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*Research Article*

# Norm and Essential Norm of an Integral-Type Operator from the Dirichlet Space to the Bloch-Type Space on the Unit Ball

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Operator norm and essential norm of an integral-type operator, recently introduced by this author, from the Dirichlet space to the Bloch-type space on the unit ball in $\mathbb{C}^n$ are calculated here.

## 1. Introduction

Let $\mathbb{B}^n = \mathbb{B}$ be the open unit ball in $\mathbb{C}^n$, $\mathbb{B}^1 = \mathbb{D}$ the open unit disk in $\mathbb{C}$, $H(\mathbb{B})$ the class of all holomorphic functions on $\mathbb{B}$, and $H^\infty(\mathbb{B})$, the space consisting of all $f \in H(\mathbb{B})$ such that $\|f\|_\infty = \sup_{z\in\mathbb{B}}|f(z)| < \infty$.

For an $f \in H(\mathbb{B})$ with the Taylor expansion $f(z) = \sum_\alpha a_\alpha z^\alpha$, let

$$\Re f(z) = \sum_\alpha |\alpha| a_\alpha z^\alpha \tag{1.1}$$

be the radial derivative of $f$, where $\alpha = (\alpha_1, \ldots, \alpha_n)$ is a multi-index, $|\alpha| = \alpha_1 + \cdots + \alpha_n$ and $z^\alpha = z_1^{\alpha_1} \cdots z_n^{\alpha_n}$. Let $\alpha! = \alpha_1! \cdots \alpha_n!$.

The Dirichlet space $\mathfrak{D}^2(\mathbb{B}) = \mathfrak{D}^2$ contains all $f(z) = \sum_\alpha a_\alpha z^\alpha \in H(\mathbb{B})$, such that

$$\|f\|_{\mathfrak{D}^2}^2 := |f(0)|^2 + \sum_\alpha |\alpha| \frac{\alpha!}{|\alpha|!} |a_\alpha|^2 < \infty. \tag{1.2}$$

The quantity $\|f\|_{\mathfrak{D}^2}$ is a norm on $\mathfrak{D}^2$ which for $n = 1$ is equal to usual norm

$$\|f\|_{\mathfrak{D}^2(\mathbb{D})} = \left( |f(0)|^2 + \int_{\mathbb{D}} |f'(z)|^2 dA(z) \right)^{1/2}, \tag{1.3}$$

where $dA(z) = (1/\pi) r\, dr\, d\theta$ is the normalized area measure on $\mathbb{D}$.

The inner product, between two functions

$$f(z) = \sum_{\alpha} a_\alpha z^\alpha, \qquad g(z) = \sum_{\alpha} b_\alpha z^\alpha, \tag{1.4}$$

on $\mathfrak{D}^2$ is defined by

$$\langle f, g \rangle := f(0)\overline{g(0)} + \sum_{\alpha} |\alpha| \frac{\alpha!}{|\alpha|!} a_\alpha \overline{b}_\alpha. \tag{1.5}$$

For $\alpha \neq 0$, let

$$e_\alpha(z) = \sqrt{\frac{|\alpha|!}{|\alpha|\alpha!}} z^\alpha, \quad z \in \mathbb{B}, \tag{1.6}$$

and $e_0(z) \equiv 1$, then it is easy to see that the family $\{e_\alpha\}$ is an orthonormal basis for $\mathfrak{D}^2$, and hence the reproducing kernel $K_w(z)$ for $\mathfrak{D}^2$ is given by ([1]) as follows:

$$K_w(z) = 1 + \sum_{\alpha \neq 0} \frac{|\alpha|!}{|\alpha|\alpha!} z^\alpha \overline{w}^\alpha = 1 + \ln \frac{1}{1 - \langle z, w \rangle}, \tag{1.7}$$

where $\langle z, w \rangle = \sum_{j=1}^{n} z_j \overline{w}_j$ is the inner product in $\mathbb{C}^n$.

