Open Econ Rev
DOI 10.1007/s11079-012-9250-8

RESEARCH ARTICLE

Nontradable Goods and the Real Exchange Rate

Pau Rabanal · Vicente Tuesta

Abstract How important are nontradable goods and distribution costs to explain real exchange rate dynamics? We answer this question by estimating a general equilibrium model with intermediate and final tradable and nontradable goods. We find that the estimated model can match characteristics of the data that are relevant in international macroeconomics, such as real exchange rate persistence and volatility, and the correlation between the real exchange rate and other variables. The distinction between tradable and nontradable goods is key to understand real exchange rate fluctuations, but the introduction of distribution costs is not. Nontradable sector technology shocks explain about one third of real exchange rate volatility. We also show that, in order to explain the low correlation between the ratio of relative consumption and the real exchange rates across countries, demand shocks are necessary.

We wish to thank Pablo Guerrón and Carlos Montoro for helpful discussions, and an anonymous referee for very helpful comments. The opinions expressed in this paper are those of the authors and should not ne attributed to the IMF or IMF policy. Any errors and omissions are our own. Additional material is available in an online appendix at www.paurabanal.net/research.html.

P. Rabanal
International Monetary Fund, 700 19th St.,
Washington DC, NW 20431, USA
e-mail: prabanal@imf.org

V. Tuesta (✉)
CENTRUM Católica, Pontificia Universidad Católica del Perú,
Lima, Peru
e-mail: vicentetuesta@gmail.com

Published online: 08 September 2012

Keywords Real exchange rates · Nontradable goods · Nominal rigidities · Bayesian estimation

JEL Classification F31 · F32 · F41 · C11

1 Introduction

A main challenge in the empirical international macroeconomics literature is the so called "real exchange rate disconnect": models with optimizing agents have difficulty in accounting for the behavior of the real exchange rate. A related problem is that these models are not able to explain key correlations between the real exchange rate and other macroeconomic variables. In addition, these models cannot capture the comovement between key macroeconomic variables across countries. In this paper, we focus on the role played by nontradable goods in explaining the behavior of the real exchange rate.

There is empirical and quantitative evidence supporting the role of nontradable goods for to understand real exchange rate dynamics. On the empirical front, Betts and Kehoe (2006) provide evidence of the important role of nontradable goods in accounting for the variance of the real exchange rate of the most important U.S. trade partners. They suggest that the larger are trade flows between two countries, the lower is the importance for deviations from the law of one price (i.e. tradable goods prices) for real exchange rate behavior. Furthermore, in the U.S., consumption of nontradable goods represents roughly 40 % of GDP and final goods also contain substantial nontradable input components (between 20 and 30 % of GDP). On the quantitative front, Stockman and Tesar (1995) show that introducing nontradable goods in macroeconomic models is crucial to explain international business cycles. More recently, Dotsey and Duarte (2008), Benigno and Thoenissen (2008) and Corsetti et al. (2008) highlight the role of nontradable goods in explaining real exchange rate behavior, and in particular, its persistence and volatility, and its correlations with other international relative prices and real variables.

In this paper, we find that nontradable goods play an important role in explaining real exchange rate dynamics and several international macroeconomics facts. Our starting point is an estimated two-country (U.S.-euro area), two-sector (tradable-nontradable goods) dynamic stochastic general equilibrium (DSGE) model with nominal rigidities, of the class that is now becoming mainstream in academic circles and policy institutions for macroeconomic analysis.[1] Our analysis is empirical and model-based, and we estimate and

[1] Empirical papers that have estimated fully specified general equilibrium international macroeconomic models include Rabanal and Tuesta (2010), Lubik and Schorfheide (2005), Adolfson et al. (2007), Justiniano and Preston (2010) and De Walque et al. (2006). None of the above mentioned studies consider the role of nontradable goods.

compare two versions of this model: in the first one, the two sectors (tradable and nontradable) produce final consumption goods. In the second one, we introduce a nontradable intermediate input that is incorporated in the production of the final tradable good. In this case, we aim at understanding the role of distribution costs in explaining features of international macroeconomics, as suggested by Dotsey and Duarte (2008). Our methodology consists in estimating each model using a Bayesian approach and eleven macroeconomic series, including both the producer price index (PPI) for finished industrial goods and the consumer price index (CPI) for the United States and the Euro Area. PPI inflation (for finished industrial goods) allows us to capture inflation in the tradable goods sector of the economy, and unlike the "goods" component of the CPI, should not include distribution costs. Also, the PPI series is for finished industrial goods, and hence it should exclude a larger proportion of nontradables than other measures.[2]

Our results can be summarized as follows. First, the parameter estimates of the baseline model are quite similar to what has been estimated or calibrated in the vast existing literature. Therefore, our likelihood-based method does not rely on implausible parameter values for structural coefficients such as the degree of nominal rigidity, the degree of backward looking behavior in inflation or consumption, the monetary policy rules in both countries, and the size and persistence of economic shocks to explain the data. Second, we find that the version of the model without distribution costs performs better than the version with distribution costs. Since the model already includes several nominal and real rigidities, the addition of distribution costs does not help in explaining the data better: in fact, model fit is worse in some dimensions, including real exchange rate persistence. Therefore, our estimates support that distribution costs should not be treated differently than other services in the production of final goods. Third, our variance decomposition exercise (using the preferred model) shows that the nontradable sector in the model does indeed help to explain real exchange rate fluctuations: nontradable sector technology shocks explain as much as 30 % of the fluctuation of the bilateral real exchange rate, while tradable sector technology shocks and monetary policy shocks together explain less than 2 %. Interestingly, demand shocks explain a great amount of real exchange rate fluctuations (45 %).

Finally, our estimated model allows us to draw important implications for the behavior of the real exchange rate, the terms of trade and the trade balance. The relative price of domestic tradables decreases under a tradable sector technology shock, which is consistent with the traditional Balassa-Samuelson effect. With a productivity improvement in either the tradable and nontradable sectors, relative output, consumption and net exports increase. Finally, following a productivity shock in either tradable or nontradable sectors, domestic prices decrease and as a result the real exchange rate depreciates.

[2]It is impossible to obtain a pure measure of tradable goods inflation. Input-output table data for the U.S. reveals that services are an intermediate input for the production of industrial goods.

The model features the usual transmission mechanism with terms of trade deterioration following increases in productivity.[3] In addition, our estimated model generates, conditional on a tradable sector productivity shock, a real exchange rate depreciation and an increase in the ratio of relative consumptions. Therefore, our results suggest that demand shocks are the ones that help to explain the negative correlation between the real exchange rate and relative consumptions observed in the data. On this regard, our findings are in contrast to those of Corsetti et al. (2008), who find that the tradable sector productivity shocks are able to explain the apparent lack of risk sharing across countries (negative correlation between relative consumptions and the real exchange rate).

In all the models we estimate, we do not include capital accumulation. We argue that this is unlikely to change our results. In estimated DSGE models that include investment in the model and in the set of observable variables, an investment-specific technology shock is also included (see Rabanal and Tuesta 2010). This shock typically explains most of the volatility of investment but it also has counterfactual implications for consumption. In particular, this shock implies a negative comovement between consumption and investment. Hence, we suspect that if we had introduced investment and investment specific technology shocks, these shocks would not have contributed to explaining RER dynamics and the correlation between consumption and the RER. Moreover, some recent papers have shown that investment-specific technology shocks are not volatile enough in the data in order to solve certain macroeconomic puzzles (see Mandelman et al. 2011 and Schmitt-Grohe and Uribe 2011). Finally, for most countries it is not possible to obtain data on tradable and nontradable investment. Given these concerns and in order to keep the transmission mechanism simple, we have chosen not to include investment.

The rest of the paper is organized as follows. In Section 2, we present the model that we estimate. In Section 3 we discuss the data, and the prior and posterior distribution of the model's parameters. In Sections 4 and 5 we discuss the implications of the estimated model for real exchange rate behavior and the transmission mechanism in open economies. In Section 6 we discuss the estimation of a model that incorporates distribution services, while Section 7 concludes.

2 The Model

In this section, we present the model that we use for analyzing real exchange rate dynamics and the international transmission of shocks. The model is a fairly standard international macro two-country, two-sector (tradable and nontradable) economy, in the spirit of Stockman and Tesar (1995) and Dotsey

[3] On the contrary, Debaere and Lee (2004), Corsetti et al. (2006) find evidence in support of terms of trade improvement after favorable productivity shocks.

and Duarte (2008). The model includes sticky prices in both sectors, and it assumes that monetary policy is conducted with an interest rate rule of the Taylor type. Based on the arguments by Benigno and Thoenissen (2008) and on the empirical results of Rabanal and Tuesta (2010), we only explore the possibility that there are incomplete markets at the international level. Finally, we assume that the law of one price holds and intermediate firms set prices in their own currency.[4]

Since our contribution is to estimate this model using Bayesian methods and eleven observable variables, in this section we briefly present its main assumptions, parameters and functional forms, and refer the reader to the online appendix for a full-blown version of the model. In the last section of the paper, we study the effects of introducing a distribution sector in the model. We follow Dotsey and Duarte (2008) and assume that the production function of final tradable goods includes a portion of nontradable inputs. Finally, to keep the exposition of the model at its minimum, we only present the equations for households and firms in the home country. The expressions for the foreign country are analogous, and obtaining them is straightforward, with the appropriate change of notation.[5]

Households Representative households in the home country are assumed to maximize the following utility function:

$$U_t = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \psi_t \left[\log \left(C_t - b\,\bar{C}_{t-1} \right) - \frac{L_t^{1+\varphi}}{1+\varphi} \right] \right\}, \tag{1}$$

subject to the following budget constraint:

$$\frac{B_t^H}{P_t R_t} + \frac{S_t B_t^F}{P_t R_t^* \Phi\left(\frac{S_t \bar{B}_t^F}{P_t Y_t}\right)} \leq \frac{B_{t-1}^H}{P_t} + \frac{S_t B_{t-1}^F}{P_t} + \frac{W_t}{P_t} L_t - C_t + \Pi_t. \tag{2}$$

E_0 denotes the conditional expectation on information available at date $t = 0$, β is the intertemporal discount factor, with $0 < \beta < 1$. C_t denotes the level of consumption in period t, L_t denotes labor supply. The utility function displays external habit formation with respect to the habit stock, which is last period's aggregate consumption of the economy $\bar{C}_{t-1}$. $b \in [0, 1]$ denotes the importance of the habit stock. $\varphi > 0$ is inverse elasticity of labor supply with respect to the real wage. ψ_t is a preference shock that follows a zero-mean $AR(1)$ process in logs:

$$\log \psi_t = \rho_\psi \log \psi_{t-1} + \varepsilon_t^{\psi}. \tag{3}$$

[4]Dotsey and Duarte (2008) show that alternative assumptions regarding pricing decisions of firms, namely producer currency pricing (PCP) and local currency pricing (LCP), are not so different for the real exchange rate dynamics.

[5]The convention will be to use an asterisk to denote the counterpart in the foreign country of a variable in the home country (i.e. if aggregate consumption is C in the home country, it will be C^* in the foreign country and so on. The same applies to the model's parameters. When there is potential for confusion we explictly clarify so.

In the budget constraint, W_t is the nominal wage, P_t is the consumer price index, and Π_t are real profits for the home consumer. For modelling simplicity, we choose to assume incomplete markets at the international level with two risk-free one-period nominal bonds denominated in domestic and foreign currency, and a cost of bond holdings is introduced to achieve stationarity (see Benigno 2009). B_t^H is the holding of the risk free domestic nominal bond and B_t^F is the holding of the foreign risk-free nominal bond expressed in units of foreign country currency. S_t is the nominal exchange rate, expressed in units of home country currency per unit of foreign country currency. R_t and R_t^* are the nominal interest rates in the home and foreign countries. The function $\Phi\,(.)$ depends on the aggregate net foreign asset position of the home country, $\bar{B}_t^F$, in percent of home-country GDP, and is taken as given by the domestic household. $\Phi\,(.)$ is a convex function that introduces the cost of undertaking positions in the international asset market, and allows to have a well-defined steady-state. In addition, it is assumed that $\Phi\,(0) = 1$ and that $\Phi(.)$ is a decreasing function. Also, while we do not make it explicit in the budget constraint 2, we assume that there are complete markets at the domestic level, such that the consumption/savings decision is the same among households in a country, and the stochastic discount factor to value future profits is also the same among households in a country.

The aggregate consumption index (C_t) is a composite of final tradable (C_t^T) and final nontradable (C_t^N) consumption goods. We define the consumption index as

$$C_t \equiv \left[\gamma_c^{1/\varepsilon}\left(C_t^T\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma_c)^{1/\varepsilon}\left(C_t^N\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4}$$

where ε is elasticity of substitution between the final tradable (C_t^T) and final nontradable (C_t^N) goods, and γ_c is the share of final tradable goods in the consumption basket at home. In this context, the consumer price index that corresponds to the previous specification is given by

$$P_t \equiv \left[\gamma_c\left(P_t^T\right)^{1-\varepsilon} + (1-\gamma_c)\left(P_t^N\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{5}$$

where all prices are for goods sold in the home country, in home currency and at the consumer level, for both tradable and nontradable goods.

