Pramāṇa - J. Phys., Vol. 30, No. 1, January 1988, pp. 15-27. © Printed in India.

Nonsingular cosmological models: the massive scalar field case

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MS received 11 May 1987; revised 8 September 1987

Abstract. The nonminimal coupling of a massive self-interacting scalar field with a gravitational field is studied. Spontaneous symmetry breaking occurs in the open universe even when the sign on the mass term is positive. In contrast to grand unified theories, symmetry breakdown is more important for the early universe and it is restored only in the limit of an infinite expansion. Symmetry breakdown is shown to occur in flat and closed universes when the mass term carries a wrong sign. The model has a naturally defined effective gravitational coupling coefficient which is rendered time-dependent due to the novel symmetry breakdown. It changes sign below a critical value of the cosmic scale factor indicating the onset of a repulsive field. The presence of the mass term severely alters the behaviour of ordinary matter and radiation in the early universe. The total energy density becomes negative in a certain domain. These features make possible a nonsingular cosmological model for an open universe. The model is also free from the horizon and the flatness problems.

Keywords. Symmetry breaking; cosmology; nonsingular cosmological models.

PACS No. 98·80

1. Introduction

The most widely accepted model of the Universe today is the Friedmann-Robertson-Walker big bang model which is based on the general theory of relativity. It is rightly titled as the standard model, being the only one which is capable of explaining satisfactorily most of the cosmological data obtained in the past few decades. The cosmological redshift, the microwave background radiation, the relative abundance of elements are but only a few things which the model can account for (Weinberg 1972). However, it is well known that the big bang cosmology can at best be a working model which would guide us in our endeavour to understand the true nature of the Universe. The reason for this belief is that some of the drawbacks of the model like the initial singularity are conceptual in nature. These obstacles are not due to any spurious assumptions made in constructing the model; rather they are generic to general relativity on which the model is based (Hawking and Penrose 1970; Hawking and Ellis 1973). So one hopes that some day we would have a theory which would most likely give us a model of the Universe which would be rid of the problems encountered in the standard model. It is believed that a quantum theory of gravity would solve many problems confronted not just in cosmological models but elsewhere in general relativity. Using De Witt's formulation of quantum gravity Hawking has shown that for a Friedmann-Robertson-Walker universe minimally coupled to a massive scalar field, there are bounce solutions without singularity (Page 1984; Narlikar and Padmanabhan 1983).

However, as of today, no formulation of quantum gravity is complete (Isham *et al* 1981; Markov and West 1984). Thus it has now become a practice to attack the impediments of the standard model in the classical and semiclassical framework with the hope that such moves might give us clue to the more desired quantum theory of gravity.

In the past few years many researchers have tried to develop the inflationary universe scenario (Guth 1981; Linde 1982, 1984) since it solves, apart from others, the horizon and the flatness problems. But it is also true that there are many problems associated with inflation (Mazenko *et al* 1985; Narlikar and Padmanabhan 1985) and moreover this scenario does not solve the most outstanding problem—the initial singularity (Vilenkin 1982, 1983). It can be shown that the initial singularity is the root cause of the horizon puzzle (Narlikar and Padmanabhan 1983) and the flatness problem (Sathyaprakash *et al* 1986). Once this is taken care of, the others automatically disappear. In a recent paper (Sathyaprakash *et al* 1986) we have shown that a nonsingular cosmological model is possible in the classical framework by modifying general relativity. The alterations made do not imply any new results at low energies and large length scales (in particular the present day physics is unchanged); but on the other hand the novel features which become important for the early universe, make possible a nonsingular cosmological model. The modification is made by nonminimally coupling the gravitational field with a self-interacting scalar field (Callan *et al* 1970).* In spite of the absence of a mass term the symmetry $\phi \to -\phi$ of the scalar field is spontaneously broken, through a non-vanishing time-dependent ground state solution (Grib and Mostepanenko 1977; Fleming and Silveira 1980; Padmanabhan 1983). Depending on the kind of model we choose, symmetry is broken either permanently (Grib and Mostepanenko 1977; Fleming and Silveira 1980; Padmanabhan 1983) or below a critical "radius" (Sathyaprakash *et al* 1984; Sathyaprakash and Sinha 1987). In either case symmetry breakdown is more important for the early universe (Abbot 1981; Sathyaprakash *et al* 1984; Gonzalez 1985) in contrast to grand unified theories where all symmetries are restored at an early epoch (or equivalently at a high temperature). The model has a naturally defined "Effective Gravitational Coupling Coefficient" (EGCC) which is rendered time-dependent due to the symmetry breakdown. It undergoes a change in sign below a critical value of the cosmic scale factor indicating the onset of repulsive gravity** (Linde 1980). Provided the repulsive forces are strong enough one could avoid the singularity. In our earlier work (Sathyaprakash 1986) it was shown that a nonsingular model can be constructed if the background metric is that of an open universe. There we had considered only a massless scalar field to be the source of geometry. But finite temperature calculations show that the scalar field acquires a mass term which varies

