

Note Concerning our Paper “On the Nature of Turbulence”

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After the paper referred to in the title was published, references to the relevant Russian literature were made available to us by Ya. G. Sinai and V. I. Arnold. From these it appears that much of the work on bifurcation described in our paper duplicates results already published by Russian authors. On the other hand the mathematical interpretation which we give of turbulence seems to remain our own responsibility!

We thank Sinai and Arnold for informing us of the Russian references. The following list has been compiled from their indications.

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