The demand functions for the final tradable and nontradable goods are given by:

$$C_t^T = \gamma_c\left(\frac{P_t^T}{P_t}\right)^{-\varepsilon} C_t, \text{ and } C_t^N = (1-\gamma_c)\left(\frac{P_t^N}{P_t}\right)^{-\varepsilon} C_t, \tag{6}$$

while consumption/savings decisions in home and foreign bonds are standard:

$$\lambda_t = \beta E_t\left\{R_t\frac{P_t}{P_{t+1}}\lambda_{t+1}\right\}, \tag{7}$$

$$\lambda_t = \Phi\left(\frac{S_t\bar{B}_t^F}{P_tY_t}\right)\beta E_t\left\{R_t^*\frac{Q_{t+1}}{Q_t}\lambda_{t+1}\right\}, \tag{8}$$

where $\lambda_t = \frac{\psi_t}{C_t - b\bar{C}_{t-1}}$ is the marginal utility of consumption, and $Q_t = \frac{S_t P_t^*}{P_t}$ is the real exchange rate. Labor supply is given by:

$$\lambda_t \frac{W_t}{P_t} = L_t^{\varphi}. \tag{9}$$

Firms There are three sectors in each country: (i) a final goods producer sector, that produces final tradable and nontradable goods for consumption by domestic households, (ii) an intermediate tradable goods sector, that produces goods that can be traded internationally to final tradable goods producers either in the home or in the foreign country, and (iii) an intermediate nontradable goods sector, that sells its production to final nontradable goods producers. We assume that the final goods producers operate under flexible prices and perfect competition, while intermediate goods producers operate under sticky prices à la Calvo with partial indexation, and monopolistic competition.

Final Goods Producers The final tradable good is consumed by domestic households. This good is produced by a continuum of firms, each producing the same variety, labelled by Y_t^T, using intermediate home $\left(X_t^h\right)$ and foreign $\left(X_t^f\right)$ goods with the following technology:

$$Y_t^T = \left\{\gamma_x^{1/\theta}\left(X_t^h\right)^{\frac{\theta-1}{\theta}} + (1-\gamma_x)^{1/\theta}\left(X_t^f\right)^{\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}}, \tag{10}$$

where θ is the elasticity of substitution between home-produced and foreign-produced imported intermediate goods. X_t^h and X_t^f denote the amount of home and foreign intermediate tradable inputs to produce the final tradable good at home, are also Dixit-Stiglitz aggregators of all types of home and foreign intermediate goods, with elasticity of substitution σ.:

$$X_t^h \equiv \left[\int_0^1 X_t^h(h)^{\frac{\sigma-1}{\sigma}} dh\right]^{\frac{\sigma}{\sigma-1}}, \text{ and } X_t^f \equiv \left[\int_0^1 X_t^f(f)^{\frac{\sigma-1}{\sigma}} df\right]^{\frac{\sigma}{\sigma-1}}.$$

Optimizing conditions by final tradable goods producers deliver the following demand functions:

$$X_t^h(h) = \gamma_x\left(\frac{P_t^h(h)}{P_t^h}\right)^{-\sigma}\left(\frac{P_t^h}{P_t^T}\right)^{-\theta} Y_t^T, \text{ and}$$

$$X_t^f(f) = (1-\gamma_x)\left(\frac{P_t^f(f)}{P_t^f}\right)^{-\sigma}\left(\frac{P_t^f}{P_t^T}\right)^{-\theta} Y_t^T, \tag{11}$$

where

$$P_t^h \equiv \left[\int_0^1 P_t^h(h)^{1-\sigma} dh\right]^{\frac{1}{1-\sigma}}, \ P_t^f \equiv \left[\int_0^1 P_t^f(f)^{1-\sigma} df\right]^{\frac{1}{1-\sigma}}.$$

Thus, the price index of tradable goods is given by:

$$P_t^T = \left[\gamma_x \left(P_t^h\right)^{1-\theta} + (1-\gamma_x)\left(P_t^f\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{12}$$

We assume that the law of one price holds for intermediate inputs, such that $P_t^h(h) = P_t^{h^*}(h)S_t$, and $P_t^f(f) = P_t^{f^*}(f)S_t$.

The production of the final nontradable good is given by:

$$Y_t^N \equiv \left[\int_0^1 X_t^N(n)^{\frac{\sigma-1}{\sigma}} dn\right]^{\frac{\sigma}{\sigma-1}}$$

where we assume the same elasticity $\sigma > 1$ than in the case of final tradable goods produced in the home country. The price level for nontradables is

$$P_t^N \equiv \left[\int_0^1 p_t^N(n)^{1-\sigma} dn\right]^{\frac{1}{1-\sigma}}$$

Intermediate Goods Producers The structure of intermediate goods producers in the two sectors is very similar. The main difference is that the intermediate nontradable sector produces differentiated goods that are aggregated by final nontradable good producing firms, and ultimately used for final consumption by domestic households only, while the intermediate tradable sector produces differentiated goods that can be sold to home and foreign final tradable good producers.

The production function in both sectors is linear in the labor input and has two technology shocks:

$$\begin{aligned} Y_t^N(n) &= A_t Z_t^N L_t^N(n), \text{ for } n \in [0,1], \text{ and} \\ Y_t^h(h) &= A_t Z_t^h L_t^h(h), \text{ for } h \in [0,1] \end{aligned} \tag{13}$$

where A_t is a labor augmenting aggregate world technology shock which has a unit root with drift:

$$\log A_t = g + \log A_{t-1} + \varepsilon_t^a \tag{14}$$

Hence, real variables in both countries grow at a rate g. Z_t^N and Z_t^h are country-specific, stationary productivity shocks to the nontradable and the tradable sector at time t, which evolve according to zero-mean, AR(1) process in logs

$$\log Z_t^N = \rho^{Z,N} \log Z_{t-1}^N + \varepsilon_t^{Z,N}, \text{ and } \log Z_t^h = \rho^{Z,h} \log Z_{t-1}^h + \varepsilon_t^{Z,h} \tag{15}$$

Firms in both sectors face a Calvo lottery with partial indexation when setting their prices. In the nontradable sector, firms receive a stochastic signal that allows them to reset prices optimally in each period, with probability $1-\alpha_N$. We assume that there is partial indexation with a coefficient φ_N to last period's sectorial inflation rate for those firms that do not get to reset prices optimally. As a result, firms maximize the following profits function:

$$Max_{P_t^N(n)} E_t \sum_{k=0}^{\infty} \alpha_N^k \Lambda_{t,t+k} \left\{ \left[\frac{P_t^N(n) \left(\frac{P_{t+k-1}^N}{P_{t-1}^N} \right)^{\varphi_N}}{P_{t+k}} - MC_{t+k}^N \right] Y_{t+k}^{N,d}(n) \right\} \tag{16}$$

subject to

$$Y_{t+k}^{N,d}(n) = \left[\left(\frac{P_t^N(n)}{P_{t+k}^N} \right) \left(\frac{P_{t+k-1}^N}{P_{t-1}^N} \right)^{\varphi_N} \right]^{-\sigma} Y_t^N \tag{17}$$

where $Y_t^{N,d}(n)$ is total individual demand for a given type of nontradable good n, and Y_t^N is aggregate demand for nontradable goods, as defined above, and $\Lambda_{t,t+k} = \beta^k \frac{\lambda_{t+k}}{\lambda_t}$ is the stochastic discount factor. MC_t^N corresponds to the real marginal cost in the nontradable sector. From cost minimization:

$$MC_t^N = \frac{W_t}{P_t Z_t^N A_t} \tag{18}$$

The evolution of the price level of nontradables is

$$P_t^N \equiv \left\{ \alpha_N \left[P_{t-1}^N \left(\Pi_{t-1}^N \right)^{\varphi_N} \right]^{1-\sigma} + (1-\alpha_N) \left(\hat{p}_t^N \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \tag{19}$$

where $\Pi_{t-1}^N = \frac{P_{t-1}^N}{P_{t-2}^N}$. Similar expressions hold for the intermediate tradable sector, where the relevant parameters for price setting are α_h and φ_h, and with the appropriate change of notation in Eqs. 16, 17, 18, and 19, and similar expressions hold for the foreign country.

Closing the Model The model includes a demand shock. One interpretation is that this shock is a government spending shock that is financed by lump-sum taxation. More generally, this demand shock is capturing movements in GDP and in consumption that cannot be explaining through interest-rate changes. Hence, we will be using both terms: demand shock and government spending shock, to refer to the shock in the market clearaing condition. We assume that the demand shock is allocated between tradable and nontradable goods in the same way that private consumption is. Hence the market clearing conditions for both types of final goods, consisting of private consumption and government spending, are:

$$Y_t^T = C_t^T + G_t^T, \tag{20}$$

and

$$Y_t^N = C_t^N + G_t^N, \tag{21}$$

where G_t^N, G_t^T follow AR(1) processes in logs. The bond market clearing conditions are:

$$B_t^H + B_t^{H^*} = 0, \tag{22}$$

and

$$B_t^F + B_t^{F^*} = 0. \tag{23}$$

For the nontradable intermediate goods, the market clearing condition is:

$$Y_t^N(n) = X_t^N(n), \text{ for all } n \in [0, 1],$$

while for the intermediate tradable goods sector it is:

$$Y_t^h(h) = X_t^h(h) + X_t^{h^*}(h), \text{ for all } h \in [0, 1].$$

Monetary policy is conducted with a Taylor rule that targets CPI inflation and output growth deviation from steady-state values:

$$R_t = \bar{R}^{(1-\rho_r)} R_{t-1}^{\rho_r} \left(\frac{P_t / P_{t-1}}{\Pi} \right)^{(1-\rho_r)\gamma_\pi} \left(\frac{Y_t / Y_{t-1}}{1+g} \right)^{(1-\rho_r)\gamma_y} \exp(\varepsilon_t^r). \tag{24}$$

3 Data, Priors and Posterior Distributions

We use Bayesian methods to estimate the parameters of the model of Section 2. Bayesian estimation of DSGE models has now become very popular, so we leave the technical details and a discussion of its benefits aside.[6] We use the following data series for each country: output (real GDP) growth per capita, consumption growth per capita, CPI inflation, interest rates on 3-month T-Bills, and PPI inflation for finished industrial goods. CPI inflation is used as a measure of overall inflation, while PPI inflation tries to measure the inflation in the tradable sector of the economy. Several authors (Engel 1999; Betts and Kehoe 2006) have emphasized that using the "goods" component of the CPI might not be a good proxy for tradable goods, because it contains distribution and retail services that are nontradable. The home country is the euro area, and the foreign country is the U.S. The last series that we use in the estimation procedure is the bilateral real exchange rate between the euro and the US dollar. To construct this series, we multiply the nominal exchange rate (in euros per U.S. dollar) by the U.S. CPI, and divide it by the euro area CPI. An increase of the real exchange rate is an euro depreciation. These eleven variables are our set of observable variables in the likelihood function. The sample period goes from 1985:02 to 2004:04. We are constrained by the availability of the PPI series for finished industrial goods in the euro area, since all other variables are available from earlier periods.

[6] See An and Schorfheide (2007), Lubik and Schorfheide (2005) and Fernandez-Villaverde and Rubio-Ramírez (2004) for detailed explanations on how to implement a Bayesian approach to estimation of fully-specified dynamic stochastic general equilibrium models.

Before we proceed to describe the prior and posterior distribution of the model's parameters, we discuss what parameters we calibrate first. We follow Dotsey and Duarte (2008) closely, and calibrate the two economies with the same parameters. We set the share of tradable goods in the CPI to $\gamma_c = 0.44$. We set the fraction of intermediate tradable inputs in the production of final tradable goods to $\gamma_x = 0.6$. Since we are not using labor market data we calibrate the value of $\varphi = 1$, which is in line with parameter estimates obtained by Rabanal and Rubio-Ramírez (2005, 2008). We set the steady-state growth rate of the economy, g, equal to 0.5 %, which implies that the world growth rate of per capita variables is about 2 % per year. In order to match a real interest rate in the steady state of about 4 % per year, we set the discount factor to $\beta = 0.99$. For reasonable parameterizations of these two variables the parameter estimates do not change significantly. Finally, the parameter χ, that measures the elasticity of the risk premium with respect to the net foreign asset position, is set equal to 0.007 based on Rabanal and Tuesta (2010).

With the previous parameters fixed in advance, Table 1 presents the prior and posterior distributions for the model's remaining parameters which are fairly standard (see Smets and Wouters 2003; Lubik and Schorfheide 2005 and Rabanal and Tuesta 2010). In order to make the table more readable, we also include a brief description of each parameter.

Since we are mostly interested in understanding the implications of the model for real exchange rate dynamics and the international transmission of shocks, we briefly comment on the parameter estimates. Overall, they are quite similar to what has been obtained in the literature that estimates open economy dynamic macroeconomic models with Bayesian methods.[7] The estimates for the degree of habit formation are quite similar in both countries, of 0.57 in the United States and of 0.58 in the euro area, respectively. The elasticity of substitution between home and foreign tradable inputs, θ, is estimated at 0.85, a value much smaller than the prior mean of 1.5, which was chosen according to Chari et al. (2002). However, this value is higher than that obtained by Rabanal and Tuesta (2010) and Lubik and Schorfheide (2005) in a model with tradable goods only. As it will become clearer later, the higher estimated value for this elasticity stems from endogenous volatility that nontradable goods adds to the model, hence making less necessary a small value of θ to account for real exchange volatility. On the other hand, the elasticity of substitution between tradable and nontradable final consumption goods, ε, is estimated to be quite low, with a posterior mean of 0.13, which is much lower than the prior mean, of 1, and the value typically used calibrated exercises in the literature of 0.44, following Stockman and Tesar (1995). The estimated Phillips Curves suggest that prices in the U.S. are reset optimally about every 2 quarters in both sectors, with a low degree of backward looking indexation ($\varphi_{N^*}, \varphi_{f^*}$), between 0.06 in the nontradable sector and 0.21 in the tradable sector. The Phillips Curves in the euro area are more heterogeneous:

[7] See Lubik and Schorfheide (2005) and Rabanal and Tuesta (2010).

