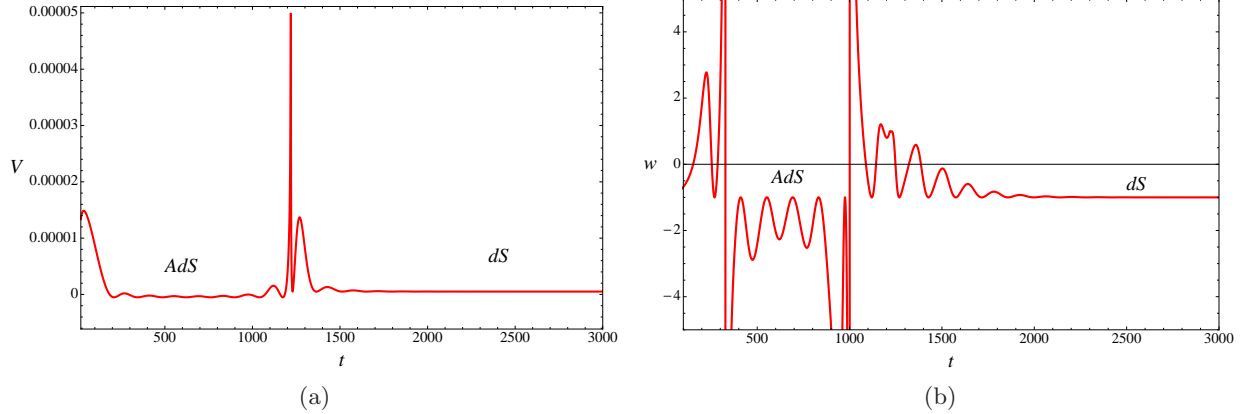


FIG. 3: Figures (a) and (b) respectively show the evolution of the potential and the equation of state of the scalar field corresponding to Fig. 2. The initial conditions for these plots are: $a(0) = 10.5$, $\phi(0) = -1.4$, $\dot{\phi}(0) = 0.14$ (in Planck units). The parameters of the potential are taken to be: $\alpha = 0.28$, $\beta = 0.16$, $\delta = 10$ and $V_o = 10^{-6}$.

Thus, it is clear from the above discussion and Fig. 2 that in the effective dynamical description of LQC, there is a non-singular and smooth transition from AdS to dS vacuum. It is also important to note that the energy density of the scalar field remains finite throughout the evolution, and scale factor remains greater than Planck length at the bounce. Fig. 4 shows the evolution of energy density of the scalar field close to the bounce as the transition from AdS to dS vacuum takes place. The horizontal curve denoted by ρ_{crit} shows the upper bound on the energy density, given as $\rho_{\text{crit}} \approx 0.41\rho_{\text{Pl}}$. It is evident from the figure that, unlike in the classical theory where the energy density diverges, in LQC the energy density remains bounded. This is a distinguished feature of LQC, that all the curvature scalars always remain finite during the evolution.

It is worth mentioning that, the initial conditions are a little fine tuned, in order for the field to roll down and spend some time at the bottom of the AdS vacuum. However, the behavior described above remains qualitatively similar for a small perturbation around the initial conditions. As shown in Fig. 5, it also turns out that the dS vacuum phase is a future attractor for all such initial conditions for which the evolution takes place according to Fig. 2. It is also evident that the

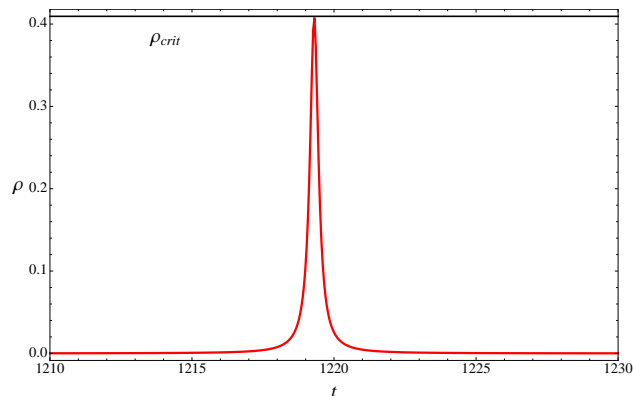


FIG. 4: This figure shows the evolution of the energy density of the scalar field, corresponding to Fig. 2, as the bounce takes place. The plot is zoomed in near the bounce in order to show the smooth evolution. It is clear to see that unlike in the classical theory where the energy density would have diverged to infinity, in LQC it remains finite throughout the evolution. The value of energy density shown in this figure is in the units of Planck density ρ_{Pl}