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* Such a coupling was first considered by Callan *et al* with the intension of improving the energy-momentum tensor of the scalar field in curved space-time, so that the resulting theory has no or lesser number of divergences.

** The appearence of repulsive gravity in the early universe was first noted by Linde (1980). Our treatment differs from Linde's in that we have exactly solved the scalar field equation in the curved space-time instead of just obtaining the extrema of the potential.

linearly with temperature (Dolan and Jackiw 1974; Linde 1979; Weinberg 1974). In this paper we shall consider a massive scalar field, couple it nonminimally with the metric field and study the consequences. Many few features emerge due to the presence of the mass term. As opposed to the massless theory where the symmetry is broken only in the open universe, in the massive case symmetry may breakdown irrespective of whether the universe is open, closed or flat. As a consequence of this, gravity becomes repulsive in the early universe in all these cases, but solutions to the gravitational field equations are nonsingular only in the open universe. A very important consequence of the mass term is that the energy-momentum tensor of the scalar field has a nonzero covariant divergence. This means that the principle of equivalence might not hold good any more. However, consistency with the principle of equivalene is guaranteed by including other matter fields and demanding that the total energy-momentum tensor is covariant-divergenceless. Such a criterion leads to a change in the behaviour of ordinary matter and radiation.

In § 2 the Lagrangian is set up and the equations of motion are derived. Spontaneous symmetry breakdown with its novel features are demonstrated by obtaining the stable ground state solutions of the scalar field equation. Implications of such solutions for EGCC is discussed in § 3. The conservation laws and their implications for the energy-momentum tensor of matter fields are dealt with in § 4. Solutions to the gravitational field equations are obtained in § 5.

2. The system

We shall consider a massive, self-interacting scalar field, nonminimally coupled to gravity, a metric field g and other matter fields. The Lagrangian density appropriate for the system is[†]

$$L = \sqrt{-g}[g_{\mu\nu}\phi^{\mu}\phi^{\nu} - \mu^2\phi^2 - \lambda\phi^4 + (\kappa^{-1} - \tfrac{1}{6}\phi^2)R + L_m]. \tag{1}$$

Hence ϕ_μ denotes $\partial\phi/\partial x^\mu$, and R is the curvature scalar of the background metric field which we shall assume to be Friedmann-Robertson-Walker universe. $\sqrt{-g}L_m$ is the Lagrangian density for the rest of matter fields and we shall assume that it does not contain ϕ explicitly. The mass of the scalar field μ and the quartic self-interaction coupling constant λ are the free parameters in the model. We shall identify the inverse of the coefficient of the curvature scalar as EGCC.

$$\boldsymbol{\kappa} = \kappa[1 - \tfrac{1}{6}\kappa\phi^2]^{-1}. \tag{2}$$

In general ϕ is a space-time-dependent quantity and so is $\boldsymbol{\kappa}$. But the choice of the homogenous and isotropic background metric dictates that ϕ be time-dependent atmost and therefore $\boldsymbol{\kappa}$ is also a time-dependent quantity. Moreover, $\boldsymbol{\kappa}$ can become negative when $\kappa\phi^2 > 6$ and this is a departure from the standard Einstein theory. We shall see an equivalent interpretation of this result in § 4.

