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Nonstatic One-Boson-Exchange Potentials*

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A list of the nonstatic one-boson-exchange potentials between two nucleons is given. The potentials are obtained exactly in the momentum space. In the coordinate space they are given up to the order M^{-2} in the M^{-1} expansion. (M is the nucleon mass.)

§ 1. Introduction

The existence of resonance or bound states with isospin and angular momentum zero or one has recently been anticipated from both experiments[1] and theoretical speculations.[2] It also results from the Sakata model,[3] especially from the hypothesis of the full symmetry between p, n and Λ.[4],[5] If such a resonance is really present, it will more or less affect the two-nucleon force. Actually several authors[6]-[9] have discussed on this problem. At present, of cource, no definite conclusion has been drawn because of our poor knowledge about the nature of such states. It may still be hoped, however, that the two-nucleon phenomena could give useful information about the above-mentioned states.

The aim of this note is to calculate extensively the nonstatic two-nucleon potentials due to the exchange of a boson, assuming that the resonance or bound state with definite spin and parity can be approximated by an actual particle with the same spin and parity. These potentials are expected to serve as a basis or guide in the more systematic analysis in the future investigations.

In this note we would like to consider spin zero boson and spin one boson. The interaction Lagrangian density is

$$L' = \begin{cases} g_S \bar{\psi}\psi\phi, & (S) \\ \dfrac{f_S}{\mu} \bar{\psi} i\gamma_\mu \psi \partial_\mu \phi, & (V) \end{cases}$$

for scalar boson,

$$L' = g_P \bar{\psi} i\gamma_5 \psi\phi + \frac{f_P}{\mu} \bar{\psi} i\gamma_5\gamma_\mu \psi \partial_\mu \phi,$$

for pseudoscalar boson,

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$$L'=g_V\bar{\psi}i\gamma_\mu\psi\phi_\mu+\frac{f_V}{2\mu}\bar{\psi}\sigma_{\mu\nu}\psi\phi_{\mu\nu},$$

for vector boson,

and

$$L'=\begin{cases} g_A\bar{\psi}i\gamma_5\gamma_\mu\psi\phi_\mu, & (PV) \\ \dfrac{f_A}{2\mu}\bar{\psi}i\gamma_5\sigma_{\mu\nu}\psi\phi_{\mu\nu}, & (PT) \end{cases}$$

for pseudovector boson,

where

$$\sigma_{\mu\nu}=(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)/2i$$

and

$$\phi_{\mu\nu}=\partial_\mu\phi_\nu-\partial_\nu\phi_\mu.$$

For iso-triplet boson, ϕ or ϕ_μ is replaced by $\tau_i\phi_i$ or $\tau_i\phi_{\mu i}$. Mixing of (S) and (V) for scalar field and mixing of (PV) and (PT) for pseudovector field are not allowed, assuming that the theory is invariant under charge conjugation or time reflection.[10)]

In the next section we give a list of the nonstatic one-boson-exchange potentials (OBEP) calculated from the above Lagrangian. These potentials are derived following the method adopted in reference 11). The two-boson-exchange potentials are left untouched throughout this paper, because they are expected to have little effects on the outside of the force range in which we are mainly interested.

§ 3 is devoted to the following discussion. First, some features of the present potentials are mentioned. Secondly, remarks about the attempts to explain the origin of $\boldsymbol{L}\cdot\boldsymbol{S}$ force by ρ meson are briefly given. Finally, we refer to a possible method of investigating the bosons which have been predicted by the full symmetry theory of the Sakata model.

§ 2. One-boson-exchange potentials

i) The potential is defined in the momentum space as the interaction kernel in the integral equation for the two-nucleon system. Then we can decompose the potential, where two incoming nucleons with momenta $\boldsymbol{p}$, $-\boldsymbol{p}$ are scattered into a state with $\boldsymbol{p}'$, $-\boldsymbol{p}'$, as

$$V=V_0+\frac{(i\boldsymbol{S}\cdot\boldsymbol{k}\times\boldsymbol{q})}{\mu^2}V_1+\frac{(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{k})(\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{k})}{\mu^2}V_2+\frac{(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{q})(\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{q})}{\mu^2}V_3$$
$$+\frac{(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{k}\times\boldsymbol{q})(\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{k}\times\boldsymbol{q})}{\mu^4}V_4+(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})V_5, \qquad (1)$$

where

$$V_i=V_i(\boldsymbol{k}^2,\ \boldsymbol{q}^2,\ (\boldsymbol{k}\times\boldsymbol{q})^2),$$

and

$$\boldsymbol{k}=\boldsymbol{p}'-\boldsymbol{p},\quad \boldsymbol{q}=(\boldsymbol{p}'+\boldsymbol{p})/2,\quad \boldsymbol{S}=(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})/2.$$

μ is the mass of a boson under consideration. The whole expression of V_i corresponding to the one-boson-exchange process is given in the Appendix.

One merit of working out the calculation in the momemtum space[12] may be to make the M^{-1} expansion of the potential unnecessary, M being the mass of a nucleon. This point would become more important when we consider heavy bosons ($\mu\lesssim M$). In spite of this advantage, however, we are still going to expand the potential with respect to $\boldsymbol{q}/M$ and $\boldsymbol{k}/M$, in order to transform it into coordinate space. (μ/M-expansion need not be made here.) The validity of this M^{-1} expansion has been discussed in reference 11). It has been concluded that the $\boldsymbol{q}/M$-expansion is valid for $q\lesssim 4\mu_\pi c$, μ_π being the mass of a pion.

