Nonstandard optics from quantum space-time

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We study light propagation in the picture of semiclassical space-time that emerges in canonical quantum gravity in the loop representation. In such a picture, where space-time exhibits a polymerlike structure at microscales, it is natural to expect departures from the perfect nondispersiveness of an ordinary vacuum. We evaluate these departures, computing the modifications to Maxwell's equations due to quantum gravity and showing that under certain circumstances nonvanishing corrections appear that depend on the helicity of propagating waves. These effects could lead to observable cosmological predictions of the discrete nature of quantum space-time. In particular, recent observations of nondispersiveness in the spectra of gamma-ray bursts at various energies could be used to constrain the type of semiclassical state that describes the universe. [S0556-2821(99)05612-X]

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The recent discovery of the cosmological nature of gamma-ray bursts opens new possibilities to use them as a laboratory to test fundamental physics. This has been emphasized by Amelino-Camelia *et al*. [1]. What these authors point out is that the light coming from gamma-ray bursts travels very large distances before being detected on Earth, and is therefore quite sensitive to departures from orthodox theories. In particular, the bursts present detailed time structures, with features smaller than 1 ms, that are received simultaneously through a broad band of frequencies, ranging from 20 to 300 keV, as reported by the BATSE detector of the Compton Gamma Ray observatory [2]. This implies stringent limits on any dispersive effects that light might suffer in traveling towards the Earth.

Various models of string quantum gravity imply dispersive frequency wavelength relations for light propagation, and in Ref. [1] it was shown that the simultaneity of time structures in the patterns of light received gamma ray bursts are possible candidates to set limits on these models. In this note we would like to probe similar issues for loop quantum gravity. An attractive feature of this approach is that it might imply a unique signature of the discrete nature of space time tantamount to an ''intrinsic birefringence'' of quantum space-time. This effect would imply a distinctive ''doubling'' of patterns observed in the time series analysis of the bursts, making it attractive from the observational point of view. We will see however, that the nature of the effects predicted by loop quantum gravity depend on the type of semiclassical state that one considers. In a sense, one can turn the argument around and suggest that rather than viewing these effects as a prediction of the theory, they can be used to constrain the type of semi-classical states one considers to represent realistic cosmologies.

Loop quantum gravity [3] is usually formulated in the canonical framework. The states of the theory are given by functions of spin networks, which are a convenient label for a basis of independent states in the loop representation. This kinematic framework is widely accepted throughout various formulations of the theory, and has led to several physical predictions associated with the ''polymerlike'' structure of quantum space-time [4]. For instance, a quite clear picture of the origin of the black hole entropy emerges [5]. The dynamics of the theory is embodied in the Hamiltonian constraint, and consistent proposals are currently being debated [6]. To show the existence of the birefringent effect we will not need too many details of the dynamics of the theory. We prefer to leave the discussion a bit loose, reflecting the state of the art in the subject, since there is no agreement on a precise dynamics. Also, the spirit of our calculation is to attempt to make contact with observational predictions, something that is importantly lacking in the canonical approach, in part as a consequence of the absence of a detailed prescription for constructing the semiclassical limit of the theory. One should therefore view the current work as a further elaboration towards probing the nature of the semiclassical limit. Initial explorations on this subject can be found in Ref. [8].

The term in the Hamiltonian constraint coupling Maxwell fields to gravity is the usual ''E^2+B^2'' term, but in a curved background:

$$H_{\rm Maxwell}=\frac{1}{2}\int d^3x \underset{\sim}{g}_{ab}(\tilde{e}^a\tilde{e}^b+\tilde{b}^a\tilde{b}^b), \tag{1}$$

where we have denoted with tildes the fact that the fields are vector densities in the canonical framework. This requires the division by the determinant of the metric, which we denoted by an under-tilde in the metric. Thiemann [7] has a concrete proposal for realizing in the loop representation the operator corresponding to the metric divided by the determinant.