Clearly for each $f \in \mathfrak{D}^2$ and $w \in \mathbb{B}$, the next reproducing formula holds:

$$f(w) = \langle f, K_w \rangle. \tag{1.8}$$

Note that for $f = K_w$ from (1.8), we obtain

$$K_w(w) = \|K_w\|_{\mathfrak{D}^2}^2 = \ln \frac{e}{1 - |w|^2}. \tag{1.9}$$

Also, by the Cauchy-Schwarz inequality and (1.9), we have that, for each $f \in \mathfrak{D}^2$ and $w \in \mathbb{B}$,

$$|f(w)| = |\langle f, K_w \rangle| \le \|f\|_{\mathfrak{D}^2} \|K_w\|_{\mathfrak{D}^2} = \|f\|_{\mathfrak{D}^2} \left( \ln \frac{e}{1 - |w|^2} \right)^{1/2}. \tag{1.10}$$

Note that inequality (1.10) is exact since it is attained for $f = K_w$.

The weighted-type space $H_\mu^\infty(\mathbb{B}) = H_\mu^\infty$ ([2, 3]) consists of all $f \in H(\mathbb{B})$ such that

$$\|f\|_{H_\mu^\infty} := \sup_{z\in\mathbb{B}} \mu(z)\left|f(z)\right| < \infty, \tag{1.11}$$

where $\mu$ is a positive continuous function on $\mathbb{B}$ (*weight*).

The Bloch-type space $\mathcal{B}_\mu(\mathbb{B}) = \mathcal{B}_\mu$ consists of all $f \in H(\mathbb{B})$ such that

$$\|f\|_{\mathcal{B}_\mu} := \left|f(0)\right| + \sup_{z\in\mathbb{B}} \mu(z)\left|\Re f(z)\right| < \infty, \tag{1.12}$$

where $\mu$ is a weight.

Let $g \in H(\mathbb{D})$, $g(0) = 0$, and $\varphi$ be a holomorphic self-map of $\mathbb{B}$, then the following integral-type operator:

$$P_\varphi^g(f)(z) = \int_0^1 f\left(\varphi(tz)\right)g(tz)\frac{dt}{t}, \quad z \in \mathbb{B},\ f \in H(\mathbb{B}) \tag{1.13}$$

has been recently introduced in [4] and considerably studied (see, e.g., [5–8] and the related references therein). For some related operators, see also [9–16] and the references therein.

Usual problem in this research area is to provide function-theoretic characterizations for when $\varphi$ and $g$ induce bounded or compact integral-type operator on spaces of holomorphic functions. Majority of papers only find asymptotics of operator norm of linear operators. Somewhat concrete but perhaps more interesting problem is to calculate operator norm of these operators between spaces of holomorphic functions on various domains. Some results on this problem can be found, for example, in [3, 17–26] (for related results see also [7, 27–32]). Having published paper [3], we started with systematic investigation of methods for calculating operator norms of concrete operators between spaces of holomorphic function.

Here, we calculate the operator norm as well as the essential norm of the operator $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$, considerably extending our recent result in note [33].

## 2. Auxiliary Results

In this section, we quote several auxiliary results which are used in the proofs of the main results.

**Lemma 2.1** (see [4]). *Let $g \in H(\mathbb{B})$, $g(0) = 0$, and $\varphi$ be a holomorphic self-map of $\mathbb{B}$, then*

$$\Re P_\varphi^g(f)(z) = g(z)f\left(\varphi(z)\right). \tag{2.1}$$

The next Schwartz-type lemma ([34]) can be proved in a standard way. Hence, we omit its proof.