This criterion may also remain valid for other boson cases. Now we tabulate OBEP up to the order of M^{-2}:

α) *scalar boson*

$$V_0=-\frac{g_S^2}{\omega^2}\left(1-\frac{\boldsymbol{q}^2}{M^2}+\frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{M^2\omega^2}\right)+O\left(\frac{1}{M^4}\right), \tag{2a}$$

$$V_1=g_S^2\frac{\mu^2}{2M^2}\frac{1}{\omega^2}+O\left(\frac{1}{M^4}\right), \tag{2b}$$

$$V_4=O\left(\frac{1}{M^4}\right). \tag{2c}$$

As is well known the vector interaction of the scalar boson has no effects on the nuclear force.

β) *pseudoscalar boson*

$$V_2=-\left(f_P+\frac{\mu}{2M}g_P\right)^2\frac{1}{\omega^2}\left\{1-\frac{\boldsymbol{k}^2}{4M^2}-\frac{\boldsymbol{q}^2}{M^2}+\frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{M^2\omega^2}+O\left(\frac{1}{M^4}\right)\right\}$$
$$+\frac{\boldsymbol{q}^2}{\mu^2}V_{kq}, \tag{3a}$$

$$V_3=\left(f_P+\frac{\mu}{2M}g_P\right)^2O\left(\frac{1}{M^4}\right)+\frac{\boldsymbol{k}^2}{\mu^2}V_{kq}, \tag{3b}$$

$$V_4=V_{kq}, \tag{3c}$$

$$V_5=-\frac{(\boldsymbol{k}\times\boldsymbol{q})^2}{\mu^4}V_{kq}, \tag{3d}$$

with

$$V_{kq}=\left(\left(\frac{\mu}{2M}g_P\right)^2-f_P^2\right)\frac{\mu^2}{2M^2}\frac{1}{\omega^2}+\frac{\mu}{2M}g_Pf_PO\left(\frac{1}{M^4}\right). \tag{3e}$$

γ) *vector boson*

$$V_0=\frac{g_V^2}{\omega^2}\left\{1-\frac{\boldsymbol{k}^2}{4M^2}+\frac{\boldsymbol{q}^2}{M^2}+\frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{M^2\omega^2}\right\}-f_Vg_V\frac{\boldsymbol{k}^2}{\mu M}\frac{1}{\omega^2}+f_V^2\frac{\boldsymbol{k}^4}{4\mu^2M^2}\frac{1}{\omega^2}$$
$$+g_V^2O\left(\frac{1}{M^4}\right)+f_Vg_VO\left(\frac{1}{M^3}\right)+f_V^2O\left(\frac{1}{M^4}\right),\tag{4a}$$

$$V_1=\frac{3}{2}g_V^2\frac{\mu^2}{M^2}\frac{1}{\omega^2}+4f_Vg_V\frac{\mu}{M}\frac{1}{\omega^2}-\frac{3}{2}f_V^2\frac{\boldsymbol{k}^2}{M^2}\frac{1}{\omega^2}$$
$$+g_V^2O\left(\frac{1}{M^4}\right)+f_Vg_VO\left(\frac{1}{M^3}\right)+f_V^2O\left(\frac{1}{M^4}\right),\tag{4b}$$

$$V_2=\frac{1}{4}g_V^2\frac{\mu^2}{M^2}\frac{1}{\omega^2}+f_Vg_V\frac{\mu}{M}\frac{1}{\omega^2}+f_V^2\frac{1}{\omega^2}\left\{1-\frac{\boldsymbol{k}^2}{4M^2}+\frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{M^2\omega^2}\right\}$$
$$+\frac{\boldsymbol{q}^2}{\mu^2}V_{kq}+g_V^2O\left(\frac{1}{M^4}\right)+f_Vg_VO\left(\frac{1}{M^3}\right)+f_V^2O\left(\frac{1}{M^4}\right),\tag{4c}$$

$$V_3=f_V^2\frac{\boldsymbol{k}^2}{M^2}\frac{1}{\omega^2}+\frac{\boldsymbol{k}^2}{\mu^2}V_{kq}+g_V^2O\left(\frac{1}{M^6}\right)+f_Vg_VO\left(\frac{1}{M^5}\right)+f_V^2O\left(\frac{1}{M^4}\right),\tag{4d}$$

$$V_4=-f_V^2\frac{\mu^2}{M^2}\frac{1}{\omega^2}+V_{kq}+g_V^2O\left(\frac{1}{M^4}\right)+f_Vg_VO\left(\frac{1}{M^3}\right)+f_V^2O\left(\frac{1}{M^4}\right),\tag{4e}$$

$$V_5=-\frac{1}{4}g_V^2\frac{\boldsymbol{k}^2}{M^2}\frac{1}{\omega^2}-f_Vg_V\frac{\boldsymbol{k}^2}{\mu M}\frac{1}{\omega^2}-f_V^2\frac{\boldsymbol{k}^2}{\mu^2\omega^2}\left\{1-\frac{\boldsymbol{k}^2}{4M^2}+\frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{M^2\omega^2}\right\}$$
$$-\frac{(\boldsymbol{k}\times\boldsymbol{q})^2}{\mu^4}V_{kq}+g_V^2O\left(\frac{1}{M^4}\right)+f_Vg_VO\left(\frac{1}{M^3}\right)+f_V^2O\left(\frac{1}{M^4}\right),\tag{4f}$$

with

$$V_{kq}=-f_V^2\frac{\mu^2}{2M^2}\frac{1}{\omega^2}+g_V^2O\left(\frac{1}{M^4}\right)+f_Vg_VO\left(\frac{1}{M^3}\right)+f_V^2O\left(\frac{1}{M^4}\right).\tag{4g}$$

