NON-LINEAR CONVOLUTION: A NEW APPROACH FOR THE AURALIZATION OF DISTORTING SYSTEMS

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Goals for Auralization

- Transform the results of objective electroacoustics measurements to audible sound samples suitable for listening tests
- Traditional auralization is based on linear convolution: this does not replicates faithfully the nonlinear behaviour of most transducers
- The new method presented here overcomes to this strong limitation, providing a simplified treatment of memory-less distortion

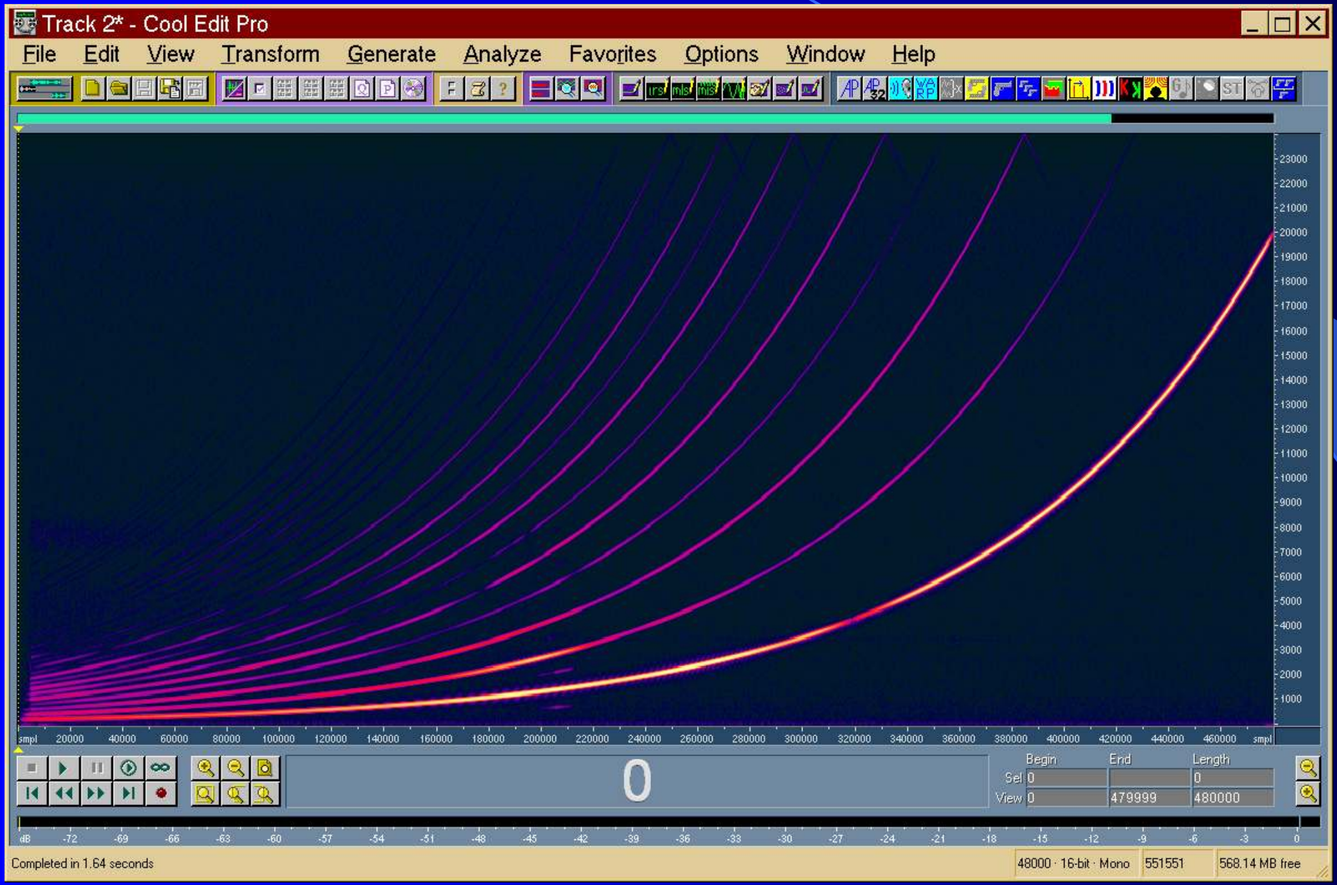