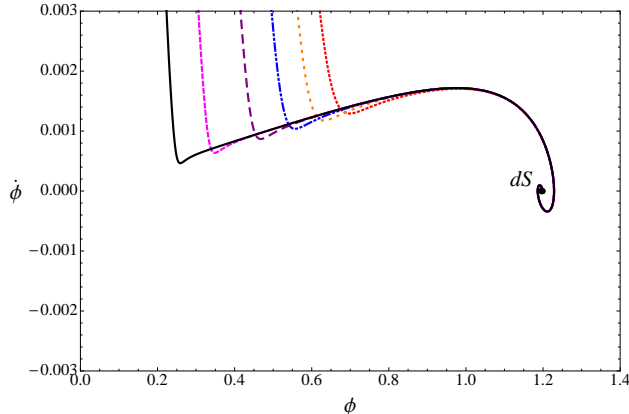


FIG. 5: This figure shows the dynamical phase trajectories of the scalar field at late times when the field climbs up to the positive part of the potential. Different trajectories correspond to different initial conditions on the value of the scalar field velocity, $\dot{\phi}$. It is clear from the plot that the terminal dS phase is obtained for a range of initial condition. This implies that dS vacuum is a future attractor for such a choice of initial conditions. The parameters of the potential are taken to be: $\alpha = 0.38$, $\beta = 0.62$, $\delta = 10$ and $V_o = 10^{-6}$.

dynamical trajectories corresponding to different initial conditions tend to the dS vacuum in their future evolution.

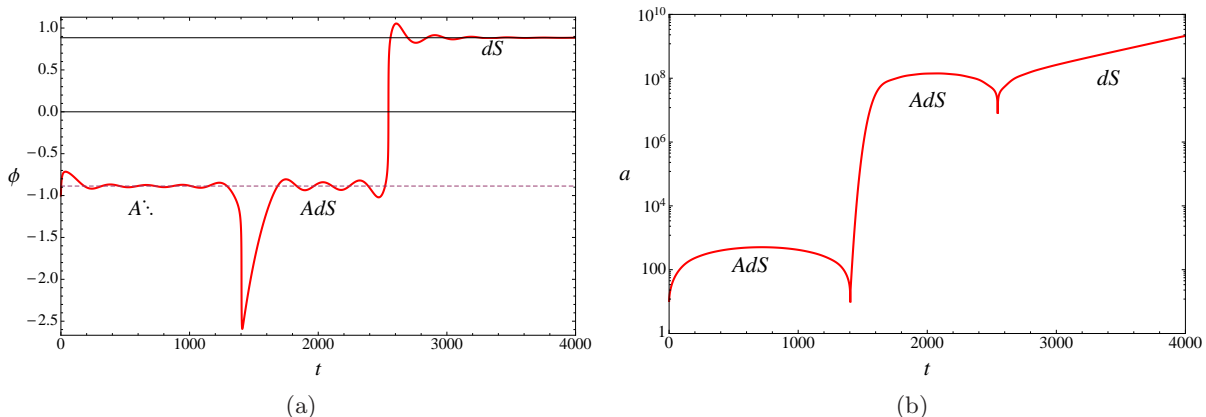


FIG. 6: This figure shows an example of evolution when the field spends more time in the AdS well while the scale factor undergoes two cycles of recollapse and bounce. After these two cycles, the field makes a transition to dS vacuum. The initial conditions for these plots are: $a(0) = 10.5$, $\phi(0) = -1.4$, $\dot{\phi}(0) = 0.07$ (in Planck units). Parameters of the potential are taken to be the same as in Fig. 2.

Depending on the initial conditions, it is possible that the scalar field may go through multiple AdS phases before making a non-singular transition to the dS vacuum. Fig. 6 shows such an interesting phenomenological case for the double well landscape potential. As compared to the evolution shown in Fig. 2, the scale factor undergoes two cycles of recollapse while the scalar field oscillates in the AdS well. It turns out that after the first bounce, the velocity of the scalar field, $\dot{\phi}$ is negative. As a result, instead of crossing the barrier between AdS and dS vacua, the field attempts to climb back towards 'A' but fails due to infinite barrier on that side. However, during the second cycle, the field has a positive velocity with enough kinetic energy to transit to the dS vacuum. Similarly, the initial conditions can be fine tuned to obtain more cycles of recollapse before transitioning to dS vacuum. By tuning the parameters of the potential, one may also obtain initial conditions such that the field never shoots off to the positive part of the potential and keeps

oscillating around the AdS vacuum. In this way the evolution of spacetime under the landscape potential in LQC has a rich phenomenology.

2. Evolution in double well potential: initial conditions in the AdS well

So far we have discussed the evolution of the field in a double well potential (as shown Fig. 1) when the initial conditions are given near the point ‘A’. That is, the field begins to roll down from a positive part of the potential, and approaches the AdS well. Let us now consider initial conditions so that the field starts its evolution from the bottom of the AdS well with a very small kinetic energy. Fig. 7 shows the evolution of the scalar field and the corresponding scale factor. Since the field is in the AdS vacua, future evolution leads to a recollapse, and the scale factor starts decreasing. In subsequent evolution, the curvature of spacetime becomes Planckian, and due to the quantum geometric effects, the scale factor goes through a non-singular bounce. If the kinetic energy is sufficient and the field has the correct sign of $\dot{\phi}$ (positive in this case), it settles in the dS well (corresponding to point ‘D’). In the further evolution, as shown in the Fig. 7, as the field settles in the dS vacuum, while the scale factor undergoes an exponentially expanding phase. Thus we obtain a non-singular transition from AdS to dS phase.