Table 1 Prior and posterior distributions

		Prior			Posterior		
		Distribution	Mean	St.Dev	Mean	Lower	Upper
Habit formation							
b	EMU	*Beta*	0.70	0.05	0.57	0.51	0.64
b^*	USA	*Beta*	0.70	0.05	0.58	0.52	0.64
Elasticities of substitution							
θ	Home and foreign tradable int. goods	*Normal*	1.50	0.25	0.85	0.80	0.93
ε	Tradable and nontradable final goods	*Gamma*	1.00	0.25	0.13	0.09	0.18
Calvo parameters							
α_h	Tradable int. goods EMU	*Beta*	0.50	0.20	0.73	0.66	0.80
α_{f^*}	Tradable int. goods USA	*Beta*	0.50	0.20	0.48	0.38	0.59
α_N	Nontradable int. goods EMU	*Beta*	0.50	0.20	0.10	0.03	0.17
α_{N^*}	Nontradable int. goods USA	*Beta*	0.50	0.20	0.40	0.34	0.46
Indexation parameters							
φ_h	Tradable int. goods EMU	*Beta*	0.50	0.20	0.30	0.07	0.51
φ_{f^*}	Tradable int. goods USA	*Beta*	0.50	0.20	0.21	0.01	0.41
φ_N	Nontradable int. goods EMU	*Beta*	0.50	0.20	0.39	0.08	0.69
φ_{N^*}	Nontradable int. goods USA	*Beta*	0.50	0.20	0.06	0.01	0.10
Taylor rule coefficients							
ρ_r	Interest rate smoothing EMU	*Uniform*	0.50	0.29	0.76	0.71	0.82
ρ_{r^*}	Interest rate smoothing USA	*Uniform*	0.50	0.29	0.88	0.85	0.90
γ_π	Response to inflation, EMU	*Normal*	1.50	0.25	2.72	2.46	2.98
γ_{π^*}	Response to inflation, USA	*Normal*	1.50	0.25	2.05	1.73	2.34
γ_y	Response to growth, EMU	*Normal*	1.00	0.25	0.56	0.40	0.71
γ_{y^*}	Response to growth, USA	*Normal*	1.00	0.25	0.91	0.68	1.15
Prior and posterior distributions of shock							
AR coefficients							
	Preference						
ρ_ψ	EMU	*Beta*	0.75	0.10	0.87	0.82	0.92
ρ_{ψ^*}	USA	*Beta*	0.75	0.10	0.88	0.83	0.93
	Technology						
$\rho^{Z,h}$	Tradable int. sector EMU	*Beta*	0.75	0.10	0.94	0.89	0.98
ρ^{Z,f^*}	Tradable int. sector USA	*Beta*	0.75	0.10	0.93	0.88	0.98
$\rho^{Z,N}$	Nontradable int. sector EMU	*Beta*	0.75	0.10	0.97	0.95	0.99
ρ^{Z,N^*}	Nontradable int. sector USA	*Beta*	0.75	0.10	0.93	0.90	0.97
	Government spending						
$\rho^{G,T}$	Tradable sector EMU	*Beta*	0.75	0.10	0.84	0.75	0.94
ρ^{G,T^*}	Tradable sector USA	*Beta*	0.75	0.10	0.73	0.56	0.89
$\rho^{G,N}$	Nontradable sector EMU	*Beta*	0.75	0.10	0.85	0.76	0.94
ρ^{G,N^*}	Nontradable sector USA	*Beta*	0.75	0.10	0.93	0.89	0.97
Standard deviations of shocks (in percent)							
	Preference						
ε_t^ψ	EMU	*Gamma*	1.00	0.50	1.89	1.43	2.36
$\varepsilon_t^{\psi^*}$	USA	*Gamma*	1.00	0.50	1.91	1.50	2.30
	Technology						
$\varepsilon^{Z,h}$	Tradable int. sector EMU	*Gamma*	0.70	0.30	1.38	1.06	1.71
ε^{Z,f^*}	Tradable int. sector USA	*Gamma*	0.70	0.30	1.72	1.37	2.05
$\varepsilon^{Z,N}$	Nontradable int. sector EMU	*Gamma*	0.70	0.30	1.81	1.54	2.08
ε^{Z,N^*}	Nontradable int. sector USA	*Gamma*	0.70	0.30	1.02	0.82	1.22
ε^a	Permanent technology shock.	*Gamma*	0.70	0.30	0.36	0.19	0.53

Table 1 (continued)

		Prior			Posterior		
		Distribution	Mean	St.Dev	Mean	Lower	Upper
	Government spending						
$\varepsilon^{G,T}$	Tradable sector EMU	*Gamma*	1.00	0.50	1.00	0.45	1.65
$\varepsilon^{G,T*}$	Tradable sector USA	*Gamma*	1.00	0.50	0.69	0.15	1.22
$\varepsilon^{G,N}$	Nontradable sector EMU	*Gamma*	1.00	0.50	3.05	2.68	3.46
$\varepsilon^{G,N*}$	Nontradable sector USA	*Gamma*	1.00	0.50	4.07	3.57	4.60
	Monetary policy						
ε^{r}	EMU	*Gamma*	0.40	0.20	0.16	0.12	0.19
ε^{r^*}	USA	*Gamma*	0.40	0.20	0.11	0.09	0.13

the estimated probability of not resetting prices is 0.73 in the tradable sector, while we obtain a surprisingly low coefficient for the nontradable sector, where the posterior mean is 0.1, much lower than the prior mean of 0.5. Backward looking behavior is higher than in the case of the U.S., with coefficients of 0.3 in the tradable sector and 0.4 in the nontradable sector. Finally, the coefficients of the Taylor rule are quite similar to previous estimates in the literature for the sample period that we use, starting in 1985, with coefficients on the response of nominal interest rates to inflation of 2 in the United States (Clarida et al. 2000) and even higher in the euro area. Regarding the exogenous processes, all shocks are estimated with high, but reasonable, persistence. The technology shock in the intermediate nontradable sector has the highest persistence, with a posterior mean of 0.97, while the persistence of all the other shocks ranges between that value and 0.73 for the demand shock in the tradable sector in the U.S. The high persistence in preference and technology shocks in the nontradable sector in U.S. might explain why the backward behavior in price setting is unimportant. Similar results have been found by Ireland (2006) for an estimated closed economy using U.S. data.

4 Implications for Real Exchange Rate Dynamics: Second Moments and Variance Decomposition

After taking a linear approximation of the equilibrium conditions around the steady state, the equations determining the real exchange rate are as follows. First, combining the consumption Euler equations for both households, we obtain that:

$$E_t\left(q_{t+1}-q_t\right)=\left[\frac{(1+g)\,E_t\Delta c_{t+1}-b\,\Delta c_t}{(1+g-b)}\right]-\left[\frac{(1+g)\,E_t\Delta c^*_{t+1}-b^*\Delta c^*_t}{(1+g-b^*)}\right]$$
$$+\left(1-\rho_\psi\right)\widehat{\psi}_t-\left(1-\rho^*_\psi\right)\widehat{\psi}^*_t+\chi b_t \tag{25}$$

where q_t is the real exchange rate, c_t and c^*_t are consumption in the euro area and in the United States, $b_t=\left(\frac{S_t\bar{B}^F_t}{P_t}\right)Y^{-1}$ is the net foreign asset position as percent of GDP, where $\chi\equiv-\Phi'(0)\,Y$, and $\widehat{\psi}_t$ and $\widehat{\psi}^*_t$ are the preference shocks (all expressed in log deviations from steady-state values).[8] Therefore, in principle, if consumption growth in both areas is not related to the real exchange rate, the preference shocks should allow us to explain the data in case of misspecification. In addition, by taking the definition of the real exchange rate as the ratio of price levels expressed in common currency, and by using the definition of the CPIs in both countries and the definitions of the price level of tradable goods, we obtain the following expression:

$$q_t = (2\gamma_x - 1)t_t + (1 - \gamma_c)[(t_t^T - t_t^N) - (t_t^{T^*} - t_t^{N^*})] \tag{26}$$

where t_t is the terms of trade, defined as the price of imports minus the price of exports, $t_t^i = p_t^i - p_t$, $i = T, N$ is the relative price between tradables and nontradables in the euro area CPI, and $t_t^{i^*} = p_t^{i^*} - p_t^*$, $i = T, N$ is the relative price between tradables and nontradables in the U.S. CPI. Therefore, the shocks that drive the terms of trade, or that move prices of tradable and nontradable goods in both countries in different directions, are also likely to affect the behavior of the real exchange rate. Indeed, the presence of nontradable goods helps in breaking the strong correlation between the real exchange rate and the terms of trade implied by a model without nontradable goods: In that particular case, $\gamma_c = 1$, and $q_t = (2\gamma_x - 1)t_t$. Furthermore, as pointed out by Dotsey and Duarte (2008), the presence of nontradable goods lowers the correlation of real variables with international relative prices, helping the model to better explain the data. In the next sub-section we analyze some second moments and evaluate how well the estimated model works in the previous mentioned dimensions.

In the Bayesian approach, assessments of the goodness of fit and model comparisons are performed using the marginal likelihood of the data, which updates the researcher's prior beliefs on which model is closer to the true one after observing the data. Fernandez-Villaverde and Rubio-Ramírez (2004) show that, in the Bayesian framework, model comparison is consistent when models are misspecified, which is typically the case. However, the marginal likelihood, which averages all possible likelihood values implied by the model across the parameter space, using the prior as a weight, is a summary statistic of overall goodness of fit. In this section, we focus instead on a subset of second moments that are key in the international macroeconomics literature. In Table 2 we present some selected posterior second moments of the raw data, while in Table 3 we report selected posterior second moments of HP-filtered real variables.

The model overpredicts the volatility of consumption and output growth, and of CPI and PPI inflation in both countries, while it underpredicts the volatility of nominal interest rates and the real exchange rate (Table 2). Yet, using a longer period that includes the 1970s makes the model fit the inflation

[8] The evolution of net foreign assets over GDP is: $\beta \widetilde{b}_t = \frac{1}{1+g}\widetilde{b}_{t-1} + \frac{X^f}{Y}\left(\widetilde{x}_t^{h^*} - \widetilde{x}_t^f - t_t\right)$ where $\frac{X^f}{Y}$ is the imports-GDP ratio, $\widetilde{x}_t^{h^*}$ is exports of intermediate tradable goods, $\widetilde{x}_t^f$ is imports, and t_t is the terms of trade. Appendix B details the full set of loglinearized conditions of the model.

Table 2 Second moments in the model and in the data

	Euro area					United States					
	Y	C	R	CPI	PPI	Y	C	R	CPI	PPI	Q
Standard Deviation (in %)											
Data	0.51	0.51	0.77	0.27	0.33	0.50	0.48	0.49	0.36	0.79	4.64
Model	0.74	0.87	0.57	0.55	0.60	0.59	0.75	0.49	0.84	1.23	3.34
Variance decomposition											
Preferences	9.6	11.0	68.3	25.3	19.8	4.1	13.2	65.9	43.5	14.4	23.6
Tech. tradable	7.4	8.4	4.3	4.1	52.9	11.9	14.0	5.0	12.1	63.8	0.9
Tech. nontradable	49.5	56.5	19.2	33.4	18.3	20.4	55.9	22.7	23.7	13.1	31.8
Demand shock	26.4	19.5	7.1	16.8	5.5	51.8	10.4	5.8	4.7	2.9	43.5
Monetary policy	0.2	0.2	0.3	16.2	3.1	0.8	1.2	0.5	14.3	5.1	0.2
Unit root Shock	7.0	4.4	0.9	4.3	0.3	11.0	5.4	0.1	1.6	0.8	0.1

Note: Y is output, C is consumption, R is nominal interest rate. Q is the real exchange rate. Moments for R are based on the level of this variable, in all other cases they are based on their quarterly growth rate

data better as documented by Rabanal and Tuesta (2010). To explain which shocks drive the behavior of macroeconomic variables, we perform a variance decomposition exercise and then add up shocks across countries.[9] Most important for the purpose of this paper, we examine what is the role of each shock in explaining real exchange rate fluctuations. In this case, demand shocks (mostly in the tradable sector, not shown) explain 43.5 % of the variance of the real exchange rate, while technology shocks in the nontradable sector explain 31.8 %, and preference shocks explain 23.6 %. The other shocks (monetary policy, innovations to the permanent technology shock, and the tradable sector technology shock) explain about the remaining 1 %. These results confirm the findings of Rabanal and Tuesta (2010) with a model with tradable goods only. Of course, in that case we were not able to tell what sector the shocks belonged to, but we assigned an important contribution (about 40 % each) to technology and demand factors. In the present estimated model, nontradable technology shocks, fiscal shocks and preference shocks are able to explain a large fraction of the volatility of most variables. Note also that the tradable sector technology shocks only explain an important fraction of tradable (PPI) inflation in both countries. Therefore, shocks arising in the nontradable sector are an important source of real exchange rate fluctuationsFinally, the monetary policy shock does explain a significant fraction of CPI inflation in both countries, about 15 %.