Methods

- We start from a measurement of the system based on exponential sine sweep (Farina, 108th AES, Paris 2000)
- Diagonal Volterra kernels are obtained by post-processing the measurement results
- These kernels are employed as FIR filters in a multiple-order convolution process (original signal, its square, its cube, and so on are convolved separately and the result is summed)

Exponential sweep measurement

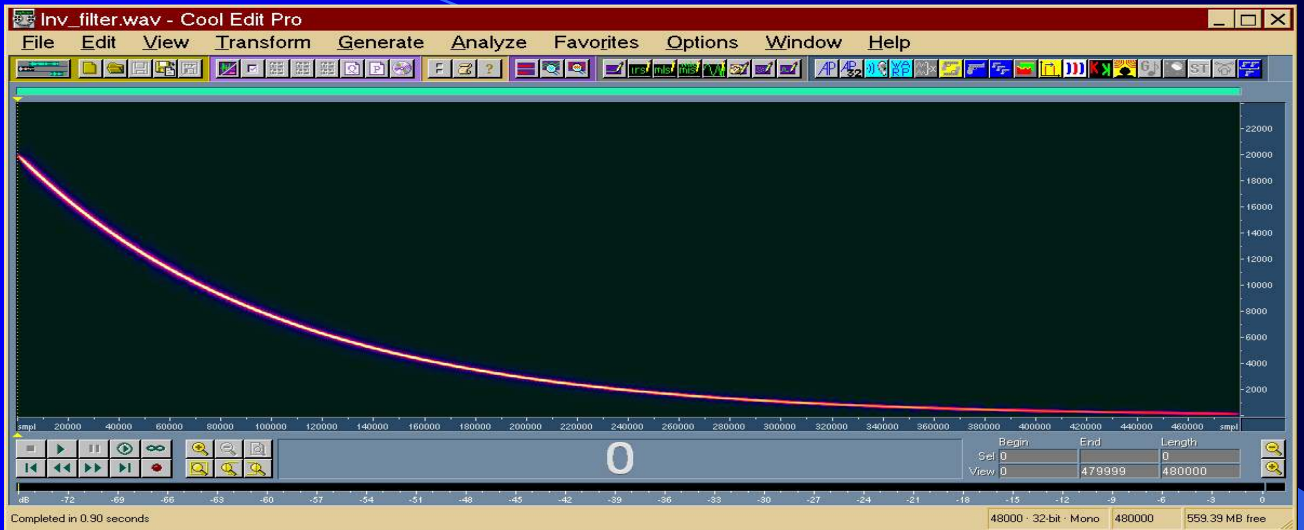
- The excitation signal is a sine sweep with constant amplitude and exponentially-increasing frequency