[†]The metric has a signature − 2. The Greek indices run from 0 to 3. The Ricci tensor is defined as in Weinberg 1972.

By constructing the action from (1) and using the variational principle we arrive at the field equations. The scalar field equation is

$$\Box\phi + \mu^2\phi + 2\lambda\phi^3 + \tfrac{1}{6}R\phi = 0, \tag{3}$$

where $\Box$ denotes the d'Alambertian in curved space-time. The metric field equations are

$$G_{\mu\nu} = -\boldsymbol{\kappa}(\Theta_{\mu\nu} + T_{\mu\nu}), \tag{4}$$

where $G_{\mu\nu}$ is the Einstein tensor, $T_{\mu\nu}$ is the energy-momentum tensor corresponding to $\sqrt{-g}L_m$ and $\Theta_{\mu\nu}$ is the effective energy-momentum tensor of the scalar field given by:

$$\Theta_{\mu\nu} = \phi_\mu\phi_\nu - \tfrac{1}{2}g_{\mu\nu}(\phi_\rho\phi^\rho - \mu^2\phi^2 - \lambda\phi^4) - \tfrac{1}{6}(\phi^2_{;\mu\nu} - g_{\mu\nu}\Box\phi^2). \tag{5}$$

Here a semicolon denotes the covariant derivative. It is interesting to note that (3) and (5) are exactly those of Hoyle-Narlikar cosmology with $\mu = 0$ and $\lambda = 0$ (Hoyle and Narlikar 1964). In (5) we have not included that contribution of the gravitational field to the energy-momentum tensor of the scalar field, since our objective is to first study the behaviour of EGCC in the following section. In §§ 4 and 5 we shall consider the excluded term also. A few comments concerning the nonminimal coupling term are in order. Taking trace of (4) we get

$$R = \kappa(\mu^2\phi^2 + T), \tag{6}$$

where T is the trace of $T_{\mu\nu}$. Substituting this in the scalar field equation we get,

$$\Box\phi + \boldsymbol{\mu}^2\phi + 2\boldsymbol{\lambda}\phi^3 = 0 \tag{7}$$

where $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ are the modified mass and self-interaction coupling constant respectively:

$$\boldsymbol{\mu}^2 = \mu^2 + \tfrac{1}{6}\kappa T \quad \text{and} \quad \boldsymbol{\lambda} = \lambda + \tfrac{1}{12}\mu^2\kappa. \tag{8}$$

Thus, when a massive scalar field is present the nonminimal coupling term has a contribution to the self-coupling also, unlike the massless case where it behaves purely as a mass term. Observe that although we had assumed L_m to be independent of ϕ, coupling between ϕ and the rest of matter fields is indeed present through geometry. Note that, unlike the usual symmetry breaking theories, (7) does not admit any nonzero constant solution, since in general T is a space-time-dependent quantity.

We shall now proceed to obtain the ground state solutions of the scalar field equation. The Friedmann–Robertson–Walker metric in conformal coordinates is given by the line element

$$ds^2 = a^2(\tau)[d\tau^2 - d\chi^2 - h^2(x)(d\theta^2 + \sin^2\theta d\phi^2)] \tag{9}$$

where $h(\chi)$ is given by

$h(\chi) = \sin\chi$ for a closed universe

$= \chi$ for a flat universe and

$= \sinh\chi$ for an open universe.

The curvature scalar for this metric is

$$R = \tfrac{1}{6}[(\ddot{a}/a) + k] \tag{10}$$

where $k = \pm 1, 0$ according as the Universe is closed, open or flat and a dot denotes differentiation with respect to the time coordinate τ. Now, let the ground state solution be denoted by η. The choice of the homogeneous and isotropic background restricts η to be atmost time-dependent. Substituting the above relation for R in (3) and using the metric (9) we get the following equation in η:

$$\ddot{\eta} + 2\frac{\dot{a}}{a}\dot{\eta} + \left(\mu^2 a^2 + \frac{\ddot{a}}{a} + k\right)\eta + 2\lambda a^2 \eta^3 = 0. \tag{11}$$

