Normal freezing and thawing degree-days from normal monthly temperatures
Boyd, D. W.

Canadian Geotechnical Journal, 13, 2, pp. 176-180, 1976-05

NATIONAL RESEARCH COUNCIL OF CANADA

CONSEIL NATIONAL DE RECHERCHES DU CANADA

NORMAL FREEZING AND THAWING DEGREE-DAYS FROM NORMAL MONTHLY TEMPERATURES

BY

D. W. BOYD

Reprinted from

CANADIAN GEOTECHNICAL JOURNAL

Vol. 13, No. 2, May 1976

p. 176–180

Research Paper No. 675

OF THE

DIVISION OF BUILDING RESEARCH

Price 10 cents OTTAWA NRCC 15245

Normal freezing and thawing degree-days from normal monthly temperatures

DONALD W. BOYD[1]

Division of Building Research, National Research Council of Canada, Ottawa, Canada K1A 0R6

Received October 1, 1975

Accepted February 11, 1976

A method has been devised for using normal monthly mean temperatures to compute normal freezing and thawing degree-days in change-over months with adequate accuracy. The tedious calculation of daily departures from freezing for 30 years can, therefore, be avoided. The method has been applied to over 800 Canadian stations and the resulting freezing and thawing indices used to prepare the charts accompanying this note.

Une méthode a été établie pour permettre le calcul, à partir des températures moyennes mensuelles normales, des degrés jours de gel ou de dégel durant les mois de transition, avec une précision convenable. Le calcul fastidieux des occurences de dégel quotidiens sur une période de 30 ans peut donc être évité. La méthode a été appliquée à plus de 800 stations canadiennes et les indices de gel et dégel résultants ont servi à préparer les graphiques inclus dans la présente note.

[Traduit par la revue]

Can. Geotech. J., **13**, 176 (1976)

The depth of freezing of the ground and the thickness of ice on lakes must often be estimated from air temperature observations. Many formulas are available for computing frost penetration in the ground, but some require a knowledge of the thermal properties. Brown (1964) has reviewed the work of several authorities and studied the difficulties associated with predicting depth of freeze or thaw. If the ground surface is free of vegetation and

[1]Seconded to Division of Building Research, National Research Council of Canada, from the Atmospheric Environment Service, Department of the Environment.

snow cover, Brown concludes that a moderately good estimate of frost penetration in the ground can be made using the accumulated degree-days of air temperature below freezing.

This note is concerned only with providing the required degree-days below freezing in a year with normal or average temperatures. In particular a method of computing the degree-days below and above freezing in change-over months is described and the results of its application to Canadian data are presented.

The degree-day is defined as a measure of the departure of the daily mean temperature from a given standard (Huschke 1959). These departures can be accumulated and used as an index of the effect of the temperature over a period of time. Negative departures (or positive departures, if one is primarily concerned with the negative ones) are treated in two different ways: they may be considered as zero, that is contributing nothing to the accumulated total, or they may be considered as having a negative value which reduces the accumulated total. Heating and growing degree-days are calculated using the first method; freezing and thawing degree-days the second.

If the departures from the freezing point are accumulated on a daily basis for several years and the accumulations plotted against time, then the resulting graph will have a minimum each spring and a maximum each autumn. The freezing index is defined as the number of degree-days between the maximum in the autumn and the minimum the next spring. Similarly, the thawing index is the number of degree-days between the minimum in the spring and the maximum the next autumn (Huschke 1959). Any spring or autumn month that includes a seasonal maximum or minimum will be called a 'change-over' month. In all other months the accumulation of degree-days above freezing is given exactly by:

[1] $$N\ (t - 32) \text{ or } NX$$

where t is the average temperature in a month of N days, and X is the difference $t - 32$. In a summer month when X is positive the freezing degree-days are zero and the thawing degree-days (T) are given by:

[2] $$T = NX$$

In a winter month when X is negative the thawing degree-days are zero and the freezing degree-days (F) are given by:

[3] $$F = -NX$$

In change-over months the situation is more complex. Markham (1960) drew up tables of freezing and thawing degree-days in change-over months based on monthly mean temperatures at coastal stations. Boyd (1975) proposed some refinements in Markham's method, including the use of an equation instead of tables read from hand-drawn curves. The proposed equation was a hyperbola of the form:

[4] $$Y^2 - NXY = N^2k^2$$

where k is a constant. The solution of this equation yields two values for Y, one positive and the other negative. The positive root is the thawing degree-days (T), and the absolute value of the negative root is the freezing degree-days (F).

Daily freezing or thawing degree-days and their accumulated values were available for 10 years for a limited number of stations. The pertinent values in change-over months were abstracted for 22 stations well distributed across Canada, and used to determine the value of k giving the minimum residual variance of the observed data. The equation for minimum residual variance cannot be solved explicitly for k and the lengthy trial and error calculations had to be done on a digital computer. With k equal to 2.5 °F (1.4 °C) the residual variance was 1.124 corresponding to a standard error of estimate of 1.06 degree-days per day or 32 or 33 degree-days per month, for individual months.