δa) *pseudovector boson* (*PV-coupling*)

$$V_0=O\left(\frac{1}{M^4}\right),\tag{5a}$$

$$V_1=\frac{1}{2}g_A^2\frac{\mu^2}{M^2}\frac{1}{\omega^2}+O\left(\frac{1}{M^4}\right),\tag{5b}$$

$$V_2=-g_A^2\frac{1}{\omega^2}\left\{1-\frac{\boldsymbol{k}^2}{4M^2}-\frac{\boldsymbol{q}^2}{M^2}-\frac{\mu^2}{4M^2}+\frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{M^2\omega^2}\right\}+\frac{\boldsymbol{q}^2}{\mu^2}V_{kq}+O\left(\frac{1}{M^4}\right),\tag{5c}$$

$$V_3=-2g_A{}^2\frac{\mu^2}{M^2}\frac{1}{\omega^2}+\frac{\boldsymbol{k}^2}{\mu^2}V_{kq}+O\left(\frac{1}{M^4}\right), \tag{5d}$$

$$V_4=V_{kq}+O\left(\frac{1}{M^4}\right), \tag{5e}$$

$$V_5=-\frac{g_A{}^2}{\omega^2}\left\{1-\frac{\boldsymbol{q}^2}{M^2}+\frac{(\boldsymbol{k}\cdot\boldsymbol{q})}{M^2\omega^2}\right\}-\frac{(\boldsymbol{k}\times\boldsymbol{q})^2}{\mu^4}V_{kq}+O\left(\frac{1}{M^4}\right), \tag{5f}$$

with

$$V_{kq}=-g_A{}^2\frac{\mu^2}{2M^2}\frac{1}{\omega^2}+O\left(\frac{1}{M^4}\right). \tag{5g}$$

δb) *pseudovector boson* (*PT-coupling*)

$$V_2=\frac{f_A{}^2}{\omega^2}\left\{1+\frac{\boldsymbol{q}^2}{M^2}+\frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{M^2\omega^2}\right\}+\frac{\boldsymbol{q}^2}{\mu^2}V_{kq}+O\left(\frac{1}{M^4}\right), \tag{6a}$$

$$V_3=\frac{\boldsymbol{k}^2}{\mu^2}V_{kq}, \tag{6b}$$

$$V_4=V_{kq}, \tag{6c}$$

$$V_5=-\frac{(\boldsymbol{k}\times\boldsymbol{q})^2}{\mu^4}V_{kq}, \tag{6d}$$

with

$$V_{kq}=-\frac{3}{2}f_A{}^2\frac{\mu^2}{M^2}\frac{1}{\omega^2}+O\left(\frac{1}{M^4}\right). \tag{6e}$$

If we consider iso-triplet bosons, all V_i's given above should be replaced by $(\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)})V_i$. In the case that the ρ meson interacts with nucleon after decaying into pions, g and f in the above equations will be replaced by $gF_1(k^2)$ and $fF_2(k^2)$, $F_\alpha(k^2)$ being the nucleon form factors.

It would be in order here to remark the following. As is known, the form of the nonstatic part of V_2, V_3, V_4 and V_5 is not unique. Because of the identity

$$(\boldsymbol{k}\cdot\boldsymbol{q})\{(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{k})(\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{q})+(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{q})(\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{k})\}=\boldsymbol{q}^2(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{k})(\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{k})$$
$$+\boldsymbol{k}^2(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{q})(\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{q})+(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{k}\times\boldsymbol{q})(\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{k}\times\boldsymbol{q})-(\boldsymbol{k}\times\boldsymbol{q})^2(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}), \tag{7}$$

the V_{kq}-terms on the right-hand side of the expressions for V_2, V_3, V_4 and V_5 can be summarized as

$$V_{kq}(\boldsymbol{k}\cdot\boldsymbol{q})\{(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{k})(\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{q})+(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{q})(\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{k})\}/\mu^4. \tag{8}$$

This term together with the retardation term, $g^2(\boldsymbol{k}\cdot\boldsymbol{q})^2/M^2\omega^4$, can be transformed into the fourth order potential by a suitable unitary transformation. The static part and $\boldsymbol{L}\cdot\boldsymbol{S}$ potential, on the other hand, are free from this ambiguity up to the second order approximation.

In this note we use, for the sake of simplicity, the representation in which the term (8) together with the retardation term, $g^2(\boldsymbol{k}\cdot\boldsymbol{q})^2/M^2\omega^4$, are transformed into the fourth order. We therefore omit these terms from Eqs. (2) ~ (6) in what follows.

ii) *List of the potentials in the coordinate representation*

The potentials in the coordinate space are given by the Fourier transforms of $V(\boldsymbol{k}, \boldsymbol{q})$, Eq. (1):

$$V(r, \boldsymbol{q}) = \int \frac{d\boldsymbol{k}}{(2\pi)^3} V(\boldsymbol{k}, \boldsymbol{q})\, e^{i\boldsymbol{k}\boldsymbol{r}}. \tag{9}$$

$\boldsymbol{q}$ in $V(r, \boldsymbol{q})$ means a differential operator on the two-nucleon wave fuction, $-i(\overrightarrow{\nabla}+\overleftarrow{\nabla})/2$. Then $V(r, \boldsymbol{q})$ can be expressed as a sum of the static part, $\boldsymbol{L}\cdot\boldsymbol{S}$ force and other nonstatic forces, each of which is given below.