On dimensional grounds we can try a solution of the form

$$\eta(\tau) = \gamma_k f(\tau)/a(\tau), \tag{12}$$

where γ_k is a constant. On substituting this in (11) we obtain

$$\ddot{f} + (\mu^2 a^2 + k)f + 2\lambda\gamma_k^2 f^3 = 0. \tag{13}$$

Unfortunately, a general solution of this equation is not available. Since we need an exact solution of (3) for an analysis of the gravitational field equations, we shall make the simplifying assumption that the mass of the scalar field varies inversely with the cosmic scale factor. This can be justified because (i) μ has the dimensions of inverse length and (ii) finite temperature calculations indicate that μ varies linearly with temperature and in an adiabatically expanding universe the "radius" of the universe is inversely proportional to the temperature. Thus, with

$$\mu^2 = b/a(\tau)^2, \tag{14}$$

where b is a dimensionless constant, (13) takes the form

$$\ddot{f} + (b + k)f + 2\lambda\gamma_k^2 f^3 = 0. \tag{15}$$

This equation can have nonzero constant solutions depending on the value of k and b.

(i) $b > 1$: In this case, symmetry remains unbroken irrespective of whether the Universe is closed, open or flat since $f = 0$ is the stable constant solution of (15). This means $\eta = 0$ is the stable vacuum solution of (11) and therefore the scalar field is in the symmetric state. This will not be of any interest to us and therefore we shall not consider this any more in our discussion.

(ii) $0 < b < 1$: As long as b is positive, symmetry is unbroken for $k = 1$ and 0. However, for an open universe symmetry is broken because now the stable solutions of (15) are $f = \pm 1$ with

$$\gamma_k^2 = -(k + b)/2\lambda, \quad k = -1, \tag{16}$$

and the vacuum solutions of (11) are

$$\eta(\tau) = \pm\gamma_k/a(\tau). \tag{17}$$

Thus the symmetry $\phi \to -\phi$ of the scalar field is spontaneously broken, not through a constant ground state solution but through a time-dependent solution. These solutions are energetically more favourable than the $\eta = 0$ solution as shown in § 4 (cf. eq. (23)). It is interesting to note that symmetry breaking becomes more and more important for the early universe and it is restored only in the limit of an infinite expansion. Notice that symmetry breakdown has occurred in spite of the fact that μ^2 is positive. This is in contrast to the usual symmetry breaking theories where μ^2 has to be necessarily negative to bring about symmetry breaking. By making the transformation $\phi \rightarrow \boldsymbol{\phi} = \phi - \eta$, we can write down the effective Lagrangian; the physical field $\boldsymbol{\phi}$ is found to have a mass $\boldsymbol{\mu}$ given by

$$\boldsymbol{\mu}^2 = (3 - 2b)/a^2(\tau). \tag{18}$$

The effective mass is found to depend inversely on the cosmic scale factor.

We have restricted μ^2 to be positive in the above analysis since its origin was supposed to be the finite temperature effects which induce a mass term of the form $\lambda T^2 \phi^2$ and λ has to be positive to ensure a lower bound for the potential. Hence, when only one scalar field is present, high temperature effects cannot facilitate symmetry breaking in the usual symmetry breaking theories. However, it is perfectly consistent with the positivity conditions, of the potential, to have a tachyonic mass induced when more than one scalar field is present (Mohapatra and Senjonovic 1979; Weinberg 1974; Zee 1980). Hereafter, let us assume μ^2 to be negative (or equivalently $b < 0$) but we shall not complicate the issue by including an extra scalar field.

(iii) $b < 0$: For open and flat universes symmetry is broken since $f = \pm 1$ are the stable solutions of (15). For the closed universe symmetry is unbroken for $b > -1$ and it is spontaneously broken for $b < -1$. Broken symmetry solutions in all these cases are given by (17) but now k can take all values, i.e. $1, 0, -1$. It should be borne in mind that γ_k^2 is positive always.

The results of this section can be summarized as follows. Symmetry breaking occurs for values of $(b + k)$ less than zero; for $k = -1$ when $b < 1$, for $k = 0$ when $b < 0$ and finally for $k = 1$ when $b < 1$. In all these cases symmetry breakdown is permanent and it is restored only in the limit of an infinite expansion.