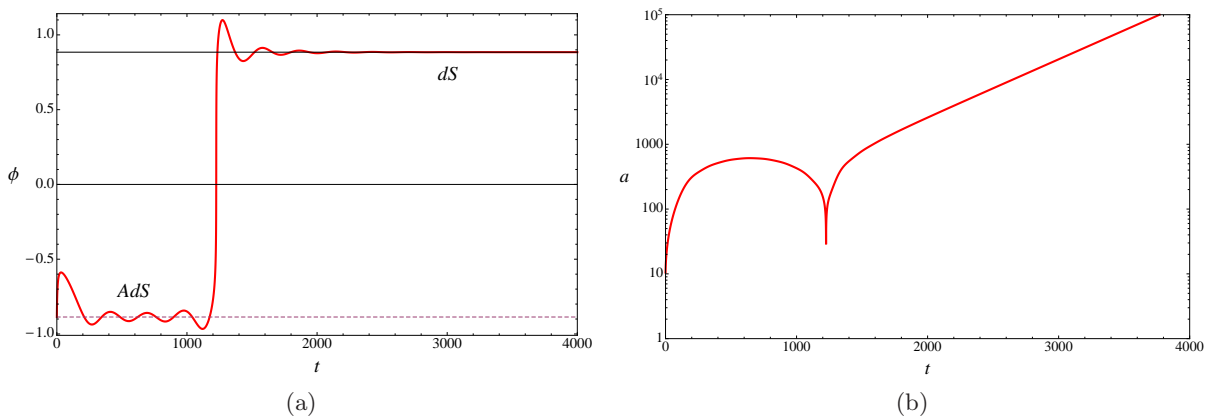


FIG. 7: This figure shows an example of evolution when the initial conditions are provided at the minimum of the AdS well corresponding to the point ‘C’ in Fig. 1. It is clear to see that the field undergoes a non-singular evolution, and ends up in the dS vacuum. During this evolution the scale factor undergoes one cycle of recollapse in the AdS phase. The initial conditions for these plots are: $a(0) = 10.5$, $\phi(0) = -1.4$, $\dot{\phi}(0) = 0.17$ (in Planck units). Parameters of the potential are taken to be the same as in Fig. 2.

B. Triple well potential

We now consider a triple well potential in the landscape scenario which has one AdS vacuum and two different dS vacua. We consider the potential of the following form:

$$V(\phi) = V_o \left(\left(\frac{\phi^2}{\alpha^2} - \delta \right)^3 + \nu \left(\frac{\phi^2}{\alpha^2} - \delta \right)^2 + \frac{\beta}{\alpha} \phi + \Omega \right) \quad (3.3)$$

where α , β , ν , δ , Ω and V_o are parameters of the potential. The landscape potential given by the above equation has the shape as shown in the Fig. 8. In this figure, point ‘A’ refers to the position of the scalar field for a class of initial conditions when the field begins its evolution away from the

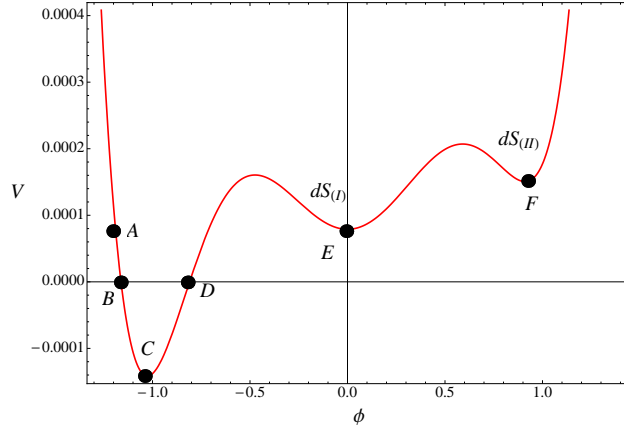


FIG. 8: This figure shows an example of a triple well landscape. The point ‘A’ refers to the position of the scalar field for a class of initial conditions when the field begins its evolution out the of the AdS well. ‘B’ and ‘D’ denote the zeros of the potential, and ‘C’ marks the AdS vacuum. Points ‘E’ and ‘F’ correspond to the de-Sitter vacua $dS_{(I)}$ and $dS_{(II)}$ respectively.

AdS well. Points ‘B’ and ‘D’ denote the zeros of the potential, ‘C’ marks the AdS vacuum, and ‘E’ and ‘F’ correspond to the de-Sitter vacua $dS_{(I)}$ and $dS_{(II)}$ respectively. The arrow in Fig. 8 depicts the case of the initial condition when the field is considered to be rolling down, and it enters the AdS well in future evolution. As compared to the double well landscape potential discussed in the previous subsection, there are two possible end states in the present case. That is, after the field comes out of the AdS well, it can either settle in the first dS vacuum ($dS_{(I)}$) or in the second dS vacuum ($dS_{(II)}$). Whether the field ends up in $dS_{(I)}$ or $dS_{(II)}$ depends on the magnitude of the potential and the initial velocity of the scalar field. We will now solve the equations of motion for the landscape potential given in eq. (3.3), and obtain transitions to both the dS vacua, namely $dS_{(I)}$ and $dS_{(II)}$.