α) The static limit $W(r)$ of $V(r, \boldsymbol{q})$ is well known. Here we reproduce it for the sake of completeness:

$$W = -\mu\frac{g_S^2}{4\pi}\phi(x), \tag{10a}$$

for scalar boson,

$$W = \mu\frac{1}{4\pi}\left(f_P + \frac{\mu}{2M}g_P\right)^2\left\{\frac{1}{3}(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\,\phi(x) + S_{12}\chi(x)\right\}, \tag{10b}$$

for pseudoscalar boson,

$$W = \mu\frac{g_V^2}{4\pi}\phi(x) + \mu\frac{f_V^2}{4\pi}\left\{\frac{2}{3}(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\,\phi(x) - S_{12}\chi(x)\right\}, \tag{10c}$$

for vector boson,

$$W = \mu\frac{g_A^2}{4\pi}\left\{-\frac{2}{3}(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\,\phi(x) + S_{12}\chi(x)\right\}, \tag{10d}$$

for pseudovector boson (PV-coupling),

and

$$W = -\mu\frac{f_A^2}{4\pi}\left\{\frac{1}{3}(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\,\phi(x) + S_{12}\chi(x)\right\}, \tag{10e}$$

for pseudovector boson (PT-coupling),

where

$$\phi(x) = \frac{e^{-x}}{x}, \quad \chi(x) = \frac{1}{3}\left(1 + \frac{3}{x} + \frac{3}{x^2}\right)\frac{e^{-x}}{x}, \tag{11}$$

with $x = \mu r$, μ being the mass of a boson under consideration.

β) The $\boldsymbol{L}\cdot\boldsymbol{S}$ force, $V_{LS}(r)\,(\boldsymbol{L}\cdot\boldsymbol{S})$, is given by

$$V_{LS} = -\mu\frac{g_S^2}{4\pi}\frac{\mu^2}{2M^2}\left(1 + \frac{1}{x}\right)\frac{e^{-x}}{x^2}, \tag{12a}$$

for scalar boson,

$$V_{LS}=-\mu\left(\frac{g_V^2}{4\pi}\frac{3}{2}\frac{\mu^2}{M^2}+\frac{f_V g_V}{4\pi}4\frac{\mu}{M}+\frac{f_V^2}{4\pi}\frac{3}{2}\frac{\mu^2}{M^2}\right)\left(1+\frac{1}{x}\right)\frac{e^{-x}}{x^2}, \tag{12b}$$

for vector boson,

and

$$V_{LS}=-\mu\frac{g_A^2}{4\pi}\frac{\mu^2}{2M^2}\left(1+\frac{1}{x}\right)\frac{e^{-x}}{x^2}, \tag{12c}$$

for pseudovector boson.

γ) Other types of the nonstatic potentials.

γ–1) The quadratic spin-orbit force, $V_{LL}(r)\,(\boldsymbol{\sigma}^{(1)}\widetilde{L})\,(\boldsymbol{\sigma}^{(2)}\widetilde{L})$* is expressed as

$$V_{LL}=\mu\frac{f_V^2}{4\pi}\frac{\mu^2}{M^2}\left(1+\frac{3}{x}+\frac{3}{x^2}\right)\frac{e^{-x}}{x^3}, \tag{13}$$

for vector boson.

γ–2) $V_q(r)\,(\boldsymbol{\sigma}^{(1)}\cdot q)\,(\boldsymbol{\sigma}^{(2)}\cdot q)/\mu^2$ is given by

$$V_q=\mu\frac{f_V^2}{4\pi}\frac{\mu^2}{M^2}\left(-1+\frac{1}{x}+\frac{1}{x^2}\right)\frac{e^{-x}}{x}, \tag{14a}$$

for vector boson,

$$V_q=-\mu\frac{g_A^2}{4\pi}\frac{2\mu^2}{M^2}\phi(x), \tag{14b}$$

for pseudovector boson.

δ) Corrections to the static central and tensor forces:

$$W'=\mu\frac{g_S^2}{4\pi}\frac{q^2}{M^2}\phi(x), \tag{15a}$$

for scalar boson,

$$W'=\mu\frac{1}{4\pi}\left(f_P+\frac{\mu}{2M}g_P\right)^2\left(\frac{\mu^2}{4M^2}-\frac{q^2}{M^2}\right)\left\{\frac{1}{3}(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\,\phi(x)+S_{12}\chi(x)\right\}, \tag{15b}$$

for pseudoscalar boson,

$$W'=\mu\left\{\frac{g_V^2}{4\pi}\left(\frac{\mu^2}{4M^2}+\frac{q^2}{M^2}\right)+\frac{f_V g_V}{4\pi}\frac{\mu}{M}+\frac{f_V^2}{4\pi}\frac{\mu^2}{4M^2}\right\}\phi(x)$$
$$+\mu\left\{\frac{g_V^2}{4\pi}\frac{\mu^2}{4M^2}+\frac{f_V g_V}{4\pi}\frac{\mu}{M}+\frac{f_V^2}{4\pi}\frac{\mu^2}{4M^2}\right\}\left\{\frac{2}{3}(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\,\phi(x)-S_{12}\chi(x)\right\}$$

* $(\boldsymbol{\sigma}^{(1)}\cdot\widetilde{L})(\boldsymbol{\sigma}^{(2)}\cdot\widetilde{L})\equiv(\boldsymbol{\sigma}^{(1)}\cdot L)(\boldsymbol{\sigma}^{(2)}\cdot L)-1/4(x\cdot\overrightarrow{\nabla}-\overleftarrow{\nabla}\cdot x)(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})+1/4\{(\boldsymbol{\sigma}^{(1)}\cdot\overrightarrow{\nabla})(\boldsymbol{\sigma}^{(2)}\cdot x)-(\boldsymbol{\sigma}^{(1)}\cdot\overleftarrow{\nabla})\times(\boldsymbol{\sigma}^{(2)}\cdot x)\}$.[11]

$$-\mu\frac{f_V^2}{4\pi}\frac{\boldsymbol{q}^2}{M^2}(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\left(1+\frac{1}{x}\right)\frac{e^{-x}}{x^2}, \tag{15c}$$

for vector boson,

$$W'=\mu\frac{g_A^2}{4\pi}\frac{\boldsymbol{q}^2}{M^2}\left\{\frac{2}{3}(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\phi(x)-S_{12}\chi(x)\right\}, \tag{15d}$$

for pseudovector boson (PV-coupling),

and

$$W'=-\mu\frac{f_A^2}{4\pi}\frac{\boldsymbol{q}^2}{M^2}\left\{\frac{1}{3}(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\phi(x)+S_{12}\chi(x)\right\}, \tag{15e}$$

for pseudovector boson (PT-coupling).