**Lemma 2.2.** *Assume that $g \in H(\mathbb{B})$, $g(0) = 0$, $\mu$ is a weight, and $\varphi$ is an analytic self-map of $\mathbb{B}$, then $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$ is compact if and only if $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$ is bounded and for any bounded sequence $(f_k)_{k\in\mathbb{N}}$ in $\mathfrak{D}^2$ converging to zero uniformly on compacts of $\mathbb{B}$ as $k \to \infty$, one has*

$$\lim_{k\to\infty} \|P_\varphi^g(f_k)\|_{\mathcal{B}_\mu} = 0. \tag{2.2}$$

**Lemma 2.3.** *Assume that $g \in H(\mathbb{B})$, $g(0) = 0$, $\mu$ is a weight, $\varphi$ is an analytic self-map of $\mathbb{B}$ such that $\|\varphi\|_\infty < 1$, and the operator $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$ is bounded, then $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$ is compact.*

*Proof.* First note that since $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$ is bounded and $f_0(z) \equiv 1 \in \mathfrak{D}^2$ by Lemma 2.1, it follows that $\Re P_\varphi^g(f_0) = g \in H_\mu^\infty$. Now, assume that $(f_k)_{k\in\mathbb{N}}$ is a bounded sequence in $\mathfrak{D}^2$ converging to zero on compacts of $\mathbb{B}$ as $k \to \infty$, then we have

$$\|P_\varphi^g(f_k)\|_{\mathcal{B}_\mu} \le \|g\|_{H_\mu^\infty} \sup_{w\in\varphi(\mathbb{B})} |f_k(w)| \longrightarrow 0, \tag{2.3}$$

as $k \to \infty$, since $\varphi(\mathbb{B})$ is contained in the ball $|w| \le \|\varphi\|_\infty$ which is a compact subset of $\mathbb{B}$, according to the assumption $\|\varphi\|_\infty < 1$. Hence, by Lemma 2.2, the operator $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$ is compact. □

## 3. Operator Norm of $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$

In this section, we calculate the operator norm of $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$.

**Theorem 3.1.** *Assume that $g \in H(\mathbb{B})$, $g(0) = 0$, and $\varphi$ is a holomorphic self-map of $\mathbb{B}$, then*

$$\|P_\varphi^g\|_{\mathfrak{D}^2\to\mathcal{B}_\mu} = \sup_{z\in\mathbb{B}} \mu(z)|g(z)| \left( \ln \frac{e}{1-|\varphi(z)|^2} \right)^{1/2} =: L. \tag{3.1}$$

*Proof.* Using Lemma 2.1, reproducing formula (1.8), the Cauchy-Schwarz inequality, and finally (1.9), we get that, for each $f \in \mathfrak{D}^2$ and $w \in \mathbb{B}$,

$$\begin{aligned} \mu(w)\left|\Re P_\varphi^g(w)\right| &= \mu(w)|g(w)||f(\varphi(w))| \\ &= \mu(w)|g(w)||\langle f, K_{\varphi(w)}\rangle| \\ &\le \mu(w)|g(w)|\|f\|_{\mathfrak{D}^2}\|K_{\varphi(w)}\|_{\mathfrak{D}^2} \\ &= \|f\|_{\mathfrak{D}^2}\mu(w)|g(w)| \left( \ln \frac{e}{1-|\varphi(w)|^2} \right)^{1/2}. \end{aligned} \tag{3.2}$$

Taking the supremum in (3.2) over $w \in \mathbb{B}$ as well as the supremum over the unit ball in $\mathfrak{D}^2$ and using the fact $P_\varphi^g(f)(0) = 0$, for each $f \in H(\mathbb{B})$, which follows from the assumption $g(0) = 0$, we get

$$\|P_\varphi^g\|_{\mathfrak{D}^2 \to \mathcal{B}_\mu} \leq L. \tag{3.3}$$

Now assume that the operator $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$ is bounded. From (1.9), we obtain that, for each $w \in \mathbb{B}$,

$$\begin{aligned}\left(\ln \frac{e}{1 - |\varphi(w)|^2}\right)^{1/2} \|P_\varphi^g\|_{\mathfrak{D}^2 \to \mathcal{B}_\mu} &= \|K_{\varphi(w)}\|_{\mathfrak{D}^2} \|P_\varphi^g\|_{\mathfrak{D}^2 \to \mathcal{B}_\mu} \\ &\geq \|P_\varphi^g(K_{\varphi(w)})\|_{\mathcal{B}_\mu} \\ &= \sup_{z \in \mathbb{B}} \mu(z)|g(z)||K_{\varphi(w)}(\varphi(z))| \\ &\geq \mu(w)|g(w)||K_{\varphi(w)}(\varphi(w))|.\end{aligned} \tag{3.4}$$