The initial evolution of the scalar field is similar to that in the case of double well potential. As the field rolls down to the AdS well, the scale factor begins to contract. In the further evolution, instead of going into big-crunch, the scale factor bounces from a finite non-zero value. During the bounce, the field gains a lot of kinetic energy owing to the anti-friction caused by negative Hubble rate. After the bounce, the field may either roll back in to the AdS well (which happens if the velocity of the scalar field becomes negative), or climb up the potential all the way to the positive hills (dS vacua). From this moment onwards, if the field has enough kinetic energy, it passes through the $dS_{(I)}$ vacuum and sits in the $dS_{(II)}$ vacuum. Otherwise, the field will settle down in the first de-Sitter well, i.e. $dS_{(I)}$. In the following subsections, we consider two types of initial conditions: (i) when the scalar field begins to roll down from the positive part of the potential (corresponding to point ‘A’ in Fig. 8) on the left of the AdS vacuum, and (ii) when the scalar field starts its evolution from the bottom of the AdS well (corresponding to point ‘C’ in Fig. 8).

1. Evolution in triple well potential: initial conditions at ‘A’

Fig. 9 shows the phase diagram of the dynamical trajectories of the scalar field, in the triple well landscape potential, when the initial conditions are provided in the positive part of the potential out of the AdS well, corresponding to point ‘A’ in Fig. 8. Different trajectories in this phase plot correspond to different initial values of the scalar field velocity, $\dot{\phi}(0)$. It is evident that in the future evolution, some of the trajectories end up in the first de-Sitter vacuum $dS_{(I)}$ (corresponding

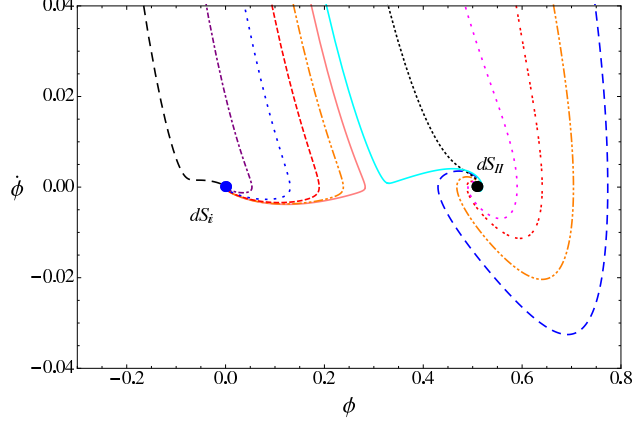


FIG. 9: This figure shows the dynamical phase trajectories of the scalar field at late times when the field climbs up to the positive part of the potential. Different trajectories correspond to different initial conditions on the value of the scalar field velocity, $\dot{\phi}$. It is evident from the figure that there are two possible future attractors for these class of solutions. Some of the trajectories end up into first dS vacuum (denoted by $dS_{(I)}$) and others into second dS vacuum (denoted by $dS_{(II)}$). The parameters of the potential are taken to be: $\alpha = 0.16$, $\beta = 129.6$, $\delta = 2.9$, $\nu = -13$, $\Omega = 180$ and $V_o = 10^{-6}$.