The δ-function terms are neglected in the above expressions.

These formulas will be valid only in the outside of the core region. Near the core region the M^{-1}-expansion cannot be allowed, because in this region the contribution from high momentum part, $k\gtrsim Mc$, is not small. Moreover, in this region, the higher order effects should be taken into account.

§ 3. Discussions

i) *Characteristics of the nonstatic OBEP*

As shown in Eqs. (12a) ~ (12c) the $\boldsymbol{L}\cdot\boldsymbol{S}$ potential is obtained as an attractive force (multiplied by $(\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)})$ in the case of iso-triplet bosons) with the magnitude of order $(\mu/M)^2$ in the case of scalar, vector and pseudovector bosons. The f_Vg_V-term in the case of vector boson is only exceptional and has the magnitude of order μ/M. No $\boldsymbol{L}\cdot\boldsymbol{S}$ force is obtained from pseudoscalar boson or pseudovector boson with pseudotensor-coupling in the f^2-approximation.

As to the other nonstatic forces we only mention that their magnitude is, in general, of the same order as that of the $\boldsymbol{L}\cdot\boldsymbol{S}$ force.

ii) *Some remarks about the attempts to explain the origin of $\boldsymbol{L}\cdot\boldsymbol{S}$ force by ρ meson*

High-energy p-p scattering seems to indicate that the large $\boldsymbol{L}\cdot\boldsymbol{S}$ type potentials are probably present. Breit[6a)] proposes to regard this $\boldsymbol{L}\cdot\boldsymbol{S}$ force and the repulsive core as originating in the interaction through vector meson with mass$\approx 9\mu_\pi\sim 12\mu_\pi$. The coupling constant, $g_V^2/4\pi$, is estimated to be 300~1700, which seems to be unreasonably large. Furthermore, Sakurai[7)] claims that this presence of a large $\boldsymbol{L}\cdot\boldsymbol{S}$ force can be completely accounted for if there exists a strongly interacting neutral vector meson (or a sharp resonance in the $I=0$, $J=1$ state of the three-pion system) of mass $\approx 3\mu_\pi\sim 4\mu_\pi$, with the vector coupling to the nucleon of the order $g_V^2/4\pi\approx 4\sim 7$. However, his $\boldsymbol{L}\cdot\boldsymbol{S}$ force, if it be present, necessarily accompanies a very strong repulsive central force (about $(M/\mu)^2$-times larger than V_{LS}) as is easily seen from Eqs. (10 c) and (12 b). The tail of this repulsive force extends into one-pion-exchange region and appears therefore to be improbably large as it has been pointed out by Breit.[8] Furthermore, Breit[8] and Ohnuma[8] criticize Sakurai's conjecture from other point of view.

Gupta[6] considers the $\mathbb{L}\cdot\mathbb{S}$ force arising from the neutral scalar meson with $\mu\approx 2\mu_\pi$ which is coupled strongly to the nucleon, $g_S^2/4\pi\approx 14$. In this case, too, the strong (attractive) central force necessarily appears in the one-pion-exchange region, and it may be inconsistent with the available low energy data.

Fujii[9] calculates the potential arising from the two-pion P-wave resonance ($I=1$, $J=1$) at an energy of about $4.3\,\mu_\pi$. This resonance corresponds to a vector meson with $I=1$ (ρ-meson). The coupling constants, g_V and f_V in Eqs. (4a) ~ (4g), can be replaced by $hF_1(k^2)$ and $hF_2(k^2)$, where $F_\alpha(k^2)$ are the nucleon electromagnetic form factors and h is the ρ meson-pion coupling constant, $h^2/4\pi\sim 5$. Then it is estimated at $k^2=0$ that $g_V^2/4\pi\sim 0.4$, $f_V^2/4\pi\sim 5.5$. In his potential, the $f_V g_V$-term in Eq. (12b) gives a main contribution. The nature of the spacial extension of his potential is influenced by $F_\alpha(k^2)$.

iii) *A possible method of investigating the bosons predicted by the full symmetry theory of the Sakata model*

In the Sakata model, p, n and Λ are assumed to be fundamental particles. Furthermore, the full symmetry between these particles has been assumed by Ogawa.[4] Then iso-singlet π_0'-(and π_0''-) meson* is predicted to exist. This boson would contribute to the nuclear force in the outside of the core region. Several resonance states** with $I=0$, 1, which are obtained in the system of two baryons and two antibaryons, would also contribute to the nuclear force. In order to investigate whether these bosons really exist or not, the following approach*** may be possible:

α) The one- and two-pion-exchange potentials with the contribution from pion-pion force neglected, (OPEP+TPEP), are subtracted from the phenomenological potential, V_{phen}, which can reproduce all the available experimental data. We then investigate whether or not the remaining part may be identified with the potentials due to the exchange of bosons mentioned above,

$$V_{phen}-(\text{OPEP}+\text{TPEP})=\Sigma(\text{OBEP}),\tag{16}$$

or more simply

β) $V_{phen}-(\text{OPEP})=\Sigma(\text{OBEP})$,

according to whether $\Sigma(\text{OBEP})\lesssim\text{TPEP}$ or $\Sigma(\text{OBEP})\gg\text{TPEP}$ in the region under consideration. Finally, we mention that this approach should be applied to the phenomena on which the potential in the core region has little effect.