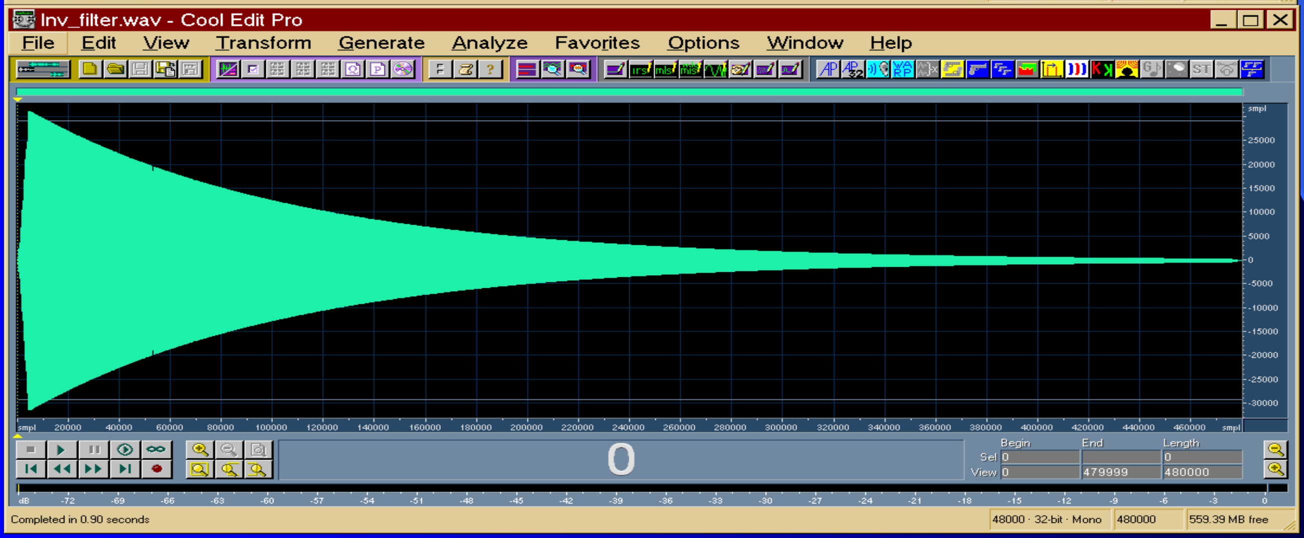
Raw response of the system



Many harmonic orders do appear as colour stripes

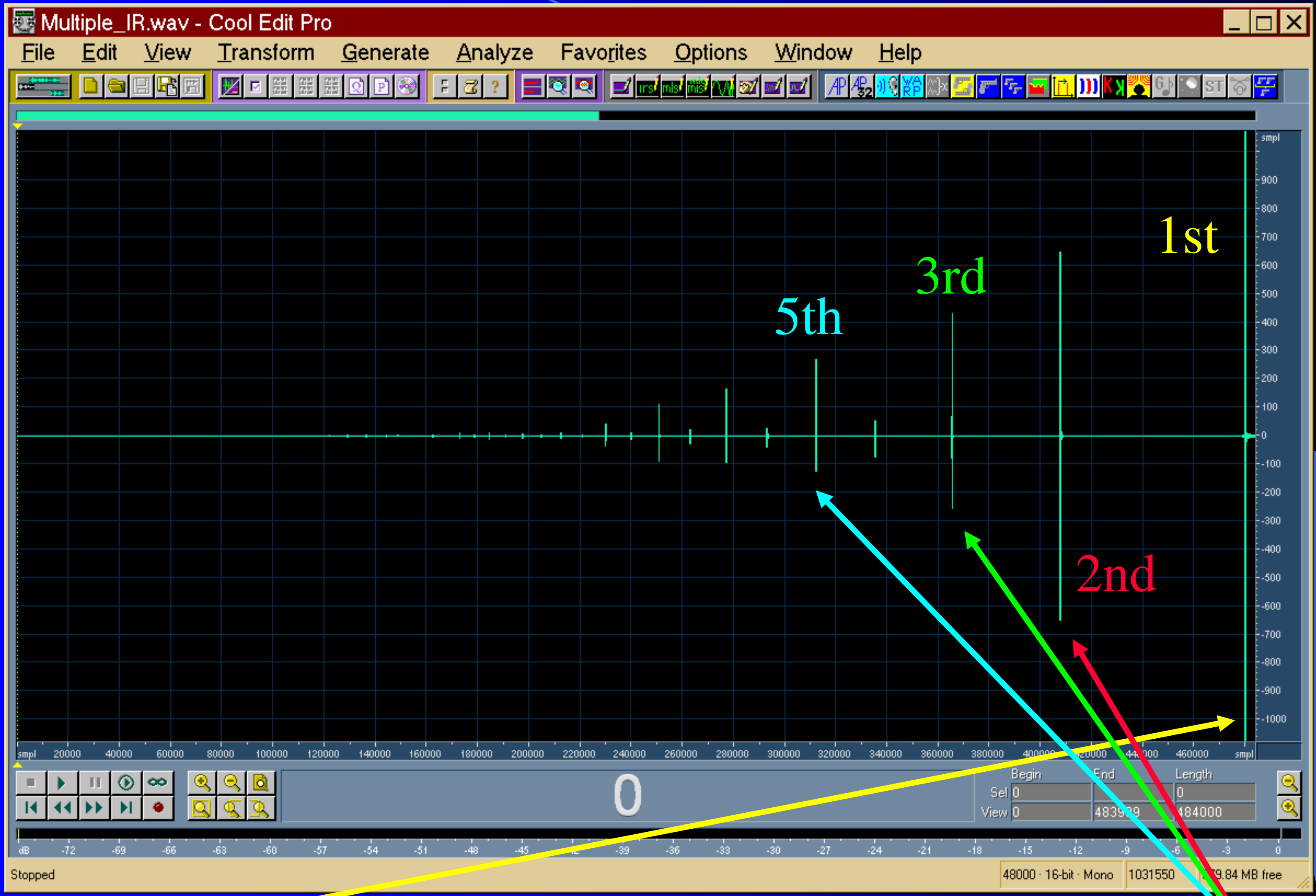
Deconvolution of system's impulse response