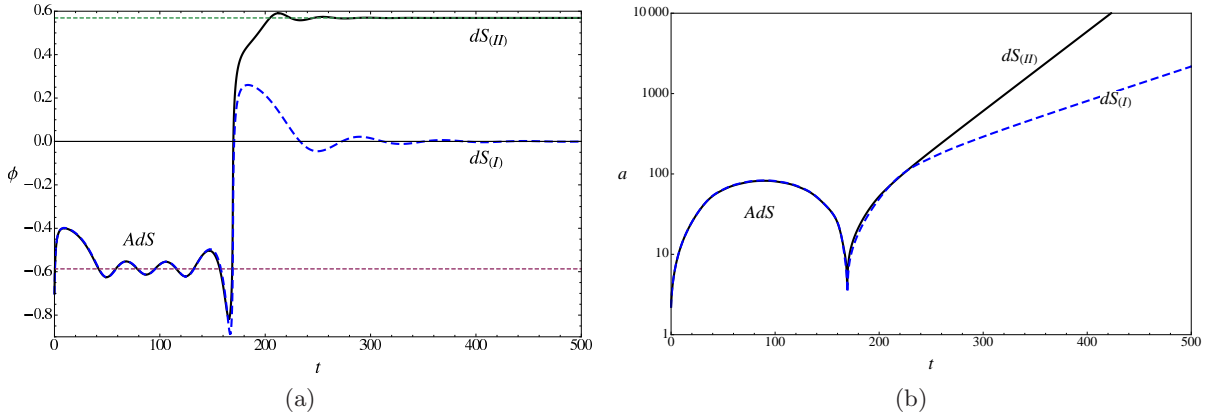


FIG. 10: This figure shows two examples of evolutions in triple well landscape potential. The solid (black) curve corresponds to the transition from AdS to $dS_{(II)}$ and the dashed (blue) curve shows the transition from AdS to $dS_{(I)}$. The figure (a) shows the evolution of the scalar field and (b) shows the corresponding scale factor which undergoes a bounce while the field makes a transition from AdS to one of the dS vacua. It is evident from the figure that in the beginning of the evolution, the scalar field spends some time in the AdS vacuum and then depending on the initial conditions and the parameters of the potential there is a non-singular transition from the AdS vacuum to one of the false dS vacua. The initial conditions for the evolution shown in these plots are: $a(0) = 7$, $\phi(0) = -0.75$, $\dot{\phi}(0) = 1.17 \times 10^{-2}$ (in Planck units) for the solid curve, and $a(0) = 7$, $\phi(0) = -0.75$, $\dot{\phi}(0) = 1.18 \times 10^{-2}$ for the dashed curve. The parameters of the potential are: $\alpha = 0.16$, $\beta = 75.78$, $\delta = 4.25$, $\nu = -13$, $\Omega = 180$ and $V_o = 10^{-6}$.

to point ‘E’ in Fig. 8) while others in the second vacuum $dS_{(II)}$ (corresponding to point ‘F’ in Fig. 8). In this way, the two de-Sitter vacua are future attractors of the corresponding dynamical trajectories. Let us now analyze the time evolution of the scalar field and the scale factor as the field passes through the landscape potential.

Fig. 10 shows the evolution of the scalar field and the corresponding scale factor for two different situations. In one of these situations, the field makes a transition from the AdS vacuum to the first dS vacuum ($dS_{(I)}$), and in the other case the field ends up in the second dS vacuum ($dS_{(II)}$).

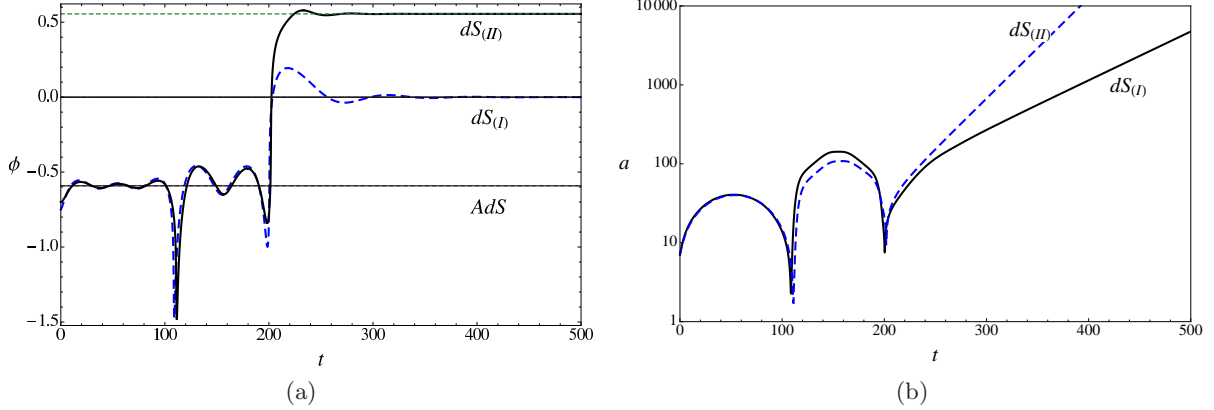


FIG. 11: This figure shows an example of evolution when the field spends more time in the AdS well while the scale factor undergoes two cycles of recollapse and bounce. The dashed curve shows the transition to $dS_{(I)}$, while the solid curve corresponds to the transition to $dS_{(II)}$. The initial conditions for the evolution shown in these plots are: $a(0) = 7$, $\phi(0) = -0.75$, $\dot{\phi}(0) = 1.2 \times 10^{-2}$ (in Planck units) for the solid curve, and $a(0) = 7$, $\phi(0) = -0.7$, $\dot{\phi}(0) = 5.6 \times 10^{-4}$ (in Planck units) for the dashed curve. The parameters of the potential are taken to be the same as in Fig. 10.

The evolution of the corresponding scale factors shows that during these transitions, the universe undergoes a non-singular bounce instead of falling into a big-crunch singularity. As a result, the scale factor bounces from a non-zero finite value. During the entire evolution, the energy density and the expansion scalar remain finite, which signals the avoidance of curvature singularity in the course of ‘AdS- $dS_{(i)}$ ’ vacuum transitions. Fig. 11 shows two examples of the evolution where the scale factor undergoes two cycles of AdS recollapse before the scalar field makes a transition from the AdS vacuum to one of the dS vacua. At the end of the first cycle, the velocity of the scalar field is negative, due to which instead of shooting off to one of the dS vacua, the field attempts to climb back in the other direction. On the other hand, the velocity of the scalar field at the end of the second cycle becomes positive and it climbs up to one of the dS vacua. The solid curve corresponds to the transitions to $dS_{(II)}$ and the dashed curves show the evolution when the end state of the field is $dS_{(I)}$. In this way, by tuning the initial conditions one can obtain more than one AdS cycles before making the transition to dS vacua.