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Since we are interested in low-energy, semiclassical effects, we will consider an approximation where the Maxwell fields are in a state that is close to a coherent state. That is, we will assume that the Maxwell fields operate as classical fields at the level of equations of motion, however, we will be careful when realizing the Hamiltonian to regulate operator products. This departs from the regularization proposed by Thiemann. In his approach, the states considered are such that the electric field operator is also discrete and finite and therefore products at the same point are acceptable. One can consider this as a feature of the full diffeomorphism-invariant context, that will disappear at an effective level when one considers semiclassical states. There one would expect to recover the usual Maxwell theory with its divergences. The coherent state chosen will be one that approximates a classical traveling wave of wavelength λ, which we assume to be much larger than the Planck length.

For the gravitational degrees of freedom we will assume we are in a ''weave'' state [8] $|\Delta\rangle$, such that

$$\langle\Delta|\hat{\underset{\sim}{g}}_{ab}|\Delta\rangle=\delta_{ab}+O\left(\frac{l_P}{\Delta}\right), \tag{2}$$

where l_P is Planck's length. Weave states [8], characterized by a length Δ, are constructed by considering collections of Planck-scale loops. They are meant to be semiclassical states such that that if one probes these states at lengths much smaller than Δ one will see features of quantum space-time, whereas if one probes at scales of the order of, or bigger than, Δ one would see a classical geometry. The weave we will consider approximates a flat geometry for lengths larger than Δ. It is worthwhile noticing that weave states were introduced some time ago in the context of the loop representation, before a variety of new techniques (cylindrical functions, spin networks) were introduced to deal with the quantum states in this representation. At the moment there is not a complete picture of how to construct weave states in the loop representation in terms of spin network states. When they were originally introduced, weave states were meant to yield semiclassical behaviors in certain operators capturing metric information of space-time. It was evident that there were many inequivalent states that could fit these requirements. If the reader wishes, this paper introduces further requirements that we need to demand from such semiclassical states. We will return to this issue after we introduce the effects we wish to discuss.

Let us now consider the action of the Hamiltonian we proposed above on a weave state. We need a few more details of the regularization of $\underset{\sim}{g}_{ab}$ that was proposed by Thiemann [7]. It consists in writing $\underset{\sim}{g}_{ab}$ as the product of two operators $\hat{w}_a(x)$, each corresponding to a commutator of the Ashtekar connection with the square root of the volume operator. The only feature we will need of these operators is that acting on spin network states they are finite and only give contributions at intersections. We now point split the operator as suggested in [7] (to shorten equations we only consider the electric part of the Hamiltonian, the magnetic portion is treated in the same way):

$$\hat{H}^E_{\rm Maxwell}=\frac{1}{2}\int d^3x\int d^3y\,\hat{w}_a(x)\hat{w}_b(y)E^a(x)E^b(y)f_\epsilon(x-y), \tag{3}$$

where $\lim_{\epsilon\to 0}f_\epsilon(x-y)=\delta(x-y)$, so it is a usual point-splitting regulator, and we have eliminated the tildes to simplify notation, and as we stated above, treat the electric fields as classical quantities. The operators $\hat{w}_a$ only act at intersections of the weave, so the integrals are replaced by discrete sums when evaluating the action of the Hamiltonian on a weave state:

$$\langle\Delta|\hat{H}^E_{\rm Maxwell}|\Delta\rangle =\frac{1}{2}\sum_{v_i,v_j}\langle\Delta|\hat{w}_a(v_i)\hat{w}_b(v_j)|\Delta\rangle E^a(v_i)E^b(v_j), \tag{4}$$

where v_i and v_j are vertices of the weave and the summation includes all vertices within the domain of characteristic length Δ. We now expand the electric field around the central point of the Δ domain, which we call P, and get