3. Repulsive gravity

We shall now investigate how the results of the previous section affect EGCC. We shall substitute for ϕ in (2) the vacuum solution $\eta(\tau)$. Then,

$$\boldsymbol{\kappa} = \kappa(1 - \tfrac{1}{6}\kappa\eta^2)^{-1}. \tag{19}$$

For values of $(b + k) > 0$, the scalar field is in the symmetric state with $\eta = 0$ the stable ground state solution. In that case EGCC is a constant and the behaviour of gravity is the same at all epochs. However, when the mass of the scalar field is sufficiently small, i.e., $(b + k) < 0$, the scalar field is in the asymmetric state with the stable ground state solutions given by (17). Then EGCC becomes

$$\boldsymbol{\kappa} = \kappa[1 - (a_c/a)^2]^{-1}, \quad \text{where} \quad a_c^2 = \kappa\gamma_k^2/6. \tag{20}$$

Here a_c is a constant and has the dimensions of length. It is the value of the cosmic scale factor below which the gravitational interaction between elementary particles becomes repulsive and we shall call it the critical "radius". For values of $a(\tau) \gg a_c$, $\boldsymbol{\kappa}$

approaches the constant κ. Observe that as $a(\tau)$ approaches a_c from below, repulsive forces increase without any limit, indicating that classical physics might not hold near the critical radius. However, as we shall see in § 5, $a(\tau) = a_c$ is not a space-time singularity as none of the curvature tensor components blow up at this epoch.

The appearance of repulsive gravity in the early universe indicates that the resulting cosmological model might be free from singularity. Incidentally, since $\gamma_{-1} > \gamma_0 > \gamma_1$, repulsive forces start operating at a larger value of the cosmic scale factor in an open universe than in a closed or a flat universe. The onset of antigravity at a larger value of $a(\tau)$ enables a nonsingular model in an open universe.

4. Conservation laws

Henceforth we shall write the gravitational field equations in the standard form:

$$G_{\mu\nu} = -\kappa(\Theta_{\mu\nu} + T_{\mu\nu}). \tag{21}$$

Here $\Theta_{\mu\nu}$ is the total energy-momentum tensor of the scalar field given by

$$\Theta_{\mu\nu} = \Theta_{\mu\nu} - \tfrac{1}{6}\phi^2 G_{\mu\nu}. \tag{22}$$

By writing the field equations in the above form the term responsible for having repulsive gravity has been absorbed in $\Theta_{\mu\nu}$ and we shall see an equivalent interpretation of antigravity presently. For the asymmetric ground state solutions (17) of the scalar field, $\Theta_{\mu\nu}$ has the following nonzero components:

$$\Theta_0^0 = \frac{\gamma_k^2}{4a^4}(b + k), \tag{23}$$

$$\Theta_j^i = -\frac{\gamma_k^2}{12a^4}(3b - k)\delta_j^i. \tag{24}$$

Notice that since γ_k^2 is positive and $(b + k)$ is negative, the energy density of the scalar field Θ_0^0 is negative. But, the symmetric solution $\eta = 0$ would have rendered all the components of $\Theta_{\mu\nu}$ zero. Thus, the asymmetric solutions (17) are not only stable when $(b + k) < 0$ but are also energetically more favourable than the unstable symmetric solution $\eta = 0$. The negative energy density accounts for the repulsive field that crops up at small length scales. The presence of a negative energy density in the model violates the energy condition of the singularity theorem and hence a singularity need not occur in the present model. However, to avert singularity, in addition to having the energy density negative, one must have the space–space components of Θ_μ^ν, which represent the pressure, positive. This cannot be satisfied if b is negative. This means a nonsingular solution is not viable in a closed or a flat universe. For $b > 0$, Θ_j^i are positive and symmetry breaking occurs for positive values of b only in an open universe provided $b < 1$. We shall assume in what follows that $0 < b < 1$ and therefore we shall consider only the open universe case for which

$$\Theta_0^0 = -\frac{\gamma_{-1}^2}{4a^4}(1 - b), \tag{25}$$

$$\Theta_j^i = \frac{\gamma_{-1}^2}{12a^4}(1 + 3b)\delta_j^i. \tag{26}$$

Let $\Sigma_{\mu\nu}$ be the total energy-momentum tensor of the scalar field and the rest of matter fields:

$$\Sigma_{\mu\nu} = \Theta_{\mu\nu} + T_{\mu\nu}. \tag{27}$$

It should obey the covariant conservation law

$$\Sigma^{\mu}_{\nu;\mu} = 0. \tag{28}$$

Now, if $\Theta_{\mu\nu}$ is the only source of geometry (i.e., $T_{\mu\nu} = 0$), then the above equations imply that $b = 0$. In other words, if b is nonzero, $\Theta_{\mu\nu}$ alone does not satisfy the conservation law. To be consistent with the principle of equivalence (28) must be satisfied. This can be assured if we have in addition to the scalar field some other matter fields as a source of the gravitational field. For simplicity we shall take $T_{\mu\nu}$ to be that of a perfect fluid.

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu - p g_{\mu\nu}, \tag{29}$$

where ρ, p and U_μ are the energy density, pressure and the four-velocity of the fluid respectively. Assuming an equation of state of the form $p = \beta\rho$, and using (25) and (26), conservation laws imply for $\beta < 1/3$ that

$$\rho = \frac{A}{a^{3(1+\beta)}} - \frac{b\gamma^2_{-1}}{(1-3\beta)}\frac{1}{a^4}, \tag{30}$$

and for $\beta = 1/3$ (i.e. radiation) that

$$\rho = \frac{A}{a^4} + \frac{b\gamma^2_{-1}}{a^4}\ln(a/a_0). \tag{31}$$

Here A and a_0 are the constants of integration. The expression for the energy density of the perfect fluid picks up an extra term due to the presence of the mass term. (In the absence of a mass term $\Theta_{\mu\nu}$ satisfies the conservation law independently and would have the usual dependence on $a(\tau)$). Even when $\beta \neq 1/3$, ρ has a term proportional to $1/a^4$. Thus, fluids other than radiation can also become important for the early universe. Notice that as $a(\tau)$ decreases, ρ becomes negative eventually.

Making the scalar field time-dependent is equivalent to including a thermal gas which behaves like radiation at very high temperatures. This is the reason why the energy density of the perfect fluid picks up an extra term which corresponds to the scalar field whose contribution is more important at higher temperatures.

In what follows, for simplicity we shall consider only the two extreme cases $\beta = 0$ and $\beta = 1/3$. Using expressions (25), (26) and (30) or (31) we get the following nonzero components of $\Sigma_{\mu\nu}$:

(i) For dust

$$\Sigma^0_0 = \frac{3}{\kappa a^4}(2qa - l^2) \tag{32}$$

$$\Sigma^i_j = \frac{1}{\kappa a^4} l^2 \delta^i_j \tag{33}$$

(ii) For radiation

$$\Sigma_0^0 = \frac{3}{\kappa a^4}(2q - m^2 + n^2 \ln(a/a_0)) \tag{34}$$

$$\Sigma_i^j = \frac{1}{\kappa a^4}(m^2 - 2q - n^2 \ln(a/a_0))\delta_j^i \tag{35}$$

where l^2, q, m^2 and n^2 are positive constants given by:

$$l^2 = (a_c^2/2)(1 + 3b).$$
$$n^2 = 2a_c^2 b,$$
$$m^2 = (a_c^2/2)(1 - 3b),$$
$$q = (A/6)\kappa.$$

5. Nonsingular solutions

We can now solve the gravitational field equations. As mentioned in the previous section, even pressure-free fluids can become important in the early universe. Thus, though we are interested in the implications of the field equations for the early universe, we should consider both dust and radiation models.