2. Evolution in triple well potential: initial condition in the AdS well

Let us now consider initial conditions so that the scalar field starts its evolution at the bottom of the AdS well (corresponding to ‘C’ in Fig. 8), with a small kinetic energy. Fig. 12 shows the evolution of the scalar field and the corresponding scale factors. The dashed curve corresponds to the transition to $dS_{(I)}$ (corresponding to point ‘E’ in Fig. 8), while the solid curve shows the transition to $dS_{(II)}$ (corresponding to point ‘F’ in Fig. 8). Since the field is inside the AdS well, in the future evolution there is a recollapse of the scale factor. As the scale factor decreases, the curvature of spacetime becomes Planckian and quantum geometric effects become prominent. As a result, instead of going into a big-crunch singularity, the scale factor undergoes a smooth non-singular bounce while the scalar field makes transition to one of the dS vacua.

Let us summarize the results of this section. We have considered a double well and a triple well potential in the effective spacetime description of the $k = -1$ model in LQC. We have considered two types of initial conditions: first, when the potential of the landscape takes a positive value

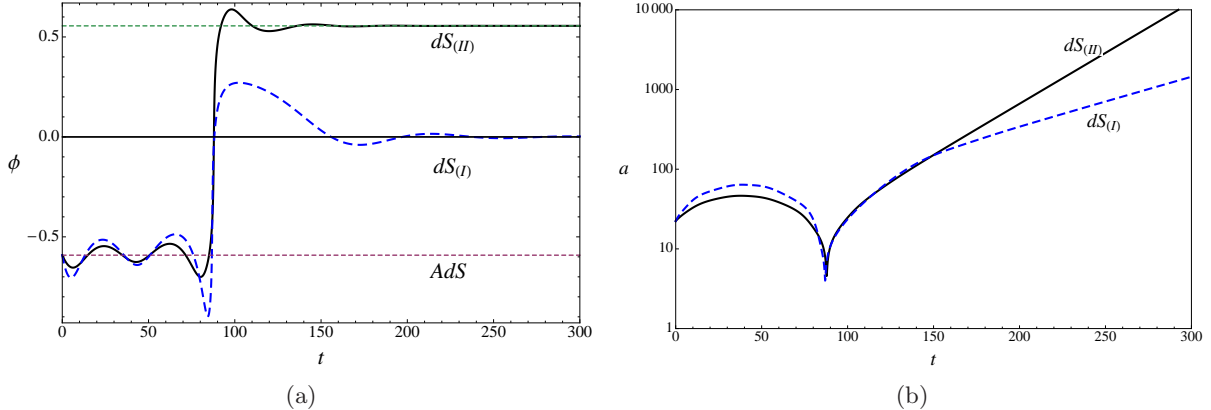


FIG. 12: This figure shows examples of evolution when the initial conditions are given at the bottom of the AdS well. The dashed curve shows the transition to $dS_{(I)}$, while the solid curve corresponds to the transition to $dS_{(II)}$. The initial conditions for the evolution shown in these plots are: $a(0) = 22.3$, $\phi(0) = -0.59$, $\dot{\phi}(0) = 2 \times 10^{-2}$ (in Planck units) for the solid curve, and $a(0) = 22.3$, $\phi(0) = -0.59$, $\dot{\phi}(0) = 4.8 \times 10^{-2}$ (in Planck units) for the dashed curve. The parameters of the potential are taken to be the same as in Fig. 10.

starting out of the AdS well and, second, when the field begins its evolution at the bottom of the AdS well. By numerically solving effective dynamical equations we show that unlike in the classical theory, there is a smooth non-singular transition from AdS vacuum to dS vacua in LQC. Depending on the initial conditions, it is possible that the field gets trapped inside the AdS well and oscillates for more than one cycle of AdS recollapse before making transition to a dS vacuum. The big crunch singularity of the classical theory is generically resolved and replaced by a big bounce. This result can now be generalized to any such landscape with more than one well. As discussed above, the LQC effects are visible only when the energy density becomes Planckian. Such a situation arises whenever the scale factor undergoes re-collapse (in our case while passing through the AdS phase). So, in any generic landscape, the LQC effects will always resolve the big-crunch singularity, as expected from the proof of resolution of all strong singularities in isotropic LQC [36].

IV. DISCUSSION

In a multiverse permitting eternal inflation, a challenging problem is to understand the way a bubble universe makes a transition from AdS phase to the dS phase. The metric inside such bubbles is that of an open FRW model. It is straightforward to see using classical theory that in the future evolution, an AdS bubble encounters a big crunch singularity. Transitions from AdS to dS are thus not possible in GR. It has been hoped that incorporation of quantum gravity effects can provide insights to this problem. Apart from understanding the global structure of multiverse, an answer to this question is also important to understand the measure problem in the multiverse. Recently, a local measure proposal based on a ‘watcher’ has been put forward [5], where probabilities are assigned to various events by an eternal observer, on a time-like geodesic, conjectured to make infinite number of transitions in the multiverse. Since the spacetime, in an AdS phase, ends with a big crunch singularity, it becomes necessary to find mechanisms to resolve singularities in the landscape scenario, so that the watcher can safely evolve from one vacua to another.