$$E^a(v_i)\sim E^a(P)+(v_i-P)_c\partial^c E^a(P)+\cdots, \tag{5}$$

and given the assumptions we made about the long wavelength nature of the electric fields involved, we will not need to consider higher order terms in the expansion at the moment. Notice that $(v_i-P)_c$ is a vector of magnitude approximately equal to Δ, whereas the partial derivative of the field is of order $1/\lambda$, that is, we are considering an expansion in Δ/λ. We now insert this expansion in the Hamiltonian and evaluate the resulting terms in the weave approximation. One gets two types of terms, one is given by the product of two electric fields evaluated at P times the sum over the vertices of the metric operator. Due to the definition of the weave state, the sum just yields the classical metric and we recover the usual Maxwell Hamiltonian in flat space.

We now consider the next terms in the expansion Δ/λ. They have the form,

$$\frac{1}{2}\sum_{v_i,v_i}\langle\Delta|\hat{w}_a(v_i)\hat{w}_b(v_j)|\Delta\rangle(v_i-P)_c\partial_c[E^a(P)]E^b(P) +(v_j-P)_cE^a(P)\partial_c[E^b(P)]. \tag{6}$$

When performing the sum over all vertices in the cell we discussed above, we end up evaluating the quantity $\langle\Delta|\hat{w}_a(v_i)\hat{w}_b(v_j)|\Delta\rangle(v_i-P)_c$. This quantity averages out to zero in a first approximation, since one is summing over an isotropic set of vertices. The value of the quantity is therefore proportional to l_P/Δ, the larger we make the box of characteristic length Δ the more isotropic the distribution of points is. We consider the leading contribution to this term, which should be a rotational invariant tensor of three indices, i.e., it is given by $\chi\epsilon_{abc}l_P/\Delta$ with χ a proportionality constant of order one (that can be positive or negative).

We have therefore found a correction to the Maxwell Hamiltonian arising from the discrete nature of the weave construction. It should be noticed that the additional term we found is rotationally invariant, i.e., it respects the original spirit of the weave construction. It is, however, parity violating. If one were to assume that the weaves are parity-invariant, the term would vanish. The term would also vanish —on average— if one assumes that the different regions of size Δ have ''random orientations'' in their parity violation. The fact that we live in a nonparity invariant universe suggests that parity invariant weaves might not necessarily be the most natural ones to consider in constructing a semiclassical state of cosmological interest.

A criticism that could be levied is how do we know that all the ''domains'' of size Δ add up their parity-violating effects as opposed to canceling out each other randomly. We are not claiming that this necessarily happens. We just point out that this parity violation is allowed within the theory. The answer to if it is plausible will depend on detailed cosmological dynamics of the weaves, which are at present unknown. It is not even clear whether the breaking of parity invariance should be viewed as a ''phase transition'' from an initially Lorentz-invariant universe, or if the initial universe was not Lorentz invariant at all, the latter being a symmetry that sets in at later stages in the evolution. Parity non-invariant dynamics have already been considered in the context of the standard model (see for instance [9]) and one could always raise the same type of objection. The purpose of this paper is not to prove that this violation is a unique consequence of the polymerlike nature of quantum space-time, rather to show that parity-violating weaves are consistent with the usual requirements that have been typically demanded of semiclassical space-times in this context.

Assuming a nonparity invariant weave, the resulting equations of motion from the above Hamiltonian can be viewed as corrections to the Maxwell equations:

$$\partial_t \vec{E} = -\nabla\times\vec{B} + 2\chi l_P \Delta^2 \vec{B}, \tag{7}$$

$$\partial_t \vec{B} = \nabla\times\vec{E} - 2\chi l_P \Delta^2 \vec{E}. \tag{8}$$

As we see the equations gain a correction proportional to the Laplacian Δ^2 of the fields, the correction is symmetrical in both fields, but is not Lorentz covariant. This already suggests that there will be modifications to the usual dispersion relation for light propagation. The lack of covariance is not surprising, since the weave selects a preferred foliation of space-time. This again is what is standardly accepted in cosmological applications as we will consider, there is a preferred set of comoving observers, and for such observers we will compute the effect to be observed.