* The mass formula of elementary particles by Matumoto[5] and by Sawada and Yonezawa[5] gives the mass of π_0' (π_0'') to be 615 Mev (377 Mev).

** The mass formula[5] predicts the energy of these levels to be 753 Mev and 849 Mev for $I=1$ state, 1135 Mev for $I=0$ state.

*** The detailed analysis is made in reference 13).

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Appendix

Exact expression for the nonstatic one-boson-exchange potentials in the momentum representation

In this Appendix we give an exact expression of V_i's in Eq. (1).

(α) *scalar boson*

$$V_0=-g_S^2\left(1-\frac{\boldsymbol{p}'\cdot\boldsymbol{p}}{(E_{p'}+M)(E_p+M)}\right)^2\frac{(E_{p'}+M)(E_p+M)}{4E_{p'}E_p}\bar{\Delta}, \tag{A·1}$$

$$V_1=g_S^2\left(1-\frac{\boldsymbol{p}'\boldsymbol{p}}{(E_{p'}+M)(E_p+M)}\right)\frac{\mu^2}{2E_{p'}E_p}\bar{\Delta}, \tag{A·2}$$

$$V_4=g_S^2\frac{\mu^4}{4E_{p'}E_p(E_{p'}+M)(E_p+M)}\bar{\Delta}, \tag{A·3}$$

where $E_p=(\boldsymbol{p}^2+M^2)^{1/2}$ and $\bar{\Delta}=P\dfrac{1}{\boldsymbol{k}^2+\mu^2-(E_{p'}-E_p)^2}$. P stands for principal value.

(β) *pseudoscalar boson*

$$V_2=-\left(f_P+\frac{\mu}{2M}g_P\right)^2\left(\frac{M}{E_{p'}+M}+\frac{M}{E_p+M}\right)^2\frac{(E_{p'}+M)(E_p+M)}{4E_{p'}E_p}\bar{\Delta}$$
$$+\frac{\boldsymbol{q}^2}{\mu^2}V_{kq}+(E_{p'}-E_p)^2V_p, \tag{A·4}$$

$$V_3=-\left(f_P+\frac{\mu}{2M}g_P\right)^2\left(\frac{M}{E_{p'}+M}-\frac{M}{E_p+M}\right)^2\frac{(E_{p'}+M)(E_p+M)}{E_{p'}E_p}\bar{\Delta}$$
$$+\frac{\boldsymbol{k}^2}{\mu^2}V_{kq}+(E_{p'}-E_p)^2 4V_p, \tag{A·5}$$

$$V_4=V_{kq}, \tag{A·6}$$

$$V_5=-\frac{(\boldsymbol{k}\times\boldsymbol{q})^2}{\mu^4}V_{kq}, \tag{A·7}$$

with

$$V_{kq}=\left\{\left(\frac{\mu}{2M}g_P\right)^2M(E_{p'}+E_p+2M)-\frac{\mu}{2M}g_Pf_P(\boldsymbol{p}'^2+\boldsymbol{p}^2)\right.$$
$$\left.-f_P^2(E_{p'}^2+E_p^2+M(E_{p'}+E_p))\right\}\frac{\mu^2M}{E_{p'}E_p(E_{p'}+M)(E_p+M)(E_{p'}+E_p)}\bar{\Delta}, \tag{A·8}$$

$$V_p=f_P\left(f_P+\frac{\mu}{2M}g_P\right)\frac{M(E_{p'}+E_p+2M)}{4E_{p'}E_p(E_{p'}+M)(E_p+M)}\bar{\Delta}. \tag{A·9}$$

(γ) *vector boson*

$$V_0=\Big[g_V{}^2\Big\{\Big(1+\frac{\boldsymbol{p}'\cdot\boldsymbol{p}}{(E_{p'}+M)(E_p+M)}\Big)^2+\Big(\frac{\boldsymbol{p}'}{E_{p'}+M}+\frac{\boldsymbol{p}}{E_p+M}\Big)^2\Big\}$$

$$+4\frac{f_Vg_V}{\mu}\Big\{-(E_{p'}+E_p-2M)+M\Big(\frac{\boldsymbol{p}'}{E_{p'}+M}+\frac{\boldsymbol{p}}{E_p+M}\Big)^2$$

$$+\frac{(E_{p'}+E_p+2M)(\boldsymbol{p}'\cdot\boldsymbol{p})^2}{(E_{p'}+M)^2(E_p+M)^2}+\frac{(E_p-E_{p'})^2M}{(E_{p'}+M)(E_p+M)}\Big\}$$

$$+\frac{f_V{}^2}{\mu^2}\Big\{\Big(\frac{(\boldsymbol{p}'\cdot\boldsymbol{p}'-\boldsymbol{p})}{E_{p'}+M}-\frac{(\boldsymbol{p}\cdot\boldsymbol{p}'-\boldsymbol{p})}{E_p+M}\Big)^2+\frac{(\boldsymbol{p}'-\boldsymbol{p})^2(\boldsymbol{p}'\times\boldsymbol{p})^2}{(E_{p'}+M)^2(E_p+M)^2}$$