As suggested by Table 3, the model does a good job in explaining the international dimension of the data, in particular to the relationship of output across countries and the correlation between relative output and the real exchange rate. The model is also able to explain the so-called *consumption-real exchange*

[9] That is, the contribution of the "Preference" shock adds up the contribution of the euro area and the U.S. preference shock. The only exception is the demand shock for which we have aggregated across countries and sectors.

Table 3 Second moments in the model and in the data

Correlation	Y,Y*	C,C*	C-C*,Q	Y-Y*,Q	Q,Q_{-1}
Data	0.30	0.18	0.01	0.15	0.78
Model	0.36	−0.28	0.05	0.20	0.78
Preferences	0.85	−0.70	−0.97	−0.96	0.77
Tech. tradable	0.96	0.87	0.84	0.89	0.62
Tech. nontradable	0.12	−0.49	0.89	0.91	0.77
Demand shocks	0.29	−0.55	−0.90	−0.38	0.79
Monetary policy	0.95	0.91	0.81	0.85	0.23
Unit root shock	1.00	1.00	0.78	−0.61	0.18

Note: Y is output, C is consumption, Q is the real exchange rate. All moments are computed by simulating the model 1,000 times with 85 periods at the posterior mean and applying the HP filter

rate anomaly. In the sample period that we use, the correlation between the ratio of relative consumptions and the real exchange rate is basically zero.[10] The fact that the model can match a basically zero correlation should not mask that the transmission mechanisms underlying this result are very different. While technology shocks in both sectors, monetary policy and unit root shocks deliver a high and positive correlation between these two variables, preference and demand shocks deliver a highly negative correlation. Therefore, any model that tries to be successful in explaining this correlation must have a combination of the two, even when the model includes nontradable goods. The same result applies when studying the correlation between relative outputs and the real exchange rate. Finally, we would like to remark that the model is able to fit real exchange rate persistence, with a first autocorrelation in the HP-filtered real exchange rate in the model and in the data of 0.78. Also, the three shocks that explain most of real exchange rate volatility are able to explain its persistence.[11]

5 Implications for the Transmission Mechanism

Having shown what are the three shocks that explain the behavior of the real exchange rate in the previous section, we now turn to discuss the impulse responses to a nontradable technology shock, a tradable sector demand shock, and a preference shock in the euro area. In Fig. 1 we depict the effects of a

[10] Adding the seventies and mid-eighties sample, as in Rabanal and Tuesta (2010), delivers a negative correlation of −0.17, that a model with incomplete markets and tradable goods can match.

[11] We use HP-filtered data to be able to compare our results with the international real business cycle literature, including Corsetti et al. (2008). The empirical literature interpreted real exchange rate persistence as the slow rate of mean reversion of the real exchange rate. Early examples of applications include Rogoff (1996) and the references therein. A typical result is the strong evidence of slow mean reversion, found by estimating first order autoregressive models for the level of the exchange rate instead of using HP filtered data. For a recent application, see Steinsson (2008).

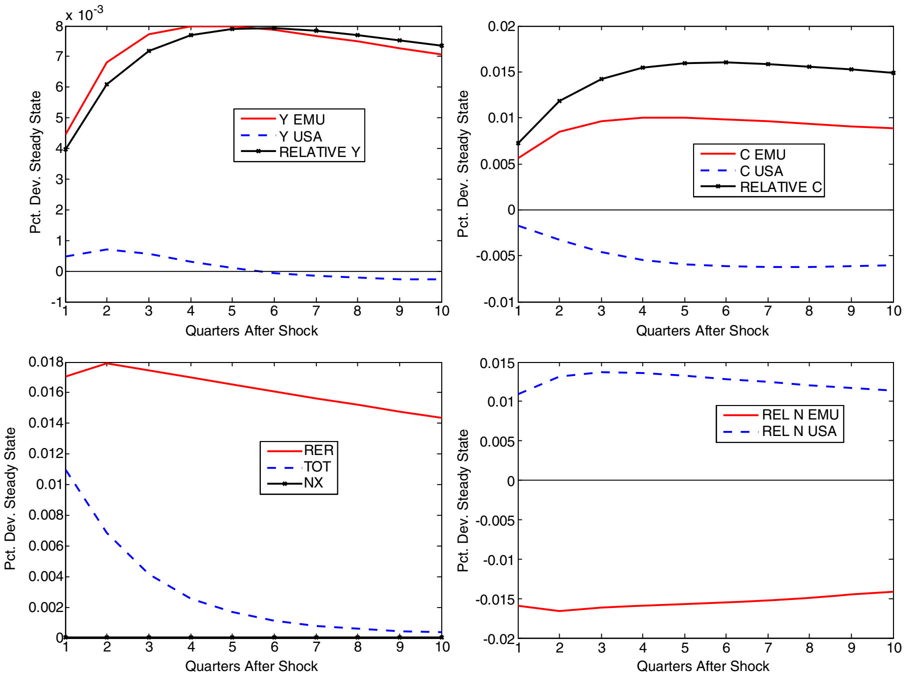

Fig. 1 Impulse response to a nontradable technology shock in the euro area

positive (one standard deviation) nontradable sector technology shock. As a result, consumption and output increase in the euro area. The real exchange rate and the terms of trade depreciate following the shock, and the relative price of nontradables ($REL^N = \frac{P^N}{P^T}$) falls in the euro area where as it increases in the USA. From Eq. 26, the RER dynamics can be decomposed in the terms-of-trade effect, $(2\gamma_x - 1)\,t_t$, and the movements of relative prices of tradable to nontradable goods in both countries. We can further rearrange Eq. 26 to get:

$$q_t = (2\gamma_x - 1)t_t + (1 - \gamma_c)(rel_t^{N^*} - rel_t^N)$$

where $rel_t^N = p_t^N - p_t^T$ and $rel_t^{N^*} = p_t^{N^*} - p_t^{T^*}$. In this case, both relative-price effects move the real exchange rate in the same direction. The terms of trade depreciate because of the associated nominal exchange rate depreciation of the euro. This causes consumption to fall in the U.S., and also the relative price of tradable goods to increase. Finally, there is a small improvement of the trade balance but of several orders of magnitude smaller than all other variables. With an estimated θ close to one, the trade balance barely moves in all the exercises that we show, because real quantities offset the movements in real prices. This shock implies a positive correlation between both the real exchange rate and the terms of trade with both relative output and consumption. The impulse response to a tradable sector technology shock (not shown) displays similar behavior of the main variables, except for the

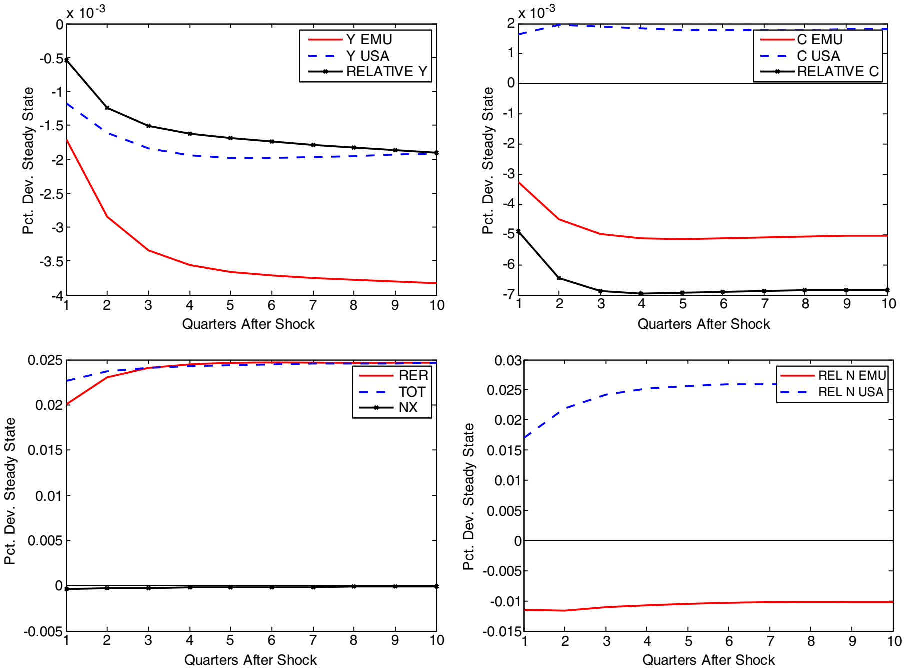

Fig. 2 Impulse response to a tradable demand shock in the euro area

relative prices of nontradable to tradable goods.[12] Our estimated impulse responses are in line with those reported by Dotsey and Duarte (2008) using a calibrated model for the U.S. and OECD countries. However, our empirical results challenge those of Corsetti et al. (2006) which find exactly the opposite.

Figure 2 displays the impulse response to a demand shock in the tradable sector in the euro area. In this case, consumption declines in the euro area and increases in the U.S., while the euro depreciates in real terms. The terms of trade also depreciates which boosts consumption in U.S. Why do both the real exchange rate and the terms of trade depreciate? Since the model features, infinitely-lived Ricardian households, the positive demand shock (which works as a fiscal shock) induces a negative wealth effect in euro area: agents work more and consume less today. Hence, the labor supply increases, causing a reduction in real wages that translates into a reduction in marginal costs in both sectors. Thus, domestic prices (tradable and nontradable) decrease, which triggers both a real exchange rate and terms of trade depreciation.

The ratio of relative consumptions decreases with the depreciation, and implies a strong negative correlation between the real exchange rate and relative consumptions across countries. Negative wealth effects cause consumption to

[12]For robustness, we have also performed an estimation using the terms of trade as an observable variable. Qualitatively, the impulse-responses do not change. Results are available upon request.

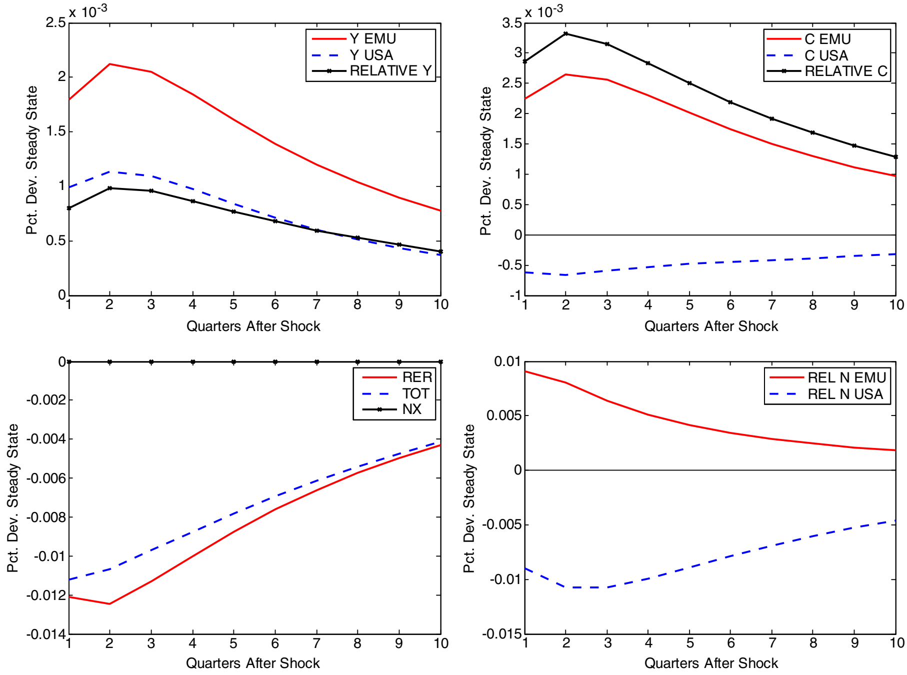

Fig. 3 Impulse response to a preference shock in the euro area

decrease in the euro area more than the reduction of consumption in the U.S. Hence, as noted above, the presence of demand shocks are necessary to explain real exchange rate dynamics, through their wealth effects on consumption, real wages and relative prices. In our model, these effects are so strong that they imply a reduction of output as well. Finally, the trade balance deteriorates slightly being consistent with the evidence reported in Monacelli and Perotti (2006). Therefore, it is crucial to have demand shocks in the model, in order to be able to explain the real exchange rate-relative consumption anomaly.[13]

Figure 3 shows the impulse response to a preference shock, which has very similar effects to the demand shock regarding the implied comovement between the real exchange rate and relative consumption. However, unlike the demand shocks it induces a positive wealth effect generating instead a real exchange rate appreciation. By increasing the marginal utility of consumption, consumption itself increases in the euro area, and the real exchange rate and terms of trade appreciate, which reduces consumption but increases output in

[13]We also estimate our model assuming non-separable preferences in line with Monacelli and Perotti (2006). Under this specification we were able to reproduce impulse responses conditional to both fiscal and tradable technology shocks that are consistent with the VAR evidence reported in Monacelli and Perotti (2006) and Corsetti et al. (2006), respectively. Yet, the likelihood decreases substantially and the overall fit of this specification underperforms our benchmark model. Results are available upon request from the authors.