The error of estimate for normal degree-days based on 30 year normal monthly mean temperature would be about 33 divided by the square root of 30 or about six degree-days in change-over months.

Normal monthly mean temperatures based on the 30 year period 1931–1960 were available for about 880 weather stations in Canada. Equation [4] was used to compute the freezing and thawing degree-days in change-over months. Equations [2] and [3] were used for the other months.

These normal monthly freezing and thawing degree-days and the annual totals (the freezing and thawing indices) have been published by the Atmospheric Environment Service (Boyd 1973).

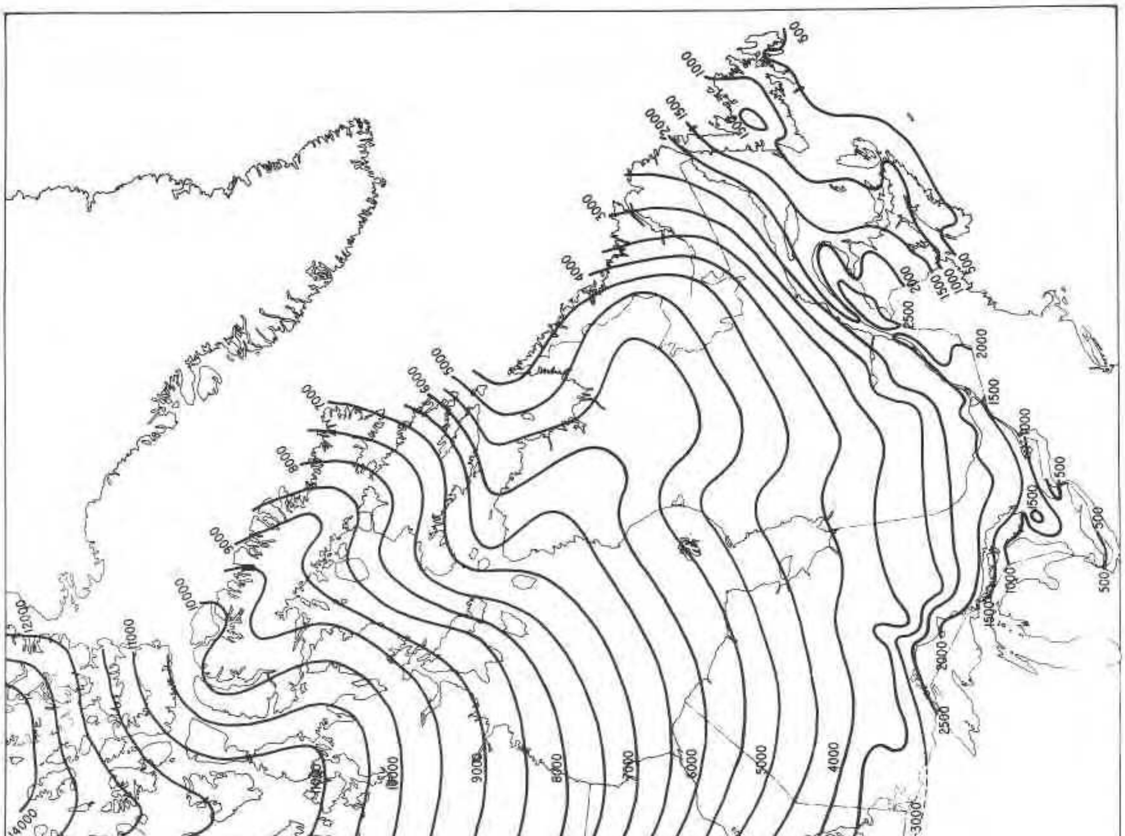

FIG. 1. Normal freezing index in degree-days (based on the period 1931 to 1960).