In this article, we addressed above problem using the effective spacetime description of LQC, which takes into account the corrections due to the quantum geometric nature of spacetime, as understood in loop quantum gravity. Due to these quantum geometric corrections, the Friedmann and Raychaudhuri equations are modified and the classical singularity is resolved. In the effective

4-dimensional picture, we consider an open FRW universe with a scalar field, with two types of self interacting potentials: a double well and a triple well potential. The form of the potentials is motivated from the landscape scenario. The problem we are interested in is to understand the transitions from the AdS to dS vacuum in the multiverse. In order to study the resolution of singularity and a smooth transition from AdS phase to a dS phase we consider potentials which have at least one AdS vacuum. With the help of explicit numerical simulations we show that, unlike in the classical theory, the big-crunch singularity is resolved in LQC and a smooth non-singular transition from AdS to dS phase can be obtained. Quantum gravitational resolution of big-crunch singularity changes the global structure of multiverse, as well as provides the watcher a safe passage from AdS to dS vacuum. It is to be noted that the quantum gravitational effects come into play only when the energy density is Planckian. Far away from bounce, when the curvature is very small and the energy density is less than a percent of the maximum allowed energy density, the quantum geometric effects are too feeble to make any difference between the effective trajectory of LQC and the classical theory.

In the numerical simulations of the landscape potentials, we considered two types of initial conditions, first when the scalar field begin to roll down from a positive part of the potential, and second when the field is already inside the AdS well. In the first type of initial conditions the field enters the AdS well in future evolution. In both the cases, the numerical simulations show that there is a non-singular smooth transition from AdS to dS phase, if the field builds up sufficient kinetic energy during the bounce. We have also shown examples of simulations where it takes two AdS cycles for the field to transit to the dS phase, giving rise to a ‘AdS-AdS-dS’ transition. In the triple well potential landscape we have considered one AdS vacuum with negative potential and two dS vacua with positive potential. As compared to the double well landscape, there are two different possible end states from the scalar field in the triple well, i.e. the two de-Sitter phases $dS_{(I)}$ and $dS_{(II)}$. The numerical simulations show that, like in the double well, there are transitions from AdS to dS phases. We have considered the transitions to both of the dS phases in the numerical simulations presented here. It turns out that there are initial conditions for which there are two AdS cycles before the field settles in one of the dS phases. For these initial conditions, there are two different future attractors corresponding to the two dS vacua, portrayed in the dynamical trajectories of the field.

Thus, we see that quantum geometric corrections in LQC resolve the big-crunch singularity occurring inside a bubble in the AdS phase. The non-singular evolution across the the big- crunch enables one to extend the geodesics across the big crunch. This provides the ‘watcher’ a safe transition from AdS to dS phase as well as opens up promising avenues to address the measure problem in the eternal inflation scenario.

Note: While this manuscript was being written, J. Garriga, A. Vilenkin, J. Zhang made us aware of their ongoing work on similar lines which is expected to appear simultaneously [60].

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Appendix A: $k = 0$ FRW universe

The goal of this appendix is to show that for $k = 0$ isotropic landscape model, though singularities are resolved, there is no AdS-dS transition. The metric for $k = 0$ FRW universe is given as

$$ds^2 = -dt^2 + a^2 (dx^2 + dy^2 + dz^2) \quad (\text{A1})$$

where a is the scale factor of the homogenous and isotropic metric and t is the proper time. In the classical theory, the flat FRW universe is governed by the following Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho. \quad (\text{A2})$$

The effective Hamiltonian for $k = 0$ FRW spacetime in terms of the symmetry reduced triad (p) and the connection (c) with the lapse being $N = 1$ is given as [13–15]

$$\mathcal{H}_{\text{eff}} = -\frac{3p^{3/2}}{8\pi G\gamma^2\lambda^2}\sin^2(\bar{\mu}c) + \frac{P_\phi^2}{2p^{3/2}} + V(\phi)p^{3/2} \quad (\text{A3})$$

where $\bar{\mu} = \lambda/\sqrt{p}$, P_ϕ is the momentum of the scalar field and $V(\phi)$ is the self interacting potential. The equations of motion of the triad and the connection can then be derived from the Hamilton's equations of motion

$$\dot{p} = \{p, \mathcal{H}_{\text{eff}}\}, \quad \dot{c} = \{c, \mathcal{H}_{\text{eff}}\}. \quad (\text{A4})$$