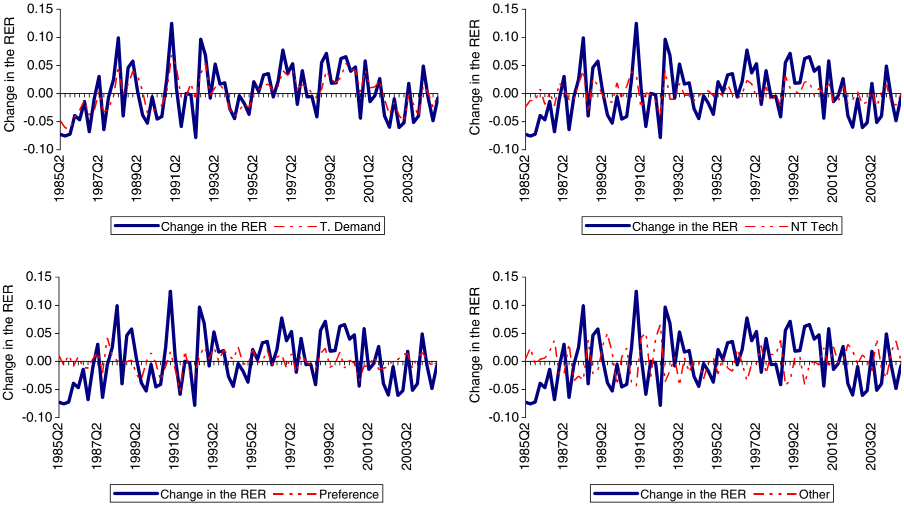

Fig. 4 Decomposition of the real exchange rate

the U.S. due to foreign demand. This also opens a small trade deficit for the euro area. Why both real exchange and the terms of trade appreciates?. The preference shock induces a positive wealth effect that is reflected in higher consumption. This increase of consumption leads to an increase in wages, marginal cost increases and consequently prices increase in both sectors. The price increase induces both a real exchange rate and terms of trade appreciation. Again, as noted above, we obtain a negative correlation between the real exchange rate and the ratio of relative consumptions, making this shock necessary to explain the data. At the same time, the relative price of nontradables increases in the euro area, but decreases in the U.S.

To further gauge the importance of the previous shocks in accounting for the historical RER dynamics, Fig. 4 displays the observed value of the variation in the real exchange rate (bold line), together with the values with only tradable demand, nontradable technology, preference, and the other shocks, according to our estimated model (dotted lines).[14] This exercise allows us to identify the nature of the shocks that have played a dominant role as a source of the real exchange rate dynamics.

It is clear that demand shocks explain a great fraction of the real exchange rate fluctuations being positive correlated with the real exchange rate, results that are consistent with the evidence illustrated above. Hence, the model

[14]We use the Kalman filter to recover the sequence of shocks. We basically obtain the cyclical components of the change in the real exchange rate associated with each shock, according to our estimated model at its posterior mean.

with demand shocks provides a very good approximation to the data. But, as we mentioned before, a model with only demand shocks would imply a too negative correlation between relative consumptions and the real exchange rate, so this is why other shocks in the model are needed. When the model is simulated with the nontradable component only, we can see that it is also able to capture some comovement with the actual series. On the other hand, when the model is simulated with preference shocks only, or the rest of shocks, the behavior of the change in the real exchange rate in the model and in the data is quite different.

6 The Role of the Distribution Sector

In recent papers, Corsetti et al. (2008) and Dotsey and Duarte (2008) have emphasized the role of the distribution sector in explaining real exchange rate dynamics. Here, we follow Dotsey and Duarte (2008) and estimate two different versions of that model. In the first one, we assume that the final tradable consumption good includes a nontradable intermediate input, and is produced under monopolistic competition (there is product differentiation). In the second case, we further assume that the final tradable good is also priced with a Calvo-type restriction.

We modify the model along the following lines. The final tradable good is consumed by domestic households. This good is produced by a continuum of firms, each producing a differentiated variety, labelled by $Y_t^T(i)$, $i \in [0, 1]$. Each firm combines a composite of home and foreign intermediate tradable goods X^T, with a composite of intermediate nontradable goods X^N with the following production function:

$$Y_t^T(i) = \left\{\gamma_y^{1/\varepsilon_Y}\left[X_t^T(i)\right]^{\frac{\varepsilon_Y - 1}{\varepsilon_Y}} + (1-\gamma_y)^{1/\varepsilon_Y}\left[X_t^N(i)\right]^{\frac{\varepsilon_Y - 1}{\varepsilon_Y}}\right\}^{\frac{\varepsilon_Y}{\varepsilon_Y - 1}}$$

where ε_y is the elasticity of substitution between tradable and nontradable intermediate goods, and γ_y is the share of tradable intermediate goods in the production function. The nontradable component can be seen as distribution services needed to bring the final consumption good to consumers. This production structure somewhat generalizes, but does not nest, Corsetti et al. (2008), and implies a wedge between the price of the CES aggregate of tradable inputs and the price paid by the final consumer, due to distribution costs. When $\gamma_y = 1$, we go back to the model of Section 2, but with product differentiation and monopolistic competition in the final tradable goods sector.

The local nontradable intermediate input is a Dixit-Stiglitz aggregate of all nontradable varieties, with the same elasticity than the consumption aggregate:

$$X_t^N(i) \equiv \left[\int_0^1 X_t^N(i,n)^{\frac{\sigma-1}{\sigma}} dn\right]^{\frac{\sigma}{\sigma-1}}$$

where $X_t^N(i,n)$ is the amount of intermediate nontradable input n by final good producer i. The price level P_t^N is the same as the one defined in Section 2.

The composite of home and foreign intermediate tradable goods is given by:

$$X_t^T(i) = \left\{\gamma_x^{1/\theta}\left[X_t^h(i)\right]^{\frac{\theta-1}{\theta}} + (1-\gamma_x)^{1/\theta}\left[X_t^f(i)\right]^{\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}}$$

The definition of the composite of home and foreign intermediate goods follows from Section 2.

Taking a linear approximation to the firm's optimizing conditions, when prices of the final tradable good are flexible, delivers the following inflation rate for the final tradable goods sector:

$$\triangle p_t^T = \gamma_y\left[\gamma_x\triangle p_t^h + (1-\gamma_x)\left(\triangle p_t^f + \Delta s_t\right)\right] + (1-\gamma_y)\triangle p_t^N, \qquad (27)$$

such that the final tradable goods sector includes a nontradable component. Further, if we assume that there are sticky prices in the final tradable good sector, inflation dynamics in the final goods tradable sector are given by:

$$\triangle p_t^T - \varphi_T\triangle p_{t-1}^T = \beta E_t\left(\triangle p_{t+1}^T - \varphi_T\triangle p_t^T\right) + \kappa_T\left(mc_t^T - t_t^T\right), \qquad (28)$$

where $\kappa_T = (1-\alpha_T)(1-\beta\alpha_T)/\alpha_T$, $mc_t^T = \gamma_y\left(t_t^X + t_t^T\right) + \left(1-\gamma_y\right)t_t^N$,and $t_t^X = [\gamma_x p_t^h + (1-\gamma_x)(p_t^f + s_t)] - p_t^T$.

Rather than presenting the full set of parameter estimates (which are available upon request) we compare how the models with a distribution sector fit the data, and in particular some selected moments of the data. In Table 4 we present the marginal likelihoods of the three models (baseline,

Table 4 Model comparison

	Data	Baseline	Distribution	Distribution with sticky prices
Marginal likelihood	–	3292.2	3222.0	3261.4
Standard Deviation(Q/Q_{-1})	4.64	3.34	3.77	4.34
Percent variance explained by				
Preference shocks	–	23.6	81.1	63.1
Nontradable tech. shocks	–	31.8	13.5	6.4
Fiscal shocks	–	43.5	0.8	0.6
Correlation (Q, Q_{-1})	0.78	0.78	0.75	0.68
Correlation $(C/C^*, Q)$	0.01	0.05	–0.14	–0.23

Note: *Standard Deviation*(Q/Q_{-1}) is based on raw data, Correlation (Q, Q_{-1}) and Correlation $(C/C^*, Q)$ is based on HP-filtered data distribution sector with final flexible prices, and distribution sector with final sticky prices).[15]

To compare overall performance, we focus on the posterior odds ratio between two models A and B:

$$\frac{\Pr(\text{model} = A|\,\{x_t\}_{t=1}^{T})}{\Pr(\text{ model} = B|\,\{x_t\}_{t=1}^{T})} = \frac{\Pr(A)}{\Pr(B)}\frac{L(\{x_t\}_{t=1}^{T}\,|\text{model} = A)}{L(\{x_t\}_{t=1}^{T}\,|\text{model} = B)}$$

If one does not have strong views about which model is the true one before observing the data, then $\Pr(A) = \Pr(B)$, and the researcher updates her beliefs on which model is the true one after observing the data according to the Bayes factor, which is the ratio of marginal likelihoods between two models $\frac{L(\{x_t\}_{t=1}^{T}|\text{model}=A)}{L(\{x_t\}_{t=1}^{T}|\text{model}=B)}$. Introducing a distribution sector in the model does not improve the model fit: a log Bayes factor of 70.2 (=3292.2−3222) implies that the researcher would need to have a prior probability that the distribution model is the true one about exp(70) times larger than the prior probability over the baseline model. When we introduce sticky prices in the final goods sector, model fit improves with respect to the model with flexible goods prices, but does not reach the value of the baseline model. We conclude that the introduction of a distribution sector in the two-sector economy does not improve its capability of explaining the data, beyond that already included in a two-sector model with tradable and nontradable goods.

Finally, Table 4 includes some additional posterior second moments that international business cycle models would want to replicate. As we can see, the addition of a distribution sector, and afterwards sticky prices in the final goods tradable sector, increases the volatility of the real exchange rate to values that are closer to those in the data. On the other hand, as we introduce these features into the models, it becomes more difficult to explain persistence. An additional unpleasant result is that, in the models with distribution costs, real exchange rate dynamics end up being explained by preference shocks, which have a more difficult interpretation than technology or demand shocks.

7 Concluding Remarks

In this paper we have examined the ability of models with tradable and nontradable goods to fit the data. Our main result is that we are able to match

[15] The additional parameters γ_y and the fraction of intermediate goods that is used to produce the final tradable good are taken from Dotsey and Duarte (2008). Hence we calibrate γ_y to 0.62, and the fraction of nontradable production that is used as an input in the production of final traded goods to $\frac{X^N}{Y^N} = 0.4$. We also estimated versions of the two distribution cost models where we estimated those parameters. The qualitative results did not change, and model fit did not improve significantly. In addition to these two parameters, in the model with a distribution sector and sticky prices, we also estimate α_T and φ_T with the same priors than the other Calvo lotteries and backward looking parameters of Table 1. We also estimate the elasticity of subtitution ε_y.

real exchange rate persistence, and to less extent, its volatility, with a medium-scale macroeconomic model estimated with Bayesian methods. We have found that it is mostly technology shocks in the nontradable sector, and demand shocks in the tradable sector the ones that explain most of the behavior of the real exchange rate. When we have estimated versions of the model with distribution services and sticky prices in the final tradable good sector, we have not obtained a better model fit. This suggests that distribution costs should not be treated differently than other nontradables in the production of final goods.

Estimation of DSGE models with several nominal and real rigidities tend to reveal that not all features are necessary to fit the data when priors are not too informative (see Galí and Rabanal 2005 or Rabanal and Tuesta 2010). On the other hand, estimated models where priors are much more informative tend to validate the rigidities in place (see Smets and Wouters 2003 and Adolfson et al. 2007). In our case, we find that distribution services on top of several other rigidities are not necessary, but this does not mean it is not a feature of relevance in international macroeconomics, or to explain the apparent deviation from the law of one price in industry-level data. In any case, we have found that a two-sector two-country model in the spirit of Stockman and Tesar (1995), complemented with nominal rigidities and habit formation, seems to do a good job in explaining the data.

Appendix A: The Baseline Model

In this appendix, we present the full version of a model with tradable and nontradable final consumption goods, in the spirit of Stockman and Tesar (1995) and Dotsey and Duarte (2008). We introduce sticky prices in both sectors to be able to study inflation dynamics and their role in affecting the real exchange rate.

A.1 Households

A.1.1 Preferences

Representative households in the home country are assumed to maximize the following utility function:

$$U_t = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \psi_t \left[\log\left(C_t - b\,\bar{C}_{t-1}\right) - \frac{L_t^{1+\varphi}}{1+\varphi} \right] \right\}, \tag{29}$$

subject to the following budget constraint:

$$\frac{B_t^H}{P_t R_t} + \frac{S_t B_t^F}{P_t R_t^* \Phi\left(\frac{S_t \bar{B}_t^F}{P_t Y_t}\right)} \leq \frac{B_{t-1}^H}{P_t} + \frac{S_t B_{t-1}^F}{P_t} + \frac{W_t}{P_t} L_t - C_t + \Pi_t \tag{30}$$

E_0 denotes the conditional expectation on information available at date $t = 0$, β is the intertemporal discount factor, with $0 < \beta < 1$. C_t denotes the level of consumption in period t, L_t denotes labor supply. The utility function displays external habit formation with respect to the habit stock, which is last period's aggregate consumption of the economy $\bar{C}_{t-1}$. $b \in [0, 1]$ denotes the importance of the habit stock. $\varphi > 0$ is inverse elasticity of labor supply with respect to the real wage. ψ_t is a preference shock that follows an $AR(1)$ process in logs

$$\log \psi_t = \rho_\psi \log \psi_{t-1} + \varepsilon_t^\psi \tag{31}$$

We define the consumption index as

$$C_t \equiv \left[\gamma_c^{1/\varepsilon} \left(C_t^T \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma_c)^{1/\varepsilon} \left(C_t^N \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where ε is elasticity of substitution between the final tradable (C_t^T) and final nontradable (C_t^N) goods, and γ_c is the share of final tradable goods in the consumption basket at home.