From (1.9) and (3.4), it follows that

$$L \leq \|P_\varphi^g\|_{\mathfrak{D}^2 \to \mathcal{B}_\mu}. \tag{3.5}$$

Hence, if $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$ is bounded, then from (3.3) and (3.5) we obtain (3.1).

In the case $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$ is unbounded, the result follows from inequality (3.3). □

## 4. Essential Norm of $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$

Let $X$ and $Y$ be Banach spaces, and let $L : X \to Y$ be a bounded linear operator. The essential norm of the operator $L : X \to Y$, $\|L\|_{e,X \to Y}$, is defined as follows:

$$\|L\|_{e,X \to Y} = \inf\{\|L + K\|_{X \to Y} : K \text{ is compact from } X \text{ to } Y\}, \tag{4.1}$$

where $\|\cdot\|_{X \to Y}$ denote the operator norm.

From this and since the set of all compact operators is a closed subset of the set of bounded operators, it follows that $L$ is compact if and only if

$$\|L\|_{e,X \to Y} = 0. \tag{4.2}$$

Here, we calculate the essential norm of the operator $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$.

**Theorem 4.1.** *Assume that $g \in H(\mathbb{B})$, $g(0) = 0$, $\mu$ is a weight, $\varphi$ is a holomorphic self-map of $\mathbb{B}$, and $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$ is bounded. If $\|\varphi\|_\infty < 1$, then $\|P_\varphi^g\|_{e,\mathfrak{D}^2 \to \mathcal{B}_\mu} = 0$, and if $\|\varphi\|_\infty = 1$, then*

$$\|P_\varphi^g\|_{e,\mathfrak{D}^2 \to \mathcal{B}_\mu} = \limsup_{|\varphi(z)| \to 1} \mu(z)\left|g(z)\right|\left(\ln \frac{e}{1 - \left|\varphi(z)\right|^2}\right)^{1/2}. \tag{4.3}$$

*Proof.* Since $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$ is bounded, for the test function $f(z) \equiv 1$, we get $g \in H_\mu^\infty$. If $\|\varphi\|_\infty < 1$, then from Lemma 2.3 it follows that $P_\varphi^g : \mathfrak{D}^2 \to \mathcal{B}_\mu$ is compact which is equivalent with $\|P_\varphi^g\|_{e,\mathfrak{D}^2 \to \mathcal{B}_\mu} = 0$. On the other hand, it is clear that in this case the condition $|\varphi(z)| \to 1$ is vacuous, so that (4.3) is vacuously satisfied.

Now assume that $\|\varphi\|_\infty = 1$, and that $(\varphi(z_k))_{k\in\mathbb{N}}$ is a sequence in $\mathbb{B}$ such that $|\varphi(z_k)| \to 1$ as $k \to \infty$. For $w \in \mathbb{B}$ fixed, set

$$f_w(z) = \frac{\ln(e/(1 - \langle z, w\rangle))}{\left(\ln\left(e/\left(1 - |w|^2\right)\right)\right)^{1/2}}, \quad z \in \mathbb{B}. \tag{4.4}$$