$$-(E_{p'}-E_p)^2\Big(\frac{\boldsymbol{p}'}{E_{p'}+M}-\frac{\boldsymbol{p}}{E_p+M}\Big)^2\Big\}\Big]\times\frac{(E_{p'}+M)(E_p+M)}{4E_{p'}E_p}\bar{\Delta}, \tag{A·10}$$

$$V_1=\Big[g_V{}^2\Big(3+\frac{\boldsymbol{p}'\cdot\boldsymbol{p}}{(E_{p'}+M)(E_p+M)}\Big)+4\frac{f_Vg_V}{\mu}\Big(2M+\frac{(E_{p'}+E_p+2M)(\boldsymbol{p}'\cdot\boldsymbol{p})}{(E_{p'}+M)(E_p+M)}\Big)$$

$$-\frac{f_V{}^2}{\mu^2}\Big\{(\boldsymbol{p}'-\boldsymbol{p})^2\Big(1+\frac{(E_{p'}+E_p+2M)^2+2\boldsymbol{p}'\cdot\boldsymbol{p}}{2(E_{p'}+M)(E_p+M)}\Big)$$

$$-(E_{p'}-E_p)^2\Big(2+\frac{(E_{p'}+E_p+2M)(E_{p'}+E_p)}{2(E_{p'}+M)(E_p+M)}\Big)\Big\}\Big]\frac{\mu^2}{2E_{p'}E_p}\bar{\Delta}, \tag{A·11}$$

$$V_2=\Big[g_V{}^2\frac{(E_{p'}+E_p+2M)^2}{4(E_{p'}+M)^2(E_p+M)^2}+\frac{f_Vg_V}{\mu}$$

$$\times\frac{M(E_{p'}+E_p+2M)^2-(E_{p'}-E_p)^2(E_{p'}+E_p+2M)/2}{(E_{p'}+M)^2(E_p+M)^2}$$

$$+\frac{f_V{}^2}{\mu^2}\Big\{\Big(1+\frac{\boldsymbol{p}'\cdot\boldsymbol{p}}{(E_{p'}+M)(E_p+M)}\Big)^2+\frac{(E_{p'}-E_p)^4}{4(E_{p'}+M)^2(E_p+M)^2}\Big\}\Big]$$

$$\times\frac{(E_{p'}+M)(E_p+M)\mu^2}{4E_{p'}E_p}\bar{\Delta}+\frac{\boldsymbol{q}^2}{\mu^2}V_{kq}, \tag{A·12}$$

$$V_3=\Big[g_V{}^2\frac{(E_{p'}-E_p)^2}{4(E_{p'}+M)(E_p+M)}-\frac{f_Vg_V}{\mu}\frac{(E_{p'}-E_p)^2(E_{p'}+E_p)}{2(E_{p'}+M)(E_p+M)}$$

$$+\frac{f_V{}^2}{\mu^2}\Big\{(\boldsymbol{p}'-\boldsymbol{p})^2\Big(1-\frac{(\boldsymbol{p}'-\boldsymbol{p})^2}{4(E_{p'}+M)(E_p+M)}\Big)$$

$$-(E_{p'}-E_p)^2\frac{M(E_{p'}+E_p+M)}{4(E_{p'}+M)(E_p+M)}\Big\}\Big]\frac{\mu^2}{E_{p'}E_p}\bar{\Delta}+\frac{\boldsymbol{k}^2}{\mu^2}V_{kq}, \tag{A·13}$$

$$V_4=-\Big[g_V{}^2+4\frac{f_Vg_V}{\mu}(E_{p'}+E_p+2M)+\frac{f_V{}^2}{\mu^2}(E_{p'}+E_p+2M)^2\Big]$$

$$\times\frac{\mu^4\bar{\Delta}}{4E_{p'}E_p(E_{p'}+M)(E_p+M)}+V_{kq}, \tag{A·14}$$

$$V_5=-\Big[g_V{}^2\Big(\frac{\boldsymbol{p}'}{E_{p'}+M}-\frac{\boldsymbol{p}}{E_p+M}\Big)^2+f_Vg_V\frac{4M}{\mu}\Big\{\Big(\frac{\boldsymbol{p}'}{E_{p'}+M}-\frac{\boldsymbol{p}}{E_p+M}\Big)^2$$
$$+\frac{(E_{p'}-E_p)^2}{(E_{p'}+M)(E_p+M)}\Big\}+\frac{f_V{}^2}{\mu^2}\Big\{(\boldsymbol{p}'-\boldsymbol{p})^2\Big(1+\frac{\boldsymbol{p}'\cdot\boldsymbol{p}}{(E_{p'}+M)(E_p+M)}\Big)^2$$
$$-(E_{p'}-E_p)^2\Big(\frac{\boldsymbol{p}'}{E_{p'}+M}+\frac{\boldsymbol{p}}{E_p+M}\Big)^2\Big\}\Big]\frac{(E_{p'}+M)(E_p+M)}{4E_{p'}E_p}\overline{\Delta}$$
$$-\frac{(\boldsymbol{k}\times\boldsymbol{q})^2}{\mu^4}V_{kq}, \qquad \text{(A·15)}$$

with

$$V_{kq}=\Big[-g_V{}^2(E_{p'}+E_p+2M)+2\frac{f_Vg_V}{\mu}(\boldsymbol{p}'^2+\boldsymbol{p}^2)-\frac{f_V{}^2}{\mu^2}$$
$$\times\Big\{2(E_{p'}+M)(E_p+M)(E_{p'}+E_p)\Big(1+\frac{\boldsymbol{p}'\cdot\boldsymbol{p}}{(E_{p'}+M)(E_p+M)}\Big)$$
$$-(E_{p'}-E_p)^2(E_{p'}+E_p+2M)\Big\}\Big]\frac{\mu^4\overline{\Delta}}{4E_{p'}E_p(E_{p'}+M)(E_p+M)(E_{p'}+E_p)}. \qquad \text{(A·16)}$$