In this context, the consumer price index that corresponds to the previous specification is given by

$$P_t \equiv \left[\gamma_c \left(P_t^T \right)^{1-\varepsilon} + (1-\gamma_c) \left(P_t^N \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}},$$

where all prices are for goods sold in the home country, in home currency and at consumer level, for both tradable and nontradable goods.

Demands for the final tradable and nontradable goods are given by:

$$C_t^T = \gamma_c \left(\frac{P_t^T}{P_t} \right)^{-\varepsilon} C_t,$$

$$C_t^N = (1-\gamma_c) \left(\frac{P_t^N}{P_t} \right)^{-\varepsilon} C_t.$$

A.1.2 Incomplete Asset Markets

For modelling simplicity, we choose to model incomplete markets with two risk-free one-period nominal bonds denominated in domestic and foreign currency, and a cost of bond holdings is introduced to achieve stationarity. Then, the budget constraint of the domestic households in real units of home currency is given by:

$$\frac{B_t^H}{P_t R_t} + \frac{S_t B_t^F}{P_t R_t^* \Phi \left(\frac{S_t B_t^F}{P_t Y_t} \right)} \leq \frac{B_{t-1}^H}{P_t} + \frac{S_t B_{t-1}^F}{P_t} + \frac{W_t}{P_t} L_t - C_t + \Pi_t \tag{32}$$

where W_t is the nominal wage, and Π_t are real profits for the home consumer. B_t^H is the holding of the risk free domestic nominal bond and B_t^F is the holding of the foreign risk-free nominal bond expressed in foreign country currency. S_t is the nominal exchange rate, expressed in units of home country currency per unit of foreign country. The function $\Phi(.)$ depends on the net liability position (i.e. the negative net foreign asset position) of the home country, $\bar{B}_t^F$, in percent of GDP in the entire economy, and is taken as given by the domestic household.[16] Φ (.) introduces a convex cost that allows to obtain a well-defined steady state, and captures the costs of undertaking positions in the international asset market.[17]

A.2 Production Sector

The production of this economy is undertaken by three sectors. First, there is a final goods sector, that uses intermediate tradable inputs from both countries and operates under perfect competition, to produce the final tradable goods. This same sector also aggregates varieties of the nontradable goods to produce a final nontradable good that is sold to households. The second sector produces intermediate tradable goods, which are used as an input for the production of final goods both in the home and in the foreign country. The third sector produces nontradable goods, that are used as inputs in the production of the final nontradable good.

A.2.1 Final Goods Sector

The final tradable good is consumed by domestic households. This good is produced by a continuum of firms, each producing the same variety, labelled by Y_t^T, using intermediate home $\left(X_t^h\right)$ and foreign $\left(X_t^f\right)$ goods with the following technology:

$$Y_t^T = \left\{\gamma_x^{1/\theta}\left(X_t^h\right)^{\frac{\theta-1}{\theta}} + (1-\gamma_x)^{1/\theta}\left(X_t^f\right)^{\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}}$$

where θ is the elasticity of substitution between home-produced and foreign-produced imported intermediate goods, and γ_x is the share of home goods in the production function. We further assume symmetric home-bias in the composite of intermediate tradable goods. The corresponding composite of home and foreign intermediate tradable goods abroad is given by

$$Y_t^{T*} = \left\{(1-\gamma_x)^{1/\theta}\left(X_t^{h*}\right)^{\frac{\theta-1}{\theta}} + \gamma_x^{1/\theta}\left(X_t^{f*}\right)^{\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}}$$

X_t^h and X_t^f, that denote the amount of home and foreign intermediate tradable inputs to produce the final tradable good at home, are also Dixit-Stiglitz

[16] As Benigno (2009) points it out, some restrictions on ϕ (.) are necessary: $\phi(0) = 1$; assumes the value 1 only if $B_{F,t} = 0$; differentiable; and decreasing in the neighborhood of zero.

[17] Another way to describe this cost is to assume the existence of intermediaries in the foreign asset market (which are owned by the foreign households) who can borrow and lend to households of country F at a rate $(1+r^*)$, but can borrow from and lend to households of country H at a rate $(1+r^*)\phi$ (.).

aggregates of all types of home and foreign final goods, with elasticity of substitution σ:

$$X_t^h \equiv \left[\int_0^1 X_t^h(h)^{\frac{\sigma-1}{\sigma}} dh\right]^{\frac{\sigma}{\sigma-1}}$$

and

$$X_t^f \equiv \left[\int_0^1 X_t^f(f)^{\frac{\sigma-1}{\sigma}} df\right]^{\frac{\sigma}{\sigma-1}}$$

where $X_t^h(h)$ and $X_t^f(f)$ denote individual quantities from intermediate tradable goods producers at home and foreign. The equivalent quantities for foreign final tradable goods producers are $X_t^{h^*}(h)$ and $X_t^{f^*}(f)$. Optimizing conditions by final tradable goods producers deliver the following demand functions:

$$X_t^h(h) = \gamma_x \left(\frac{P_t^h(h)}{P_t^h}\right)^{-\sigma} \left(\frac{P_t^h}{P_t^T}\right)^{-\theta} Y_t^T;$$

$$X_t^{h^*}(h) = (1-\gamma_x) \left(\frac{P_t^{h^*}(h)}{P_t^{h^*}}\right)^{-\sigma} \left(\frac{P_t^{h^*}}{P_t^{T^*}}\right)^{-\theta} Y_t^{T^*}$$

$$X_t^f(f) = (1-\gamma_x) \left(\frac{P_t^f(f)}{P_t^f}\right)^{-\sigma} \left(\frac{P_t^f}{P_t^T}\right)^{-\theta} Y_t^T;$$

$$X_t^{f^*}(f) = \gamma_x \left(\frac{P_t^{f^*}(f)}{P_t^{f^*}}\right)^{-\sigma} \left(\frac{P_t^{f^*}}{P_t^{T^*}}\right)^{-\theta} Y_t^{T^*}$$

where

$$P_t^h \equiv \left[\int_0^1 P_t^h(h)^{1-\sigma} dh\right]^{\frac{1}{1-\sigma}}, \quad P_t^f \equiv \left[\int_0^1 P_t^f(f)^{1-\sigma} df\right]^{\frac{1}{1-\sigma}}.$$

and

$$P_t^T = \left[\gamma_x \left(P_t^h\right)^{1-\theta} + (1-\gamma_x)\left(P_t^f\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

We assume that the law of one price holds for intermediate inputs, such that $P_t^h(h) = P_t^{h^*}(h) S_t$, and $P_t^f(f) = P_t^{f^*}(f) S_t$, where S_t is the nominal exchange rate.

The production of the final nontradable good is given by:

$$Y_t^N \equiv \left[\int_0^1 X_t^N(n)^{\frac{\sigma-1}{\sigma}} dn\right]^{\frac{\sigma}{\sigma-1}}$$

where we assume the same elasticity $\sigma > 1$ than in the case of final tradable goods produced within country H. The price level for nontradables is

$$P_t^N \equiv \left[\int_0^1 p_t^N(n)^{1-\sigma} dn\right]^{\frac{1}{1-\sigma}}$$

A.2.2 Intermediate Non-Tradable Goods Sector

The intermediate nontradable sector produces differentiated goods that are aggregated by final good producing firms, and ultimately used for final consumption by domestic households only. Each firm produces intermediate nontradable goods according to the following production function

$$Y_t^N(n) = A_t Z_t^N L_t^N(n) \tag{33}$$

where A_t is a labor augmenting aggregate world technology shock which has a unit root with drift, as in Galí and Rabanal (2005):

$$\log A_t = g + \log A_{t-1} + \varepsilon_t^a \tag{34}$$

This shock also affects the intermediate tradable sector production function. Hence, real variables in both countries grow at a rate g. Z_t^N is the country-specific productivity shock to the nontradable sector at time t which evolves according to an AR(1) process in logs

$$\log Z_t^N = (1-\rho^N)\log(\bar{Z}^N) + \rho^{Z,N} \log Z_{t-1}^N + \varepsilon_t^{Z,N} \tag{35}$$

Firms in the nontradable sector face a Calvo lottery when setting their prices. Each period, with probability $1-\alpha_N$, firms receive a stochastic signal that allows them to reset prices optimally. We assume that there is partial indexation with a coefficient φ_N to last period's sectorial inflation rate for those firms that do not get to reset prices. As a result, firms maximize the following profits function:

$$Max_{P_t^N(n)} E_t \sum_{k=0}^{\infty} \alpha_N^k \Lambda_{t,t+k} \left\{ \left[\frac{P_t^N(n)\left(\frac{P_{t+k-1}^N}{P_{t-1}^N}\right)^{\varphi_N}}{P_{t+k}} - MC_{t+k}^N \right] Y_{t+k}^{N,d}(n) \right\} \tag{36}$$

subject to

$$Y_{t+k}^{N,d}(n) = \left[\left(\frac{P_t^N(n)}{P_{t+k}^N}\right)\left(\frac{P_{t+k-1}^N}{P_{t-1}^N}\right)^{\varphi_N}\right]^{-\sigma} Y_t^N \tag{37}$$

where $Y_t^{N,d}(n)$ is total individual demand for a given type of nontradable good n, and Y_t^N is aggregate demand for nontradable goods, as defined above. $\Lambda_{t,t+k} = \beta^k \frac{\lambda_{t+k}}{\lambda_t}$ is the stochastic discount factor, where $\lambda_t = \frac{\psi_t}{C_t - bC_{t-1}}$ is the marginal utility of consumption. MC_t^N corresponds to the real marginal cost in the nontradable sector. From cost minimization:

$$MC_t^N = \frac{W_t}{P_t Z_t^N A_t}$$

A.2.3 Intermediate Tradable Goods Sector

The intermediate tradable sector produces differentiated goods that are sold to the final sector goods producers in the home and foreign countries. Most functional forms are similar to those presented for the nontradable sector.

Each firm produces tradable intermediate goods according to the following production function

$$Y_t^h(h) = A_t Z_t^h L_t^h(h) \tag{38}$$

where Z_t^h is the country-specific productivity shock to the intermediate goods tradable sector at time t which evolves according to an AR(1) process in logs

$$\log Z_t^h = (1-\rho^h)\log(\bar{Z}^h) + \rho^{Z,h}\log Z_{t-1}^h + \varepsilon_t^{Z,h} \tag{39}$$

Firms in the intermediate tradable sector face the same Calvo lottery as firms in the intermediate nontradable sector, with relevant parameters α_h and φ_h:

$$Max_{P_t^h(h)}\, E_t \sum_{k=0}^{\infty} \alpha_h^k \Lambda_{t,t+k} \left\{ \left[\frac{P_t^h(h)\left(\frac{P_{t+k-1}^h}{P_{t-1}^h}\right)^{\varphi_h}}{P_{t+k}} - MC_{t+k}^h \right] Y_{t+k}^{h,d}(h) \right\} \tag{40}$$

subject to

$$\begin{aligned} Y_{t+k}^{h,d}(h) &= X_{t+k}^h(h) + X_{t+k}^{h^*}(h) \\ &= \left[\left(\frac{P_t^h(h)}{P_{t+k}^h}\right) \left(\frac{P_{t+k-1}^h}{P_{t-1}^h}\right)^{\varphi_h} \right]^{-\sigma} X_t^h \end{aligned} \tag{41}$$

where $Y_t^{h,d}(h)$ is total individual demand for a given type of tradable intermediate good h, and X_t^h is aggregate demand for intermediate good h, consisting of home demand, and foreign demand:

$$X_t^h = \left[\gamma_x \left(\frac{P_t^h}{P_t^T}\right)^{-\theta} Y_t^T + (1-\gamma_x)\left(\frac{P_t^{h^*}}{P_t^{T^*}}\right)^{-\theta} Y_t^{T^*} \right]$$

MC_t^h corresponds to the real marginal cost in the nontradable sector. From cost minimization:

$$MC_t^h = \frac{W_t}{P_t Z_t^h A_t}$$

A.2.4 Market Clearing

We assume that the demand shock is allocated between tradable and nontradable goods in the same way that private consumption is. Hence the market clearing conditions for both types of final goods, consisting of private consumption and the demand shock in the tradadable sector, are:

$$Y_t^T = C_t^T + G_t^T$$

$$Y_t^N = C_t^N + G_t^N$$

where G_t^N, G_t^T follow AR(1) processes in logs. The bond market clearing conditions are

$$B_t^H + B_t^{H*} = 0 \tag{42}$$

$$B_t^F + B_t^{F*} = 0 \tag{43}$$

For the nontradable intermediate goods, the market clearing condition is:

$$Y_t^N(n) = X_t^N, \text{ for all } n \in [0, 1] \tag{44}$$

while for the intermediate tradable goods sector it is:

$$Y_t^h(h) = X_t^h(h) + X_t^{h^*}(h), \text{ for all } h \in [0, 1] \tag{45}$$

For the labor market:

$$\begin{aligned} L_t &= L_t^h + L_t^N = \\ &= \int_0^1 L_t^h(h)dh + \int_0^1 L_t^N(n)dn \end{aligned} \tag{46}$$

A.3 Optimizing, Market Clearing Conditions, and Monetary Policy

In this subsection we present the full set of equations characterizing the symmetric equilibrium. Since all agents in each economy are equal, then the per capita and aggregate consumption levels are equal ($C_t = \bar{C}_t$), as well as the net foreign assets levels ($B_t^F = \bar{B}_t^F$).