By (1.9), we have that $\|f_w\|_{\mathfrak{D}^2} = 1$, for each $w \in \mathbb{B}$. Hence, the sequence $(f_{\varphi(z_k)})_{k\in\mathbb{N}}$ is such that $\|f_{\varphi(z_k)}\|_{\mathfrak{D}^2} = 1$, for each $k \in \mathbb{N}$, and clearly it converges to zero uniformly on compacts of $\mathbb{B}$. From this and by [35, Theorems 6.19], it easily follows that $f_{\varphi(z_k)} \to 0$ weakly in $\mathfrak{D}^2$, as $k \to \infty$. Hence, for every compact operator $K : \mathfrak{D}^2 \to \mathcal{B}_\mu$, we have that

$$\lim_{k\to\infty} \|K f_{\varphi(z_k)}\|_{\mathcal{B}_\mu} = 0. \tag{4.5}$$

Thus, for every such sequence and for every compact operator $K : \mathfrak{D}^2 \to \mathcal{B}_\mu$, we have that

$$\begin{aligned}
\|P_\varphi^g + K\|_{\mathfrak{D}^2 \to \mathcal{B}_\mu} &\ge \limsup_{k\to\infty} \frac{\|P_\varphi^g f_{\varphi(z_k)}\|_{\mathcal{B}_\mu} - \|K f_{\varphi(z_k)}\|_{\mathcal{B}_\mu}}{\|f_{\varphi(z_k)}\|_{\mathfrak{D}^2}} \\
&= \limsup_{k\to\infty} \|P_\varphi^g f_{\varphi(z_k)}\|_{\mathcal{B}_\mu} \\
&\ge \limsup_{k\to\infty} \mu(z_k)\left|g(z_k) f_{\varphi(z_k)}(\varphi(z_k))\right| \\
&= \limsup_{n\to\infty} \mu(z_k)\left|g(z_k)\right|\left(\ln \frac{e}{1 - \left|\varphi(z_k)\right|^2}\right)^{1/2}.
\end{aligned} \tag{4.6}$$

Taking the infimum in (4.6) over the set of all compact operators $K : \mathfrak{D}^2 \to \mathcal{B}_\mu$, we obtain

$$\|P_\varphi^g\|_{e,\mathfrak{D}^2 \to \mathcal{B}_\mu} \ge \limsup_{n\to\infty} \mu(z_k)\left|g(z_k)\right|\left(\ln \frac{e}{1 - \left|\varphi(z_k)\right|^2}\right)^{1/2}, \tag{4.7}$$

from which an inequality in (4.3) follows.

In the sequel, we prove the reverse inequality. Assume that $(r_l)_{l\in\mathbb{N}}$ is a sequence of positive numbers which increasingly converges to 1. Consider the operators defined by

$$\left(P^g_{r_l\varphi}f\right)(z)=\int_0^1 f\left(r_l\varphi(tz)\right)g(tz)\frac{dt}{t},\quad l\in\mathbb{N}. \tag{4.8}$$

Since $\|r_l\varphi\|_\infty<1$, by Lemma 2.3, we have that these operators are compact.

Since $P^g_\varphi:\mathfrak{D}^2\to\mathcal{B}_\mu$ is bounded, then $g\in H^\infty_\mu$. Let $\rho\in(0,1)$ be fixed for a moment. By Lemma 2.1, we get

$$\begin{aligned}\|P^g_\varphi-P^g_{r_l\varphi}\|_{\mathfrak{D}^2\to\mathcal{B}_\mu}&=\sup_{\|f\|_{\mathfrak{D}^2}\le1}\sup_{z\in\mathbb{B}}\mu(z)\left|g(z)\right|\left|f\left(\varphi(z)\right)-f\left(r_l\varphi(z)\right)\right|\\&\le\sup_{\|f\|_{\mathfrak{D}^2}\le1}\sup_{|\varphi(z)|\le\rho}\mu(z)\left|g(z)\right|\left|f\left(\varphi(z)\right)-f\left(r_l\varphi(z)\right)\right|\\&\quad+\sup_{\|f\|_{\mathfrak{D}^2}\le1}\sup_{|\varphi(z)|>\rho}\mu(z)\left|g(z)\right|\left|f\left(\varphi(z)\right)-f\left(r_l\varphi(z)\right)\right|\end{aligned}\tag{4.9}$$

$$\begin{aligned}&\le\|g\|_{H^\infty_\mu}\sup_{\|f\|_{\mathfrak{D}^2}\le1}\sup_{|\varphi(z)|\le\rho}\left|f\left(\varphi(z)\right)-f\left(r_l\varphi(z)\right)\right|\\&\quad+\sup_{\|f\|_{\mathfrak{D}^2}\le1}\sup_{|\varphi(z)|>\rho}\mu(z)\left|g(z)\right|\left|f\left(\varphi(z)\right)-f\left(r_l\varphi(z)\right)\right|.\end{aligned}\tag{4.10}$$