The deconvolution is obtained by convolving the raw response with a suitable inverse filter

Multiple impulse response obtained



The last peak is the linear impulse response, the preceding ones are the harmonic distortion orders

Auralization by linear convolution

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Convolving a suitable sound sample with the linear IR, the frequency response and temporal transient effects of the system can be simulated properly

What’s missing in linear convolution ?

- No harmonic distortion, nor other nonlinear effects are being reproduced.
- From a perceptual point of view, the sound is judged “cold” and “innatural”
- A comparative test between a strongly nonlinear device and an almost linear one does not reveal any audible difference, because the nonlinear behavior is removed for both

Theory of nonlinear convolution

- The basic approach is to convolve separately, and then add the result, the linear IR, the second order IR, the third order IR, and so on.
- Each order IR is convolved with the input signal raised at the corresponding power:

$$y(n) = \sum_{i=0}^{M-1} h_1(i)\cdot x(n-i) + \sum_{i=0}^{M-1} h_2(i)\cdot x^2(n-i) + \sum_{i=0}^{M-1} h_3(i)\cdot x^3(n-i) + \ldots..$$

The problem is that the required multiple IRs **are not** the results of the measurements: they are instead the diagonal terms of Volterra kernels

Volterra kernels and simplification

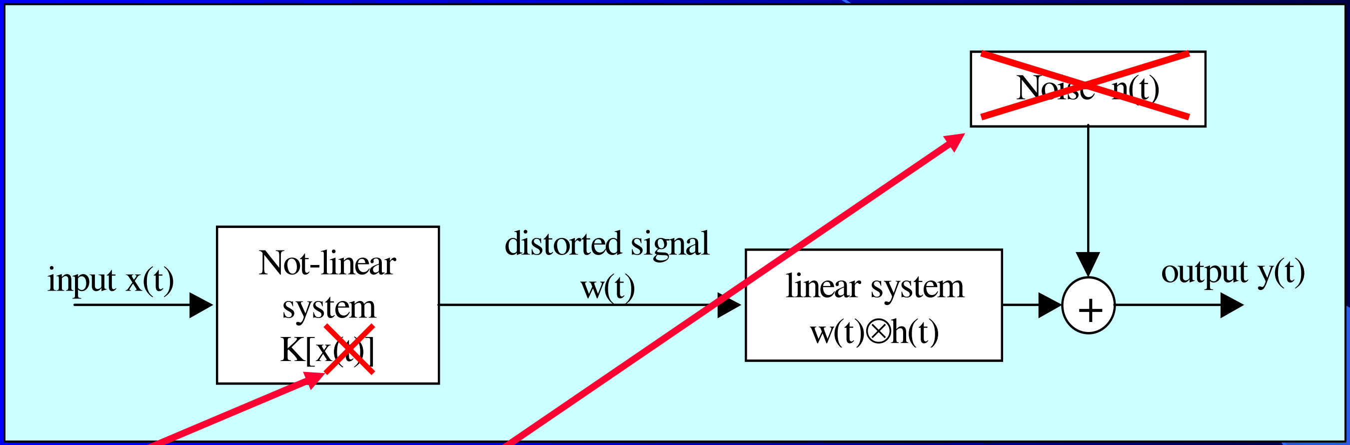
- The general Volterra series expansion is defined as:

$$y(n) = \sum_{i_1=0}^{M-1} h_1(i_1)\cdot x(n-i_1) + \sum_{i_1=0}^{M-1}\sum_{i_2=0}^{M-1} h_2(i_1,i_2)\cdot x(n-i_1)\cdot x(n-i_2) +$$

$$+ \sum_{i_1=0}^{M-1}\sum_{i_2=0}^{M-1}\sum_{i_3=0}^{M-1} h_3(i_1,i_2,i_3)\cdot x(n-i_1)\cdot x(n-i_2)\cdot x(n-i_3) + \ldots..$$

This explains also nonlinear effect with memory, as the system output contains also products of previous sample values with different delays

Memoryless distortion followed by a linear system with memory



- The first nonlinear system is assumed to be memory-less, so only the diagonal terms of the Volterra kernels need to be taken into account.
- Furthermore, we neglect the noise, which is efficiently rejected by the sine sweep measurement method.

Volterra kernels from the measurement results

The measured multiple IRs h' can be defined as:

$$y(t) = h'_1 \otimes \sin[\omega_{var}] + h'_2 \otimes \sin[2 \cdot \omega_{var}] + h'_3 \otimes \sin[3 \cdot \omega_{var}] + \ldots$$

We need to relate them to the simplified Volterra kernels h:

$$y(t) = h_1 \otimes \sin[\omega_{var}] + h_2 \otimes \sin^2[\omega_{var}] + h_3 \otimes \sin^3[\omega_{var}] + \ldots$$

Trigonometry can be used to expand the powers of the sinusoidal terms:

$$\sin^2(\omega \cdot \tau) = \frac{1}{2} - \frac{1}{2} \cdot \cos(2 \cdot \omega \cdot \tau)$$

$$\sin^3(\omega \cdot \tau) = \frac{3}{4} \cdot \sin(\omega \cdot \tau) - \frac{1}{4} \cdot \sin(3 \cdot \omega \cdot \tau)$$

$$\sin^4(\omega \cdot \tau) = \frac{3}{8} - \frac{1}{2} \cdot \cos(2 \cdot \omega \cdot \tau) + \frac{1}{8} \cdot \cos(4 \cdot \omega \cdot \tau)$$

$$\sin^5(\omega \cdot \tau) = \frac{5}{8} \cdot \sin(\omega \cdot \tau) - \frac{5}{16} \cdot \sin(3 \cdot \omega \cdot \tau) + \frac{1}{16} \cdot \sin(5 \cdot \omega \cdot \tau)$$

Finding the connection point

Going to frequency domain by taking the FFT, the first equation becomes:

$$Y(\omega) = \overline{H'_1}[\omega]\cdot X[\omega] + \overline{H'_2}[\omega]\cdot X[\omega/2] + \overline{H'_3}[\omega]\cdot X[\omega/3] + \ldots$$

Doing the same in the second equation, and substituting the trigonometric expressions for power of sines, we get:

$$Y(\omega) = \left[\overline{H_1} + \frac{3}{4}\cdot\overline{H_3} + \frac{5}{8}\cdot\overline{H_5}\right]\cdot X[\omega] + \left[-\frac{1}{2}\cdot\overline{H_2} - \frac{1}{2}\cdot\overline{H_4}\right]\cdot j\cdot X[\omega/2] +$$
$$+\left[-\frac{1}{4}\cdot\overline{H_3} - \frac{5}{16}\cdot\overline{H_5}\right]\cdot X[\omega/3] + \frac{1}{8}\cdot\overline{H_4}\cdot j\cdot X[\omega/4] + \frac{1}{16}\cdot\overline{H_5}\cdot X[\omega/5] + \ldots$$