The equations of motion hence derived give rise to modified Friedmann equations, which can be written in terms of the energy density of the matter field and the Hubble rate as follows

$$H^2 = \frac{8\pi G}{3}\rho \left(1 - \frac{\rho}{\rho_c}\right), \quad (\text{A5})$$

where ρ is the energy density of the matter field, $\rho_c = 0.41\rho_{\text{Pl}}$ is the maximum value of the energy density at the bounce, and H is the Hubble rate given via $H = \dot{a}/a$. For a scalar field the energy density (ρ) and the pressure (P) of the matter field is given via

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad \text{and} \quad P = \frac{1}{2}\dot{\phi}^2 - V(\phi). \quad (\text{A6})$$

The conservation of energy momentum tensor further gives

$$\dot{\rho} = -3H(\rho + P). \quad (\text{A7})$$

In order for the field to mimic an AdS phase inside the negative part of the landscape potential, the velocity of the scalar field should be very small $\dot{\phi} \approx 0$ and the potential should dominate to give rise to a negative total energy density, in which case $\rho \approx V(\phi)$ where $V(\phi) < 0$. The pressure of the scalar field would then be $P \approx -V(\phi)$, making the equation of state of the scalar field inside the negative part of potential $w \approx -1$. However, it is important to emphasize that in the $k = 0$ model the energy density can not be negative. From eq. (A5) it is clear that for any physical solution one must have $H^2 \geq 0$, which leads to the following inequality on the energy density

$$0 \leq \rho \leq \rho_{\text{crit}}. \quad (\text{A8})$$

It is now clear from the above inequality that in a homogeneous and isotropic $k = 0$ FRW universe it is impossible to have an AdS phase, which would require $\rho < 0$ during the evolution of a scalar

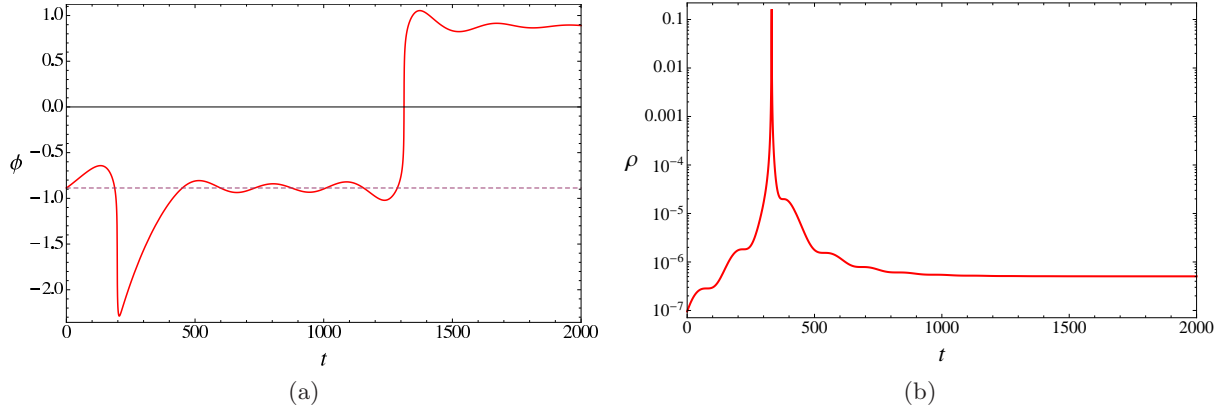


FIG. 13: These figures show the evolution of a scalar field in a double well landscape potential (eq. (3.1)) in a $k = 0$ FRW universe. Figures (a) and (b) respectively show the evolution of the scalar field and the energy density. The dashed curves in Fig. (a) denotes the value of the scalar field where the potential is negative minimum. It is clear from these figures that during the entire evolution the energy density always remains positive, even when the field evolves through the negative minimum of the potential. The initial conditions for these plots are: $a(0) = 70.7$, $\phi(0) = -0.887$, $\dot{\phi}(0) = 0.002$ (in Planck units). The parameters of the potential are taken to be: $\alpha = 0.28$, $\beta = 0.16$, $\delta = 10$ and $V_o = 10^{-6}$.

field through the negative part of the potential. Further, it is interesting to note that unlike the case of $k = -1$ model, at $\rho = 0$, the Hubble rate in $k = 0$ model in LQC vanishes. Eq.(A7) then implies that $\dot{\rho} = 0$.

Hence, the energy density can never be negative in the $k = 0$ model and there can not be change in its sign. In contrast to the $k = -1$ model, this is dynamically forbidden.

Fig. 13 shows the evolution of the scalar field and energy density in a double well landscape potential given by eq. (3.1). It is evident from the figure that even though the scalar field evolves through the negative minimum of the potential (denoted by dashed horizontal line in panel (a)), the energy density remains positive through out, implying that the evolution never becomes AdS in a $k = 0$ FRW universe. However, it is possible to achieve $\rho < 0$ and still satisfy $H^2 > 0$ if there is an additional positive term on the right hand side of eq. (A2) and eq. (A5), which is exactly the case in an open $k = -1$ FRW universe. Thus, the analysis of the flat FRW model in the landscape scenario fails to capture the relevant physics of AdS-dS transitions.

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