(i) *Dust models*: Using expressions (32) and (33) metric field equations (21) take the form

$$\dot{a}^2 - a^2 = 2qa - l^2 \tag{36}$$

$$2a\ddot{a} - \dot{a}^2 - a^2 = l^2. \tag{37}$$

A consistent solution of the above equations is

$$a(\tau) = (l^2 + q^2)^{1/2} \cosh(\tau + \tau_0) - q, \tag{38}$$

where τ_0 is a constant of integration which sets the origin of the time coordinate. The choice $\tau_0 = 0$, renders $a(\tau)$ to be minimum at $\tau = 0$:

$$a(0) = (l^2 + q^2)^{1/2} - q. \tag{39}$$

The constant l^2 involves the parameters λ and b and will be zero if the scalar field is absent; then $a(0) = 0$. Thus, it is the presence of a scalar field that leads to a nonzero value for the minimum size of the Universe. The solution we have obtained will be meaningful only if the minimum size of the Universe is larger than the Planck length since, then one can be assured that the quantum gravity effects will not alter the results obtained. The problems that arise at Planck lengths, ℓ_P can be averted if $a(0) > \ell_P$. This can be achieved by choosing l^2 to be sufficiently large. Since l^2 depends inversely on the coupling constant λ, smaller the λ larger is the initial size of the Universe. For large τ the expansion of the Universe is exponential as in the inflationary universe models. The inflation here is not due to a constant energy density of a supercooled symmetric state of a scalar field, but instead, it is due to a rapidly decreasing ground state energy density of a scalar field in the asymmetric state. As a result, the exponential expansion proceeds only as long as the negative energy density part dominates over other terms. Thereafter the evolution of the Universe is akin to the usual Friedmann phase. Thus, in this model the Friedmann phase appears naturally after an exponential phase of expansion. By choosing appropriate values for the parameters b and λ one can transit to the Friedmann phase at an early epoch such that, such late time processes as helium synthesis are not affected by the present model.

The "velocity" at $\tau = 0$ given by $\dot{a}(0)$ is zero. Thus, the Universe does not start with a big bang but instead there is a smooth bounce at $\tau = 0$.

A nonzero minimum size for the Universe does not necessarily mean that the space-time is devoid of a singularity at all epochs. In the present model, for instance, the critical radius at which the magnitude of EGCC becomes infinite can be suspected to be a singular point. We shall now see that this is not so by showing (a) finiteness of curvature tensor components and (b) completeness of the null geodesic.

(a) The nontrivial independent components of the Riemann–Christoffel curvature tensor are

$$R^1_{010} = (a\ddot{a} - \dot{a}^2)/a^2 \tag{40}$$

$$R^2_{121} = (a^2 - \dot{a}^2)/a^2. \tag{41}$$

For the solution (38) these components are

$$R^1_{010} = (\omega^2 - q\omega \cosh \tau)/J(\tau)^2 \tag{42}$$

$$R^2_{212} = (\omega^2 + q^2 - 2\omega q \cosh \tau)/J(\tau)^2, \tag{43}$$

where $J(\tau) = (\omega \cosh \tau - q)$ and ω is a constant given by $\omega^2 = l^2 + q^2$. These components of the curvature tensor remain finite for all values of τ and in particular at the critical time τ_c given by:

$$a_c = \omega \cosh \tau_c - q. \tag{44}$$

Thus, we are assured that the epoch at which EGCC changes sign is not singular and that the curvature tensor components remain finite at all finite times.

(b) Consider a null geodesic travelling radially. Then the line element satisfies

$$ds^2 = 0 = d\tau^2 - d\chi^2. \tag{45}$$

Hence the affine parameter length from the event (τ_0, χ_0) to the event (τ, χ) denoted by $\ell(\tau, \tau_0)$ is given by:

$$\ell(\tau, \tau_0) = a(\tau) \int_{\tau_0}^{\tau} d\tau. \tag{46}$$

Now the geodesic is said to be complete if the above integral diverges in the limit of $\tau_0 \to 0$ ($\tau_0 \to -\infty$) in the case of a singular (nonsingular) cosmological model. From solution (38) we have

$$\ell(\tau, \tau_0) = a(\tau)(\tau - \tau_0). \tag{47}$$

Clearly, the above integral diverges in the limit $\tau_0 \to -\infty$ and hence the geodesic is complete. Since the Universe has been existing for all times and also since the geodesic is complete, there has been enough time for different parts of the Universe to come into contact with each other. Thus the horizon problem does not occur in the present model. Notice also that one does not have to bother about the initial conditions since there is no such thing as initial time and as a consequence the flatness problem does not arise.