The terms in square brackets have to be equal to the corresponding measured transfer functions H’ of the first equation

Solution

- Thus we obtain a linear equation system:

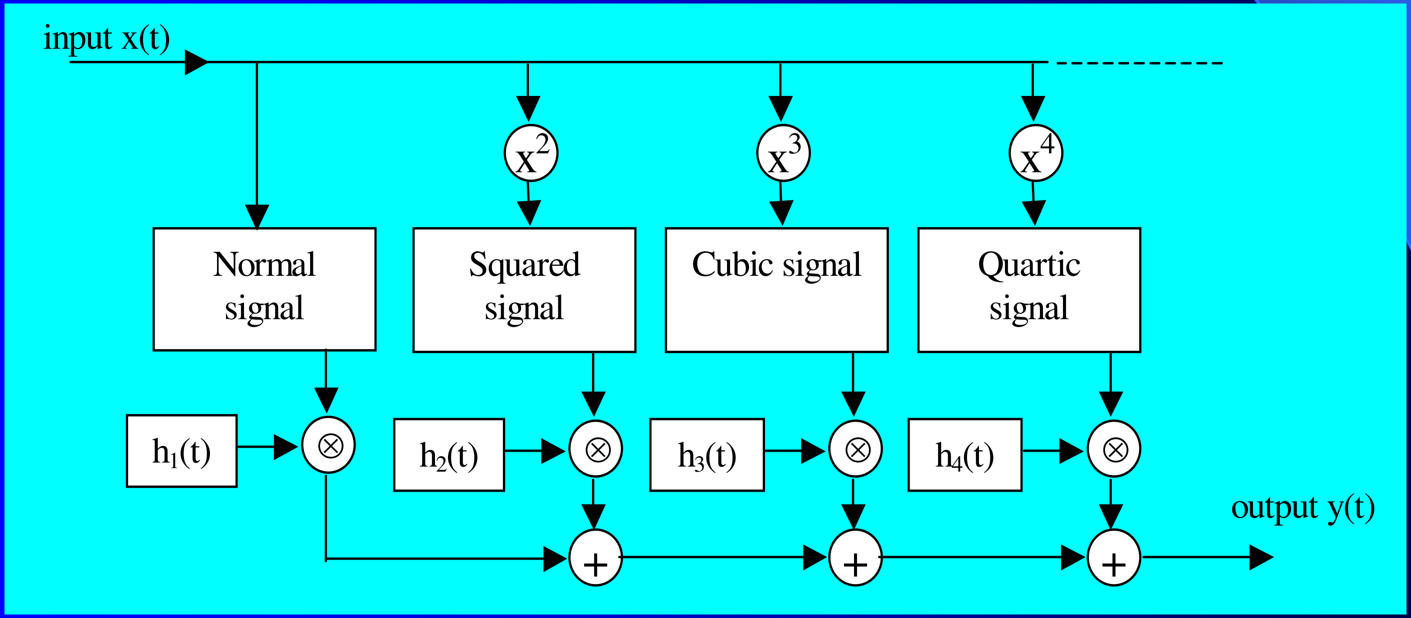
$$\begin{cases} \overline{H'}_1 = \overline{H}_1 + \frac{3}{4}\cdot\overline{H}_3 + \frac{5}{8}\cdot\overline{H}_5 \\ \overline{H'}_2 = -j\cdot\frac{1}{2}\cdot\left[\overline{H}_2 + \overline{H}_4\right] \\ \overline{H'}_3 = -\frac{1}{4}\cdot\overline{H}_3 - \frac{5}{16}\cdot\overline{H}_5 \\ \overline{H'}_4 = j\cdot\frac{1}{8}\cdot\overline{H}_4 \\ \overline{H'}_5 = \frac{1}{16}\cdot\overline{H}_5 \end{cases}$$