If one now combines the above equations to study wave propagation, we get

$$\partial_t^2 \vec{E} - \Delta^2 \vec{E} - 4\chi l_P \Delta^2 (\nabla\times\vec{E}) \tag{9}$$

and similarly for $\vec{B}$. We now seek solutions with a given helicity:

$$\vec{E}_\pm = \mathrm{Re}((\hat{e}_1 \pm i\hat{e}_2) e^{i(\Omega_\pm t - \vec{k}\cdot\vec{x})}). \tag{10}$$

Substituting in the above equations, we get

$$\Omega_\pm = \sqrt{k^2 \mp 4\chi l_P k^3} \sim |k|(1 \mp 2\chi l_P |k|). \tag{11}$$

We therefore see the emergence of a birefringence effect, associated with quantum gravity corrections. The group velocity has two branches, and the effect is of the order of a shift of one Planck length per wavelength.

This effect is distinct from other effects that have been discussed in the past. If we compare with the proposals considered by Amelino-Camelia *et al*. [1], in their case they find only a change in the dispersion relation, whereas here we in addition see a helicity-dependent effect. Our effect is also absent for scalar fields, whereas other quantum gravity corrections are all-encompassing (they can be viewed as corrections to quantum mechanics itself). Birefringence was also considered in the context of modifications of electromagnetism and also in non-symmetric gravity [10]. In those cases the effect was not frequency dependent. This is because the kind of corrective terms we are introducing in Maxwell theory, although linear in the fields, are higher order in the derivatives. This is in line with the observations of Ref. [1], that quantum gravity effects will increase with the frequency, the opposite being expected for other more standard sources of cosmological dispersion or birefringence.

To quantify the magnitude of these effects, if one considers a gamma-ray burster at cosmological distances (about 10^{10} light years) and frequencies of the order of 200 keV (like the channels of the BATSE detector), this implies a delay between the two group velocities of both polarizations that compose a plane wave of 10^{-5} s. The observed width of the bursts appears to be of the order of 0.1 s, with features like a rising edge as small as 1 ms. We therefore see that with such observations one is two orders of magnitude away of observing these effects. This is fairly impressive given that this is an effect due to quantum gravity. The intention of this note is not to present a detailed calculation of the magnitude of the effects, however, one could envision a more subtle program to seek for the effect, given its distinctive signature, and its specific dependence on frequency, using data from more than one channel and more sophisticated pattern matching techniques.

How did a birefringence appear? In the construction of the weave, we have assumed that rotational invariance is locally preserved. However, we have not assumed that parity invariance is preserved, and in the model considered it is violated. That is, one can envisage a fundamental, Planck-level violation of parity in the weave approach, without detriment to the ability of the weave to approximate a given metric. Which weave to choose (parity preserving or violating) is a reasonable issue to settle experimentally. The measurements of spectra of gamma-ray bursts might provide a mechanism for this. It is intriguing to see if other symmetries might be violated and which observational consequences it might have. It is in this sense that this paper can be viewed as further conditions that must be met by the semiclassical states of the theory.

In general, without further input from the dynamics of the theory, one would expect that a weave structure would lead to the loss of Poincaré invariance of the Maxwell equations. In the example considered we see that this invariance is broken simultaneously with parity invariance. It is interesting to notice that if the weave is parity preserving Poincaré invariance is preserved as well.

Another viewpoint could be that if at some point a complete dynamical theory is established that determines the evolution of the weaves, one could presumably construct quantum toy cosmological models. In such a situation the final weave describing our current universe would be prescribed and one could determine if the theory predicts the presence of birefringence or not along the lines discussed in this paper.

Finally, at a more formal level, the appearance of corrections to the propagation of light might allow to study effects concerning information loss in black hole systems. These considerations are currently under study.

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