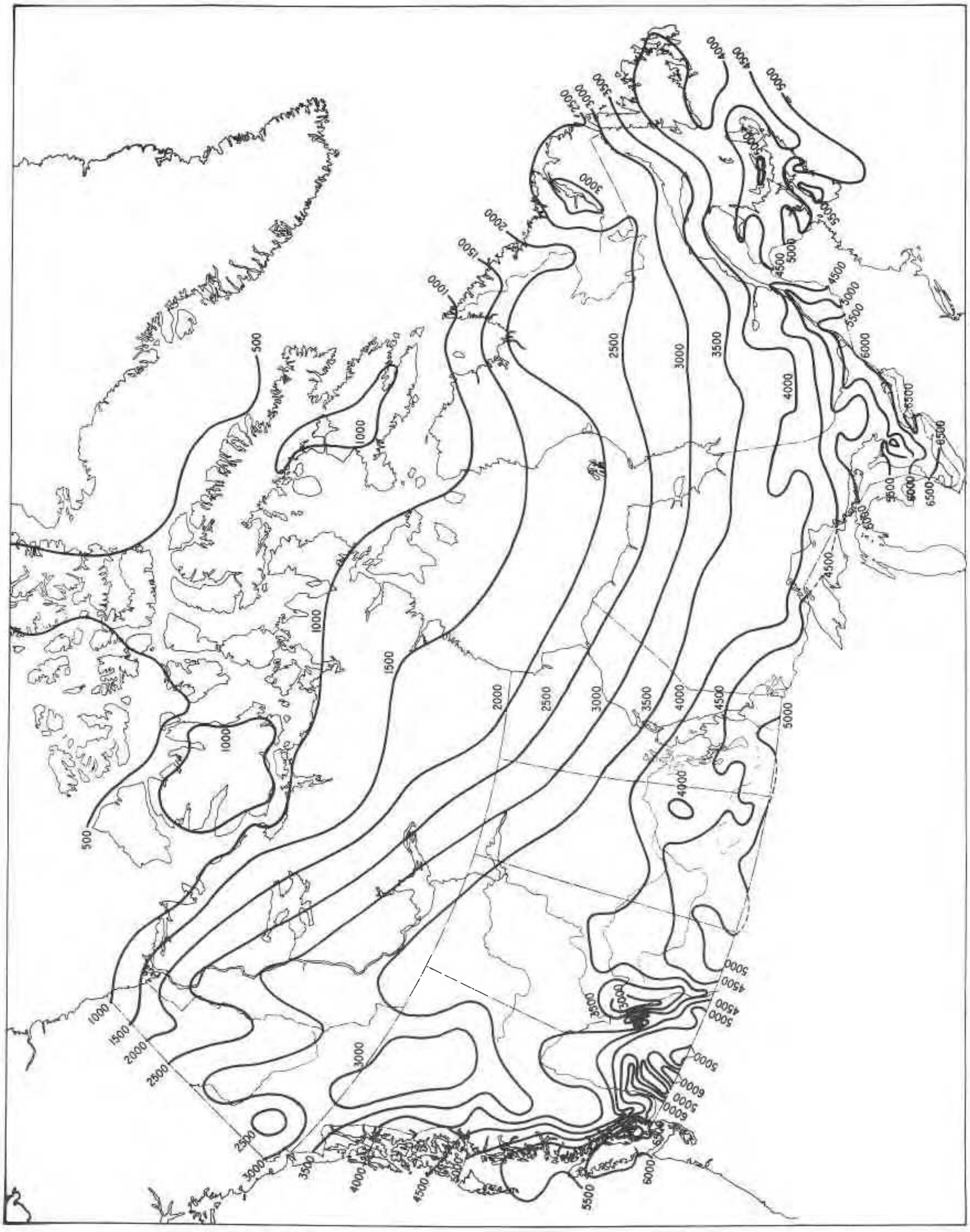

FIG. 2. Normal thawing index in degree-days (based on the period 1931 to 1960).

The normal freezing and thawing indices were plotted on maps of Canada and isopleths drawn to show the general distribution of these indices (Figs. 1 and 2). Charts on this small scale cannot indicate local variations which in some cases are significant. Values for particular locations should be obtained directly from the table in Boyd (1973) or, if not listed, should be estimated from values for nearby stations that are listed.

It was noted previously that the standard deviation of the degree-day values was about six degree-days in change-over months. Rounding-off errors amount to about one degree-day in other months. This results in a standard deviation of about nine degree-days in the annual totals. It should be noted, however, that the normal annual mean temperatures at stations only a few miles apart often differ by about 2 °F (1 °C) this usually makes a difference of 100 to 200 in the freezing and thawing indices. This means that no matter how accurate the indices may be for the weather observing site itself, the probable error at sites within a few miles is at least 100 degree-days.

Conclusion

The relationship between freezing and thawing degree-days per day in change-over months and monthly mean temperatures is well represented by a hyperbola. The dependence of the relationship on the variability of temperature, geographical location and season all appear to be insignificant. The accuracy of the annual values is better than the reliability of observed values over distances of a few miles. The application of this relationship to normal monthly mean temperatures has provided extensive tables of normal freezing and thawing degree-days and the accompanying charts of freezing and thawing indices in Canada.

Acknowledgment

This paper is a contribution from the division of Building Research, National Research Council of Canada, and is published with the approval of the director of the Division.

Boyd, D. W. 1973. Normal freezing and thawing degree-days for Canada 1931–1960. Atmos. Environ. Serv., Downsview, Can., CLI4-73.

——— 1975. Computing freezing and thawing degree-days from monthly temperatures. Div. Build. Res., Nat. Res. Counc. Can., Ottawa, Can. Tech. Pap. No. 444, NRCC 14791.

Brown, W. G. 1964. Difficulties associated with predicting depth of freeze or thaw. Can. Geotech. J. **1**, p. 215–226.

Huschke, R. E. (*Ed.*) 1959. Glossary of meteorology. Am. Meteorol. Soc., Boston, Mass.

Markham, W. E. 1960. Freezing and melting degree-day computations in spring and fall months. Meteorol. Branch, Dep. Transp., Toronto, Can., CIR-3350.