We can easily solve it, obtaining the required Volterra kernels as a function of the measured multiple-order IRs:

$$\begin{cases} \overline{H}_1 = \overline{H'}_1 + 3\cdot\overline{H'}_3 + 5\cdot\overline{H'}_5 \\ \overline{H}_2 = 2\cdot j\cdot\overline{H'}_2 + 8\cdot j\cdot\overline{H'}_4 \\ \overline{H}_3 = -4\cdot\overline{H'}_3 - 20\cdot\overline{H'}_5 \\ \overline{H}_4 = -8\cdot j\cdot\overline{H'}_4 \\ \overline{H}_5 = 16\cdot\overline{H'}_5 \end{cases}$$

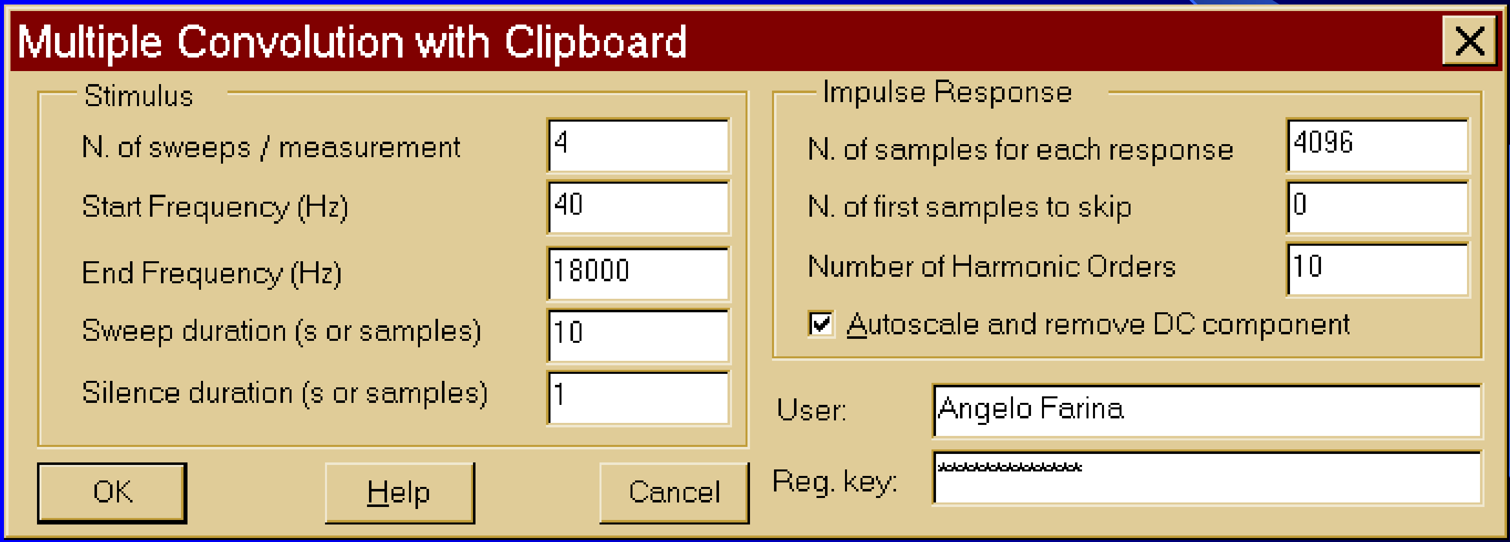
Non-linear convolution

As we have got the Volterra kernels already in frequency domain, we can efficiently use them in a multiple convolution algorithm implemented by overlap-and-save of the partitioned input signal:



Software implementation

Although today the algorithm is working off-line (as a mix of manual CoolEdit operations and some Matlab processing), a more efficient implementation as a CoolEdit plugin is being worked out:



This will allow for real-time operation even with a very large number of filter coefficients