A.3.1 Households

The Euler equations for home and foreign households, and the optimal condition of holdings by home household of the foreign bond are:

$$\lambda_t = \beta E_t \left\{ R_t \frac{P_t}{P_{t+1}} \lambda_{t+1} \right\}$$

$$\lambda_t^* = \beta E_t \left\{ R_t^* \frac{P_t^*}{P_{t+1}^*} \lambda_{t+1}^* \right\}$$

$$\lambda_t = \Phi \left(\frac{S_t B_t^F}{P_t Y_t} \right) \beta E_t \left\{ R_t^* \frac{Q_{t+1}}{Q_t} \lambda_{t+1} \right\}$$

where λ_t is the marginal utility of consumption:

$$\lambda_t = U_C(C_t) = \frac{\psi_t}{C_t - b\,C_{t-1}}$$

$$\lambda_t^* = U_C\left(C_t^*\right) = \frac{\psi_t^*}{C_t^* - b^* C_{t-1}^*}$$

The labor supply decisions in each country are:

$$\lambda_t \frac{W_t}{P_t} = L_t^{\varphi}$$

$$\lambda_t^* \frac{W_t^*}{P_t^*} = \left(L_t^*\right)^{\varphi^*}$$

where:

$$L_t = L_t^h + L_t^N$$

and

$$L_t^* = L_t^{h^*} + L_t^{N^*}$$

Household demand for final tradable and nontradable goods are given by:

$$C_t^T = \gamma_c \left(\frac{P_t^T}{P_t}\right)^{-\varepsilon} C_t,$$

$$C_t^N = (1-\gamma_c)\left(\frac{P_t^N}{P_t}\right)^{-\varepsilon} C_t.$$

$$C_t^{T^*} = \gamma_c^* \left(\frac{P_t^{T^*}}{P_t^*}\right)^{-\varepsilon^*} C_t^*,$$

$$C_t^{N^*} = (1-\gamma_c^*)\left(\frac{P_t^{N^*}}{P_t^*}\right)^{-\varepsilon} C_t^*$$

and the CPI's in each country are given by:

$$P_t \equiv \left[\gamma_c \left(P_t^T\right)^{1-\varepsilon} + (1-\gamma_c)\left(P_t^N\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}},$$

$$P_t^* \equiv \left[\gamma_c^* \left(P_t^{T^*}\right)^{1-\varepsilon} + (1-\gamma_c^*)\left(P_t^{N^*}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

The real exchange rate is

$$Q_t = \frac{S_t P_t^*}{P_t}$$

A.3.2 Final Goods Producers

The production of final tradable goods in both countries is given by:

$$Y_t^T = \left\{\gamma_x^{1/\theta}\left(X_t^h\right)^{\frac{\theta-1}{\theta}} + (1-\gamma_x)^{1/\theta}\left(X_t^f\right)^{\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}}$$

and

$$Y_t^{T^*} = \left\{(1-\gamma_x)^{1/\theta}\left(X_t^{h^*}\right)^{\frac{\theta-1}{\theta}} + \gamma_x^{1/\theta}\left(X_t^{f^*}\right)^{\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}}$$

Demand for intermediate tradable goods is:

$$X_t^h = \gamma_x\left(\frac{P_t^h}{P_t^T}\right)^{-\theta} Y_t^T;\; X_t^{h^*} = (1-\gamma_x)\left(\frac{P_t^{h^*}}{P_t^{T^*}}\right)^{-\theta} Y_t^{T^*}$$

$$X_t^f = (1-\gamma_x)\left(\frac{P_t^f}{P_t^T}\right)^{-\theta} Y_t^T;\; X_t^{f^*}(f) = \gamma_x\left(\frac{P_t^{f^*}}{P_t^{T^*}}\right)^{-\theta} Y_t^{T^*}$$

where

$$P_t^h \equiv \left[\int_0^1 P_t^h(h)^{1-\sigma}dh\right]^{\frac{1}{1-\sigma}},\; P_t^f \equiv \left[\int_0^1 P_t^f(f)^{1-\sigma}df\right]^{\frac{1}{1-\sigma}}.$$

The price of final tradable goods is:

$$P_t^T = \left[\gamma_x\left(P_t^h\right)^{1-\theta} + (1-\gamma_x)\left(P_t^f\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

and

$$P_t^{T^*} = \left[\gamma_x^*\left(P_t^{h^*}\right)^{1-\theta} + (1-\gamma_x^*)\left(P_t^{f^*}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

Since we assumed that the law of one price holds for intermediate goods, it also holds in the aggregate, such that $P_t^h = P_t^{h^*}S_t$, and $P_t^f = P_t^{f^*}S_t$, where S_t is the nominal exchange rate.

A.3.3 Nontradable Goods Producers

The price setting equations are given by the following optimal expressions:

$$\frac{\hat{p}_t^N}{P_t^N} = \frac{\sigma}{(\sigma-1)}E_t\left\{\frac{\sum_{k=0}^{\infty}\beta^k\alpha_N^k\lambda_{t+k}\left(\prod_{s=1}^{k}\frac{\left(\Pi_{t+s-1}^N\right)^{\varphi_N}}{\Pi_{t+s}^N}\right)^{-\sigma} MC_{t+k}^N Y_{t+k}^N}{\sum_{k=0}^{\infty}\beta^k\alpha_N^k\lambda_{t+k}\left(\prod_{s=1}^{k}\frac{\left(\Pi_{t+s-1}^N\right)^{\varphi_N}}{\Pi_{t+s}^N}\right)^{1-\sigma}\frac{P_{t+k}^N}{P_{t+k}}Y_{t+k}^N}\right\}$$

where

$$MC_t^N = \frac{W_t}{P_t Z_t^N A_t},$$

$$Y_t^N = C_t^N + G_t^N$$

The evolution of the price level of nontradables is

$$P_t^N \equiv \left[\alpha_N \left(P_{t-1}^N \left(\Pi_{t-1}^N\right)^{\varphi_N}\right)^{1-\sigma} + (1-\alpha_N)\left(\hat{p}_t^N\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

where $\Pi_{t-1}^N = \frac{P_{t-1}^N}{P_{t-2}^N}$.

The production function is:

$$Y_t^N = A_t Z_t^N L_t^N.$$

In the foreign country these expressions are:

$$\frac{\hat{p}_t^{N^*}}{P_t^{N^*}} = \frac{\sigma}{(\sigma-1)} E_t \left\{ \frac{\sum_{k=0}^{\infty} \beta^k \alpha_{N^*}^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{N^*}\right)^{\varphi_{N^*}}}{\Pi_{t+s}^{N^*}}\right)^{-\sigma} MC_{t+k}^{N^*} Y_{t+k}^{N^*}}{\sum_{k=0}^{\infty} \beta^k \alpha_{N^*}^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{N^*}\right)^{\varphi_{N^*}}}{\Pi_{t+s}^{N^*}}\right)^{1-\sigma} \frac{P_{t+k}^{N^*}}{P_{t+k}} Y_{t+k}^{N^*}} \right\}$$

where

$$MC_t^{N^*} = \frac{W_t}{P_t Z_t^{N^*} A_t},$$

$$Y_t^{N^*} = C_t^{N^*} + G_t^{N^*}$$

The evolution of the price level of nontradables is

$$P_t^{N^*} \equiv \left[\alpha_{N^*} \left(P_{t-1}^{N^*} \left(\Pi_{t-1}^{N^*}\right)^{\varphi_{N^*}}\right)^{1-\sigma} + (1-\alpha_{N^*})\left(\hat{p}_t^{N^*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

where $\Pi_{t-1}^{N^*} = \frac{P_{t-1}^{N^*}}{P_{t-2}^{N^*}}$.

The production function is:

$$Y_t^{N^*} = A_t Z_t^{N^*} L_t^{N^*}.$$

A.3.4 Intermediate Traded Goods Producers

The price setting equations are given by the following optimal expressions:

$$\frac{\hat{p}_t^h}{P_t^h} = \frac{\sigma}{(\sigma-1)} E_t \left\{ \frac{\sum_{k=0}^{\infty} \beta^k \alpha_h^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{h}\right)^{\varphi_h}}{\Pi_{t+s}^{h}}\right)^{-\sigma} MC_{t+k}^{h} Y_{t+k}^{h}}{\sum_{k=0}^{\infty} \beta^k \alpha_h^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{h}\right)^{\varphi_h}}{\Pi_{t+s}^{h}}\right)^{1-\sigma} \frac{P_{t+k}^{h}}{P_{t+k}} Y_{t+k}^{h}} \right\}$$

where

$$MC_t^h = \frac{W_t}{P_t Z_t^h A_t},$$

$$Y_t^h = X_t^h + X_t^{h^*}.$$

The evolution of the price level of final tradables is

$$P_t^h \equiv \left[\alpha_h \left(P_{t-1}^h \left(\Pi_{t-1}^h \right)^{\varphi_h} \right)^{1-\sigma} + (1-\alpha_h) \left(\hat{p}_t^h \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

where $\Pi_{t-1}^h = \frac{P_{t-1}^h}{P_{t-2}^h}$.

The production function is:

$$Y_t^h = A_t Z_t^h L_t^h$$

In the foreign country, this expressions are:

$$\frac{\hat{p}_t^{f^*}}{P_t^{f^*}} = \frac{\sigma}{(\sigma-1)} E_t \left\{ \frac{\sum_{k=0}^{\infty} \beta^k \alpha_{f^*}^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{f^*} \right)^{\varphi_{f^*}}}{\Pi_{t+s}^{f^*}} \right)^{-\sigma} MC_{t+k}^{f^*} Y_{t+k}^{f^*}}{\sum_{k=0}^{\infty} \beta^k \alpha_{f^*}^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{f^*} \right)^{\varphi_{f^*}}}{\Pi_{t+s}^{f^*}} \right)^{1-\sigma} \frac{P_{t+k}^{f^*}}{P_{t+k}} Y_{t+k}^{f^*}} \right\}$$

where

$$MC_t^{f^*} = \frac{W_t}{P_t Z_t^{f^*} A_t},$$

$$Y_t^{f^*} = X_t^f + X_t^{f^*}.$$

The evolution of the price level of final tradables is

$$P_t^{f^*} \equiv \left[\alpha_{f^*} \left(P_{t-1}^{f^*} \left(\Pi_{t-1}^{f^*} \right)^{\varphi_{f^*}} \right)^{1-\sigma} + (1-\alpha_{f^*}) \left(\hat{p}_t^{f^*} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

where $\Pi_{t-1}^{f^*} = \frac{P_{t-1}^{f^*}}{P_{t-2}^{f^*}}$.

The production function is:

$$Y_t^{f^*} = A_t Z_t^{f^*} L_t^{f^*}$$

A.3.5 Monetary Policy

Monetary policy in both countries is conducted with a Taylor rule that targets CPI inflation and output growth deviation from steady-state values:

$$R_t = \bar{R}^{(1-\rho_r)} R_{t-1}^{\rho_r} \left(\frac{P_t/P_{t-1}}{\Pi}\right)^{(1-\rho_r)\gamma_\pi} \left(\frac{Y_t/Y_{t-1}}{1+g}\right)^{(1-\rho_r)\gamma_y} \exp(\varepsilon_t^r)$$

$$R_t^* = \bar{R}^{*(1-\rho_r^*)} \left(R_{t-1}^*\right)^{\rho_r^*} \left(\frac{P_t^*/P_{t-1}^*}{\Pi^*}\right)^{(1-\rho_r^*)\gamma_\pi^*} \left(\frac{Y_t^*/Y_{t-1}^*}{1+g}\right)^{(1-\rho_r^*)\gamma_y^*} \exp(\varepsilon_t^{r^*})$$

A.3.6 Demand Shocks

$$G_t^T = (\bar{G}^T)^{(1-\rho_{G^T})} (G_{t-1}^T)^{\rho_{G^T}} \exp(\varepsilon_t^{G^T})$$

$$G_t^N = (\bar{G}^N)^{(1-\rho_{G^N})} (G_{t-1}^N)^{\rho_{G^N}} \exp(\varepsilon_t^{G^N})$$

$$G_t^{T^*} = (\bar{G}^{T^*})^{(1-\rho_{G^{T*}})} (G_{t-1}^{T^*})^{\rho_{G^{T*}}} \exp(\varepsilon_t^{G^{T^*}})$$

$$G_t^{N^*} = (\bar{G}^{N^*})^{(1-\rho_{G^{N*}})} (G_{t-1}^{N^*})^{\rho_{G^{N*}}} \exp(\varepsilon_t^{G^{N^*}})$$

A.3.7 Trade Balance and Net Foreign Asset Dynamics

We present the evolution of the trade balance and net foreign assets of the home country, since the definition those in the foreign country will mirror those in the home country. Holdings of foreign bonds depend on the trade balance (NX_t) as follows

$$\frac{S_t B_t^F}{P_t R_t^* \Phi\left(\frac{S_t B_t^F}{P_t Y_t}\right)} = \frac{S_t B_{t-1}^F}{P_t} + NX_t$$

Since international trade only occurs at the intermediate goods level, net exports equal exports minus imports of intermediate goods:

$$NX_t = \frac{P_t^h X_t^{h^*} - P_t^f X_t^f}{P_t}$$

Finally, we define nominal GDP to be equal to aggregate nominal private and public consumption, hence $P_t Y_t = P_t^T (C_t^T + G_t^T) + P_t^N (C_t^N + G_t^N)$.