Further, we have

$$\|f-f_r\|^2_{\mathfrak{D}^2}=\sum_\alpha|\alpha|\frac{\alpha!}{|\alpha|!}|a_\alpha|^2\left(1-r^{|\alpha|}\right)^2\le\sum_\alpha|\alpha|\frac{\alpha!}{|\alpha|!}|a_\alpha|^2\le\|f\|_{\mathfrak{D}^2}. \tag{4.11}$$

From (1.10), (4.11), and the fact $f(z)-f(rz)\in\mathfrak{D}^2$, we obtain

$$\left|f(z)-f(rz)\right|\le\|f\|_{\mathfrak{D}^2}\left(\ln\frac{e}{1-|z|^2}\right)^{1/2}, \tag{4.12}$$

in particular

$$\left|f\left(\varphi(z)\right)-f\left(r_l\varphi(z)\right)\right|\le\|f\|_{\mathfrak{D}^2}\left(\ln\frac{e}{1-\left|\varphi(z)\right|^2}\right)^{1/2}. \tag{4.13}$$

Let

$$I_l:=\sup_{\|f\|_{\mathfrak{D}^2}\le1}\sup_{|\varphi(z)|\le\rho}\left|f\left(\varphi(z)\right)-f\left(r_l\varphi(z)\right)\right|. \tag{4.14}$$

The mean value theorem along with the subharmonicity of the moduli of partial derivatives of $f$, well-known estimates among the partial derivatives of analytic functions, Theorem 6.2, and Proposition 6.2 in [35], yield

$$\begin{aligned}
I_l &\leq \sup_{\|f\|_{\mathfrak{D}^2}\leq 1}\ \sup_{|\varphi(z)|\leq\rho}(1-r_l)\left|\varphi(z)\right|\sup_{|w|\leq\rho}\left|\nabla f(w)\right| \\
&\leq C_\rho(1-r_l)\sup_{\|f\|_{\mathfrak{D}^2}\leq 1}\left(\sum_{j=1}^{[n/p]}\left|\nabla^j f(0)\right| + \sup_{|w|\leq(1+\rho)/2}\left|\nabla^{[n/p]+1}f(w)\right|\right) \\
&\leq C_\rho(1-r_l)\sup_{\|f\|_{\mathfrak{D}^2}\leq 1}\left(\sum_{j=1}^{[n/p]}\left|\nabla^j f(0)\right|\right. \\
&\qquad\qquad\left. +\left(\int_{|w|\leq(3+\rho)/4}\left|\nabla^{[n/p]+1}f(w)\right|^2\left(1-|w|^2\right)^{2([n/p]+1)}d\tau(w)\right)^{1/2}\right) \\
&\leq C_\rho(1-r_l)\sup_{\|f\|_{\mathfrak{D}^2}\leq 1}\left(\sum_{j=1}^{[n/p]}\left|\nabla^j f(0)\right| + \int_{\mathbb{B}}\left|\nabla^{[n/p]+1}f(w)\right|^2\left(1-|w|^2\right)^{2([n/p]+1)}d\tau(w)\right)^{1/2} \\
&\leq C_\rho(1-r_l)\longrightarrow 0,\quad \text{as } l\longrightarrow\infty,
\end{aligned}\tag{4.15}$$

where $d\tau(z) = dV(z)/(1-|z|^2)^{n+1}$ and $dV(z)$ is the Lebesgue volume measure on $\mathbb{B}$.

Using (4.13) in (4.10), letting $l \to \infty$ in (4.9), using (4.15), and then letting $\rho \to 1$, the reverse inequality follows, finishing the proof of the theorem. □

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