Audible evaluation of the performance

Original signal

Linear convolution

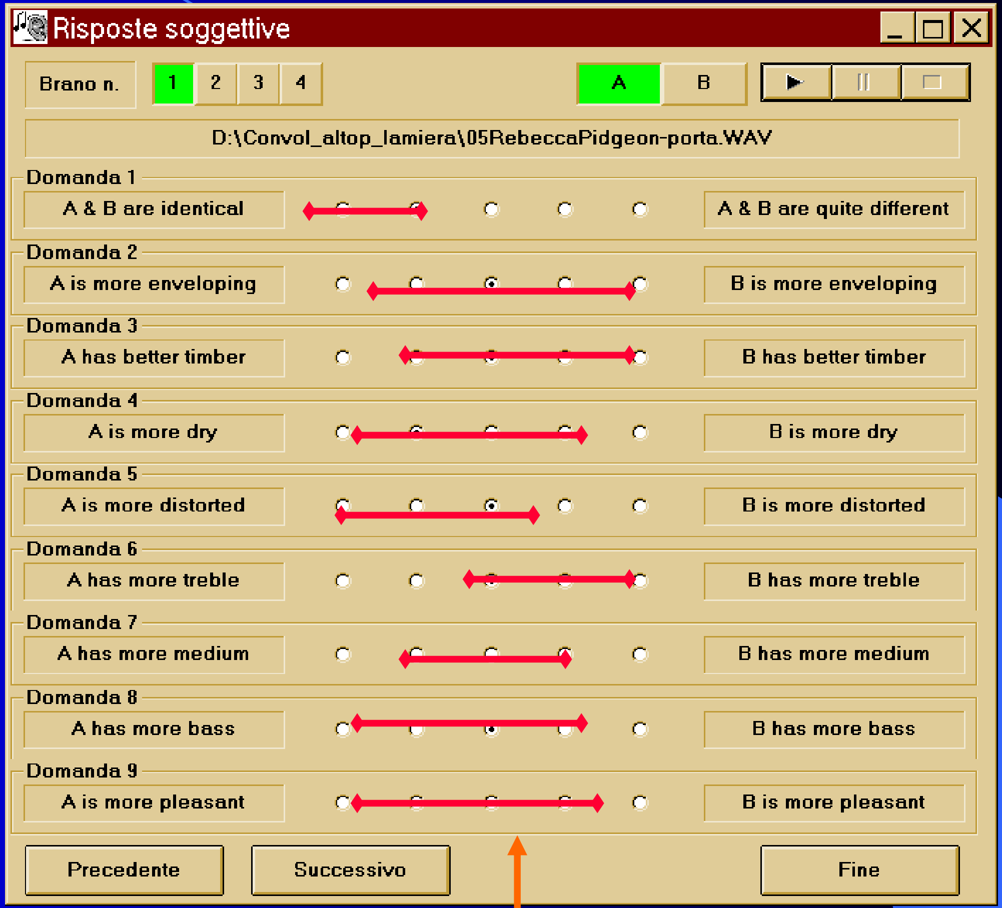
These last two were compared in a formalized blind listening test

Live recording

Non-linear multi convolution

Subjective listening test

- A/B comparison
- Live recording & non-linear auralization
- 12 selected subjects
- 4 music samples
- 9 questions
- 5-dots horizontal scale
- Simple statistical analysis of the results
- A was the live recording, B was the auralization, but the listener did not know this



95% confidence intervals of the responses

Conclusion

Statistical parameters – more advanced statistical methods would be advisable for getting more significant results

Question Number	Average score	2.67 * Std. Dev.
1 (identical-different)	1.25	0.76
3 (better timber)	3.45	1.96
5 (more distorted)	2.05	1.34
9 (more pleasant)	3.30	2.16

Final remarks

- The CoolEdit plugin is planned to be released in two months – it will be downloadable from HTTP://www.ramsete.com/aurora

- The sound samples employed for the subjective test are available for download at HTTP://pcangelo.eng.unipr.it/public/AES110

- The new method will be employed for realistic reproduction in a listening room of the behaviour of car sound systems