Appendix B: Log-Linear Version of the Model

Euler equations

$$b\,\Delta c_t = -(1+g-b)(r_t - E_t \triangle p_{t+1}) + (1+g)\,E_t \Delta c_{t+1} + (1+g-b)\left(1-\rho_\psi\right)\widehat{\psi}_t \tag{47}$$

$$b^*\Delta c_t^* = -\left(1+g-b^*\right)\left(r_t^* - E_t \triangle p_{t+1}^*\right) + (1+g)\,E_t \Delta c_{t+1}^* + \left(1+g-b^*\right)\left(1-\rho_\psi^*\right)\widehat{\psi}_t^* \tag{48}$$

Risk sharing

$$E_t\left(q_{t+1}-q_t\right) = \left[\frac{(1+g)\,E_t\Delta c_{t+1} - b\,\Delta c_t}{(1+g-b)}\right] - \left[\frac{(1+g)\,E_t\Delta c_{t+1}^* - b^*\Delta c_t^*}{(1+g-b^*)}\right] + \left(1-\rho_\psi\right)\widehat{\psi}_t - \left(1-\rho_\psi^*\right)\widehat{\psi}_t^* + \chi b_t \tag{49}$$

where $\chi \equiv -\Phi'(0)\,Y$, $b_t = \left(\frac{S_t B_t^F}{P_t}\right) Y^{-1}$.

The labor supply schedules are given by:

$$\tilde{\omega}_t = \varphi l_t + \left[\frac{1+g}{1+g-b}\right]\widetilde{c}_t - \frac{b}{(1+g-b)}\widetilde{c}_{t-1} + \frac{b}{(1+g-b)}\varepsilon_t^a \tag{50}$$

$$\tilde{\omega}_t^* = \varphi l_t^* + \left[\frac{1+g}{1+g-b^*}\right]\widetilde{c}_t^* - \frac{b^*}{(1+g-b^*)}\widetilde{c}_{t-1}^* + \frac{b^*}{(1+g-b^*)}\varepsilon_t^a \tag{51}$$

Technology

$$\widetilde{y}_t^h = l_t^h + z_t^h - \varepsilon_t^a \tag{52}$$

$$\widetilde{y}_t^{h^*} = l_t^{h^*} + z_t^{h^*} - \varepsilon_t^a \tag{53}$$

$$\widetilde{y}_t^N = l_t^N + z_t^N - \varepsilon_t^a \tag{54}$$

$$\widetilde{y}_t^{N^*} = l_t^{N^*} + z_t^{N^*} - \varepsilon_t^a \tag{55}$$

Consumer price inflation

$$\triangle p_t = \gamma_c \triangle p_t^T + (1-\gamma_c)\triangle p_t^N \tag{56}$$

$$\triangle p_t = \gamma_{c^*} \triangle p_t^{T^*} + (1-\gamma_{c^*})\triangle p_t^{N^*} \tag{57}$$

Tradable inflation

$$\triangle p_t^T = \gamma_x \triangle p_t^h + (1-\gamma_x)\left(\triangle p_t^{f^*} + \Delta s_t\right) \tag{58}$$

$$\triangle p_t^{T^*} = \gamma_{x^*}\left(\triangle p_t^h - \Delta s_t\right) + (1-\gamma_{x^*})\triangle p_t^{f^*} \tag{59}$$

Price setting in the nontradable sector

$$\triangle p_t^N - \varphi_N \triangle p_{t-1}^N = \beta E_t \left(\triangle p_{t+1}^N - \varphi_N \triangle p_t^N\right) + \kappa_N \left(\widetilde{w}_t - z_t^N - t_t^N\right) \tag{60}$$

$$\triangle p_t^{N^*} - \varphi_{N^*} \triangle p_{t-1}^{N^*} = \beta E_t \left(\triangle p_{t+1}^{N^*} - \varphi_N^* \triangle p_t^{N^*}\right) + \kappa_{N^*} \left(\widetilde{w}_t^* - z_t^{N^*} - t_t^{N^*}\right) \tag{61}$$

where $\kappa_N = (1-\alpha_N)(1-\beta\alpha_N)/\alpha_N$, $\kappa_{N^*} = (1-\alpha_{N^*})(1-\beta\alpha_{N^*})/\alpha_{N^*}$, $t^N = p_t^N - p_t$, and $t^{N^*} = p_t^{N^*} - p_t$.

Price setting in the intermediate tradable good sector

$$\triangle p_t^h - \varphi_h \triangle p_{t-1}^h = \beta E_t \left(\triangle p_{t+1}^h - \varphi_h \triangle p_t^h\right) + \kappa_h \left(\widetilde{w}_t - z_t^h - t_t^h - t_t^T\right) \tag{62}$$

$$\triangle p_t^{f^*} - \varphi_{f^*} \triangle p_{t-1}^{f^*} = \beta E_t \left(\triangle p_{t+1}^{f^*} - \varphi_f^* \triangle p_t^{f^*}\right) + \kappa_{f^*} \left(\widetilde{w}_t^* - z_t^{f^*} - t_t^{f^*} - t_t^{T^*}\right) \tag{63}$$

where $\kappa_h = (1-\alpha_h)(1-\beta\alpha_h)/\alpha_h$, $\kappa_{f^*} = \left(1-\alpha_{f^*}\right)\left(1-\beta\alpha_{f^*}\right)/\alpha_{f^*}$, $t_t^h = p_t^h - p_t^T$, $t_t^{f^*} = p_t^{f^*} - p_t^{T^*}$, $t^T = p_t^T - p_t$, and $t^{T^*} = p_t^{T^*} - p_t$.

Final consumption demand

$$\widetilde{c}_t^T = -\varepsilon t_t^T + \widetilde{c}_t \tag{64}$$

$$\widetilde{c}_t^{T^*} = -\varepsilon^* t_t^{T^*} + \widetilde{c}_t^* \tag{65}$$

$$\widetilde{c}_t^N = -\varepsilon t_t^N + \widetilde{c}_t \tag{66}$$

$$\widetilde{c}_t^{N^*} = -\varepsilon^* t_t^{N^*} + \widetilde{c}_t^* \tag{67}$$

Intermediate tradable and nontradable demand

$$\widetilde{x}_t^h = -\theta t_t^h + \widetilde{y}_t^T \tag{68}$$

$$\widetilde{x}_t^{h^*} = -\theta t_t^{h^*} + \widetilde{y}_t^{T^*} \tag{69}$$

$$\widetilde{x}_t^f = -\theta t_t^f + \widetilde{y}_t^T \tag{70}$$

$$\widetilde{x}_t^{f^*} = -\theta t_t^{f^*} + \widetilde{y}_t^{T^*} \tag{71}$$

Relative Price Index

Let's define $t_t = \frac{p_t^f}{p_t^h}$, since the law of one price holds $t_t = -t_t^* = \frac{p_t^{h^*}}{p_t^{f^*}}$, then we can write the following relative prices as a function of t_t.

$$t_t^h = -(1-\gamma_x)\, t_t \tag{72}$$

$$t_t^{h^*} = -\gamma_x t_t \tag{73}$$

$$t_t^f = \gamma_x t_t \tag{74}$$

$$t_t^{f^*} = (1-\gamma_x)\, t_t \tag{75}$$

Relative prices

$$t_t = t_{t-1} + \triangle s_t + \triangle p_t^{f*} - \triangle p_t^h \tag{76}$$

$$t_t^T = t_{t-1}^T + \triangle p_t^T - \triangle p_t \tag{77}$$

$$t_t^N = -\frac{\gamma_c}{1-\gamma_c} t_t^T \tag{78}$$

$$t_t^{T*} = t_{t-1}^{T*} + \triangle p_t^{T*} - \triangle p_t^* \tag{79}$$

$$t_t^{N*} = -\frac{\gamma_c^*}{1-\gamma_c^*} t_t^{T*} \tag{80}$$

$$q_t = q_{t-1} + \triangle s_t + \triangle p_t^* - \triangle p_t \tag{81}$$

Taylor rules

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\,\gamma_\pi \triangle p_t + \gamma_y \triangle y_t + \varepsilon_t^r \tag{82}$$

$$r_t^* = \rho_r^* r_{t-1}^* + (1-\rho_{r^*})\,\gamma_\pi^* \triangle p_t^* + \gamma_y^* \triangle y_t + \varepsilon_t^{r^*} \tag{83}$$

Net foreign assets and net exports

$$\beta \widetilde{b}_t - \frac{1}{1+g} \widetilde{b}_{t-1} = \widetilde{nx}_t \tag{84}$$

$$\widetilde{nx}_t = \frac{X^f}{Y}\left(\widetilde{x}_t^{h*} - \widetilde{x}_t^f - t_t\right) \tag{85}$$

where $\widetilde{b}_t = \frac{\bar{B}_t^F S_t}{P_t Y}$ is the debt to GDP ratio, and $\widetilde{nx}_t = \frac{NX_t}{Y}$, and where we have assumed balanced trade in the steady state. To solve for the steady-state ratios, we have that:

$$\frac{Y^T}{Y} = \frac{G^T}{G} = \frac{C^T}{C} = \gamma_c$$

$$\frac{Y^N}{Y} = \frac{G^N}{G} = \frac{C^N}{C} = 1 - \gamma_c$$

$$\frac{X^h}{Y^T} = \gamma_x, \;\; \frac{X^f}{Y^T} = 1 - \gamma_x$$

Therefore tradable GDP over total GDP is

$$\frac{X^h}{Y} = \frac{X^h}{Y^T}\frac{Y^T}{Y} = \gamma_x \gamma_c.$$

Hence

$$\frac{X^f}{Y} = \frac{X^f}{X^T}\frac{X^T}{Y^T}\frac{Y^T}{Y} = (1-\gamma_x)\gamma_c.$$

Market clearing:

$$\tilde{y}_t^T = (1-\gamma)\tilde{c}_t^T + \gamma\tilde{g}_t^T \tag{86}$$

where γ equals the fraction of government spending over total output. For the nontradable good, the market clearing condition is:

$$\tilde{y}_t^N = (1-\gamma)\tilde{c}_t^N + \gamma\tilde{g}_t^N \tag{87}$$

Finally, for the intermediate tradable goods sector:

$$\widetilde{y}_t^h = \gamma_x\widetilde{x}_t^h + (1-\gamma_x)\,\widetilde{x}_t^{h^*} \tag{88}$$

Total real GDP

$$\widetilde{y}_t = \gamma_c(t_t^T + \widetilde{y}_t^T) + (1-\gamma_c)\,(t_t^N + \widetilde{y}_t^N) \tag{89}$$

total labor

$$l_t = \frac{X^h}{X^h + Y^N}l_t^h + \frac{Y^N}{X^h + Y^N}l_t^N \tag{90}$$

For the foreign country:

$$\tilde{y}_t^{T^*} = (1-\gamma^*)\tilde{c}_t^{T^*} + \gamma^*\tilde{g}_t^{T^*} \tag{91}$$

$$\tilde{y}_t^{N^*} = (1-\gamma^*)\tilde{c}_t^{N^*} + \gamma^*\tilde{g}_t^{N^*} \tag{92}$$

$$\widetilde{y}_t^{f^*} = \gamma_x^*\widetilde{x}_t^{f^*} + \left(1-\gamma_x^*\right)\widetilde{x}_t^{f} \tag{93}$$

$$\widetilde{y}_t^* = \gamma_c^*(t_t^{T^*} + \widetilde{y}_t^{T^*}) + \left(1-\gamma_c^*\right)(t_t^{N^*} + \widetilde{y}_t^{N^*}) \tag{94}$$

$$l_t = \frac{Y^{f^*}}{Y^h + Y^N}l_t^{f^*} + \frac{Y^{N^*}}{Y^{f^*} + Y^{N^*}}l_t^{N*} \tag{95}$$

Mapping variables in the model with observable variables.

$$\widetilde{c}_t - \widetilde{c}_{t-1} = \triangle c_t - \varepsilon_t^a \tag{96}$$

$$\widetilde{c}_t^* - \widetilde{c}_{t-1}^* = \triangle c_t^* - \varepsilon_t^a \tag{97}$$

$$\widetilde{y}_t - \widetilde{y}_{t-1} = \triangle y_t - \varepsilon_t^a \tag{98}$$

$$\widetilde{y}_t^* - \widetilde{y}_{t-1}^* = \triangle y_t^* - \varepsilon_t^a \tag{99}$$

Shocks

$$\psi_t = \rho_\psi \psi_{t-1} + \varepsilon_t^\psi$$

$$\psi_t^* = \rho_{\psi^*} \psi_{t-1}^* + \varepsilon_t^{\psi^*}$$

$$z_t^h = \rho^{Z,h} z_{t-1}^h + \varepsilon_t^{Z,h}$$

$$z_t^{f^*} = \rho^{Z,f^*} z_{t-1}^{f^*} + \varepsilon_t^{Z,f^*}$$

$$z_t^N = \rho^{Z,N} z_{t-1}^N + \varepsilon_t^{Z,N}$$

$$z_t^{N^*} = \rho^{Z,N^*} z_{t-1}^{N^*} + \varepsilon_t^{Z,N^*}$$

$$g_t^T = \rho^{G,T} g_{t-1}^T + \varepsilon_t^{G,T}$$

$$g_t^N = \rho^{G,N} g_{t-1}^N + \varepsilon_t^{G,N}$$

$$g_t^{T^*} = \rho^{G,T^*} g_{t-1}^{T^*} + \varepsilon_t^{G,T^*}$$

$$g_t^{N^*} = \rho^{G,N^*} g_{t-1}^{N^*} + \varepsilon_t^{G,N^*}$$

and $\varepsilon_t^a, \varepsilon_t^r, \varepsilon_t^{r^*}$ are iid shocks.

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