(δa) *pseudovector boson (PV-coupling)*

$$V_0=g_A{}^2\frac{(\boldsymbol{p}'\times\boldsymbol{p})^2}{4E_{p'}E_p(E_{p'}+M)(E_p+M)}\overline{\Delta}, \qquad \text{(A·17)}$$

$$V_1=g_A{}^2\Big\{1-\frac{\boldsymbol{p}'\cdot\boldsymbol{p}}{(E_{p'}+M)(E_p+M)}\Big\}\frac{\mu^2}{2E_{p'}E_p}\overline{\Delta}, \qquad \text{(A·18)}$$

$$V_2=-g_A{}^2\Big\{\frac{M^2(E_{p'}+E_p+2M)^2}{\mu^2(E_{p'}+M)(E_p+M)}-1$$
$$+\frac{(\boldsymbol{p}'+\boldsymbol{p})^2+(E_{p'}-E_p)^2(1-4M(E_{p'}+E_p+2M)/\mu^2)}{4(E_{p'}+M)(E_p+M)}\Big\}\frac{\mu^2}{4E_{p'}E_p}\overline{\Delta}+\frac{\boldsymbol{q}^2}{\mu^2}V_{kq}, \qquad \text{(A·19)}$$

$$V_3=-g_A{}^2\Big\{1+\frac{(E_{p'}+E_p+2M)^2}{4(E_{p'}+M)(E_p+M)}+\frac{(\boldsymbol{p}'-\boldsymbol{p})^2}{(E_{p'}+M)(E_p+M)}$$
$$-\frac{M(E_{p'}+E_p+M)(E_{p'}-E_p)^2}{\mu^2(E_{p'}+M)(E_p+M)}\Big\}\frac{\mu^2}{E_{p'}E_p}\overline{\Delta}+\frac{\boldsymbol{k}^2}{\mu^2}V_{kq}, \qquad \text{(A·20)}$$

$$V_4=V_{kq}, \qquad \text{(A·21)}$$

$$V_5=-g_A{}^2\Big\{1-\frac{\boldsymbol{p}'\cdot\boldsymbol{p}}{(E_{p'}+M)(E_p+M)}\Big\}^2\frac{(E_{p'}+M)(E_p+M)}{4E_{p'}E_p}\overline{\Delta}-\frac{(\boldsymbol{k}\times\boldsymbol{q})^2}{\mu^4}V_{kq}, \qquad \text{(A·22)}$$

with

$$V_{kq}=-g_A{}^2\frac{\mu^2[2\{E_{p'}^2+E_p{}^2+M(E_{p'}+E_p)\}M-(E_{p'}+E_p+M)\mu^2]}{2E_{p'}E_p(E_{p'}+M)(E_p+M)(E_{p'}+E_p)}\overline{\Delta}. \qquad \text{(A·23)}$$

(δb) *pseudovector boson* (*PT-coupling*)

$$V_2=f_A^2\left\{\frac{1}{2}(E_{p'}+E_p)(E_{p'}+E_p+2M)-(E_{p'}-E_p)^2\right\}$$
$$\times\frac{(E_{p'}+E_p)(E_{p'}+E_p+2M)}{8E_{p'}E_p(E_{p'}+M)(E_p+M)}\bar{\Delta}+\frac{q^2}{\mu^2}V_{kq}, \qquad (\text{A}\cdot 24)$$

$$V_3=-f_A^2\left[\frac{(\boldsymbol{p}'-\boldsymbol{p})^2(E_{p'}+E_p+2M)^2}{4(E_{p'}+M)(E_p+M)}-(E_{p'}-E_p)^2\right.$$
$$\left.\times\left\{1-\frac{(\boldsymbol{p}'-\boldsymbol{p})^2-(E_{p'}+E_p)^2/4}{4(E_{p'}+M)(E_p+M)}\right\}\right]\frac{1}{E_{p'}E_p}\bar{\Delta}+\frac{k^2}{\mu^2}V_{kq}, \qquad (\text{A}\cdot 25)$$

$$V_4=-f_A^2\frac{\mu^2(E_{p'}+E_p+2M)^2}{4E_{p'}E_p(E_{p'}+M)(E_p+M)}\bar{\Delta}+V_{kq}, \qquad (\text{A}\cdot 26)$$

$$V_5=\frac{f_A^2}{\mu^2}\left\{(\boldsymbol{p}'-\boldsymbol{p})^2\left(\frac{\boldsymbol{p}'}{E_{p'}+M}+\frac{\boldsymbol{p}}{E_p+M}\right)^2-(E_{p'}-E_p)^2\right.$$
$$\left.\times\left(1+\frac{\boldsymbol{p}'\cdot\boldsymbol{p}}{(E_{p'}+M)(E_p+M)}\right)^2\right\}\frac{(E_{p'}+M)(E_p+M)}{4E_{p'}E_p}\bar{\Delta}-\frac{(\boldsymbol{k}\times\boldsymbol{q})^2}{\mu^4}V_{kq}, \qquad (\text{A}\cdot 27)$$

with

$$V_{kq}=-f_A^2\left[(E_{p'}+E_p)(E_{p'}+E_p+2M)\left\{1-\frac{(\boldsymbol{p}'-\boldsymbol{p})^2}{(E_{p'}+E_p)^2}\right\}-(E_{p'}-E_p)^2\right]$$
$$\times\frac{\mu^2\bar{\Delta}}{4E_{p'}E_p(E_{p'}+M)(E_p+M)}. \qquad (\text{A}\cdot 28)$$

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