(ii) *Radiation models*: The gravitational field equations in this case are

$$\dot{a}^2 - a^2 = 2q - m^2 + n^2 \ln(a/a_0) \tag{48}$$

$$2a\,\ddot{a} - \dot{a}^2 - a^2 = 2q + m^2 - n^2 \ln(a/a_0). \tag{49}$$

These equations cannot be solved exactly. However, some qualitative arguments can still be made. Self consistency of the above equations can be established very easily. Hence we shall analyse only (48) but the features that emerge will be respected by (49) also. For convenience we shall write (48) as

$$\dot{a}^2 = a^2 + w^2 + n^2 \ln(a/a_0), \tag{50}$$

where $w^2 = 2q - m^2$. Since the left hand side is always a positive quantity so should the right hand side. For $a(\tau) < a_0$ the last term in the above equation becomes negative and as $a(\tau)$ decreases further the right hand side will itself become negative. Therefore $a(\tau)$ cannot become zero and its minimum value is determined by the transcendental equation

$$a^2 + w^2 + \ln(a/a_0) = 0. \tag{51}$$

When $a(\tau)$ reaches its minimum value, $\dot{a}(0)$ is obviously zero. The main features of the 'dust' model are reflected in this case also.

At first it would appear that since the effective gravitational "constant" approaches $-\infty$ as the universe expands from $a = a(0)$ to $a = a_c$ there is a big bang at the critical "radius". But when we look at the solutions for the cosmic scale factor we see that the "velocity" given by $\dot{a}(\tau)$ does not blow up at any time and in particular at the critical time. Thus the effective gravitational coupling coefficient should only be considered as a means to understand why singularity is avoided in the present model. It is true that we have added new terms to the general relativistic theory. But that singularity avoided is proved not only by proving the finiteness of the curvature tensor invariants but also by showing that geodesics are complete. In fact we have shown that past directed null geodesics are complete.

6. Conclusions

In this paper we have obtained a nonsingular model for an open universe in the presence of a massive scalar field. The nonminimal coupling makes contributions to the mass term as well as to the self-interaction. Though the scalar field and the rest of matter fields were assumed to be independent, the coupling indeed shows up through geometry. This is also evident from the fact that a nonzero mass can be accommodated in the model only if the energy-momentum tensor corresponding to the rest of the world has a contribution from the scalar field.

In contrast to grand unified theories, the scalar field "inversion" symmetry is permanently broken and the consequences are more important for the early universe. A very peculiar feature is that symmetry breakdown occurs, in the open universe, even when the mass term carries the right sign. It is found that a singularity-free model is viable only if the mass term carries a positive sign. In a massless theory, symmetry breaking occurs only in an open universe whereas in the present model it occurs in flat and closed universes also, when the mass is tachyonic. As a result of this gravity becomes repulsive in a certain domain in all these cases, with the 'critical radius', at which gravitational interaction changes sign, being the largest for the open universe. The appearence of antigravity can be attributed to the fact that the scalar field energy density is negative.

The reason why singularity-free models are possible only for an open universe, is the following. The energy density corresponding to the total energy-momentum tensor becomes negative below a certain value of the cosmic scale factor only for $k = -1$. But it stays positive for $k = 0$, 1 except when $T_{\mu\nu}$ corresponds to radiation, in which case it becomes negative above a certain value of $a(\tau)$. This only enables to set an upper limit on the size of the universe in these two cases, but the initial singularity continues to be there. Even for $k = 0, 1$ the initial velocity $\dot{a}(0)$ is finite and thus the birth is not explosive.

At late times the expansion of the universe is exponential, as in the inflationary universe models. In inflationary universe models, the right value for density perturbations, which are needed to explain the formation of small scale structures, can be obtained only if the self-interaction coupling constant is chosen to be $\sim 10^{-14}$. However, in the present model one can possibly do away with a reasonable value of λ since we have two parameters instead of one. Work is in progress to see whether this can be accomplished.

Acknowledgements

One of the authors (BSS) would like to thank Mr Madan Rao and Mr Yatin Marathe for helpful discussions, and is grateful to the UGC for financial support.

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