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Note*

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NOTE ON "OPTIMAL ORDERING QUANTITY TO REALIZE A PRE-DETERMINED LEVEL OF PROFIT"†

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In this note we consider the one-period inventory model in which it is required to determine the order quantity which maximizes the probability of realizing a predetermined level of profit R . We give a condition for determining the optimal order quantity and provide explicit expressions for the optimal order quantity in three special cases.
(INVENTORY/PRODUCTION/STOCHASTIC MODELS)

In this note we consider the so-called static or one-period model (see, for example, Wagner [1], Hillier and Liebermann [2]) in which only a single inventory decision is made in anticipation of demand which is treated as a random variable. We assume that the initial inventory is zero. The problem is to determine the order quantity, I , which maximizes the probability of reaching or exceeding a given profit R when the other parameters of the system like selling price, cost price, shortage cost and scrap price, are known. The profit Z , being dependent on the random demand, is itself a random variable. The probability distribution of Z will naturally depend on the other quantity I and the probability distribution of the demand. Hence the probability that the profit will be greater than the pre-determined level R will be a function of I once the distribution of demand is given. It is reasonable to expect that the chances of realizing the profit R will be more if we choose I so that this probability is a maximum. We have given the methodology for arriving at the optimal order quantity using the criterion, Probability[Profit $\geq R$] is a maximum, and we have arrived at explicit expressions for the order quantity in a few special cases. We give below the notation used in this note.

X : quantity demanded,

$f(x)$: probability density function of X ,

p : unit selling price,

c : unit cost price,

s : unit shortage cost (i.e., ill-will cost over and above lost sale opportunity cost),

q : unit scrap price,

I : the order quantity which is the decision variable,

Z : the profit which depends on the demand X and hence is a random variable.

The expression for Z , when a quantity I is ordered and when X is the demand, is easily seen to be

$$\begin{aligned} Z &= pX + q(I - X) - cI, & X < I \\ &= pI - s(X - I) - cI, & X > I. \end{aligned} \quad (1)$$

Z increases for $X < I$ and decreases for $X > I$. A predetermined level of profit R will

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be realized when X is equal to either x_1 or x_2 where

$$x_1 = \frac{R + (c - q)I}{p - q} \quad \text{and} \quad (2)$$

$$x_2 = \frac{(p + s - c)I - R}{s}. \quad (3)$$

The profit Z will be greater than R whenever $x_1 < X < x_2$. So the probability that Z is greater than or equal to R is given by

$$P[Z \geq R] = P[x_1 \leq X \leq x_2] = \int_{x_1}^{x_2} f(x) dx, \quad (4)$$

the right-hand side integral being a function of I . The optimal order quantity is obtained from the condition that $\int_{x_1}^{x_2} f(x) dx$ is a maximum, i.e.,

$$\frac{d}{dI} \int_{x_1}^{x_2} f(x) dx = 0 \quad \text{or} \quad (5)$$

$$f(x_2) \frac{dx_2}{dI} - f(x_1) \frac{dx_1}{dI} = 0. \quad (6)$$

Using (2) and (3), the condition (6) can be written as

$$f(x_2) \frac{(p + s - c)}{s} - f(x_1) \frac{(c - q)}{p - q} = 0. \quad (7)$$

We now consider three special cases and give explicit expressions for I , the optimal order quantity.

Case I. Exponential Demand. $f(x) = \mu e^{-\mu x}$, $\mu > 0$, $x \geq 0$. We get by (7)

$$\mu e^{-\mu x_2} \frac{(p + s - c)}{s} = \mu e^{-\mu x_1} \frac{c - q}{p - q}. \quad (8)$$

Setting

$$\frac{(p + s - c)(p - q)}{s(c - q)} = e^{\mu \delta} \quad (9)$$

(8) can be written as

$$e^{-\mu x_2} e^{\mu \delta} = e^{-\mu x_1} \quad \text{or} \quad x_2 - \delta = x_1. \quad (10)$$

Substituting the values of x_1 and x_2 from (2) and (3) in (10), we get, after some algebraic simplification, the optimal order quantity I^* as

$$I^* = \frac{1}{(p - c)} \left[\frac{(p - q)s\delta}{p - q + s} + R \right]. \quad (11)$$

Case II. When the commodity marketed is a luxury item, it is reasonable to assume that the demand is proportional to the income y of an individual, i.e., demand = ky . Therefore

$$\begin{aligned} P[\text{Number of demands} > x] &= P[\text{Number of people whose income} > \frac{x}{k}] \\ &= \frac{c}{(x/k)^m}, \end{aligned}$$

assuming Pareto's law (Jan Pen [3], Allen [4]) for income distribution where $m > 1.5$.

The density function for the demand $f(x)$ can, then, be taken as

$$f(x) = A/x^\alpha \quad \text{where } \alpha > 2.5. \quad (12)$$

We get from (7)

$$\frac{1}{x_2^\alpha} \frac{p+s-c}{s} = \frac{1}{x_1^\alpha} \frac{c-q}{p-q}. \quad (13)$$

Setting

$$\frac{(p+s-c)(p-q)}{s(c-q)} = \lambda^\alpha \quad (14)$$

(13) reduces to

$$\lambda x_1 = x_2. \quad (15)$$

Substituting the values of x_1 and x_2 from (2) and (3) we see that the probability of achieving the desired profit is maximum when

$$\frac{\lambda[R+(c-q)I]}{p-q} = \frac{(p+s-c)I-R}{x}. \quad (16)$$

This yields the optimal order quantity I^* as

$$I^* = \frac{R[\lambda s + p - q]}{(p+s-c)(p-q) - \lambda s(c-q)}. \quad (17)$$

Case III. Here the demand follows a uniform distribution,

$$f(x) = \frac{1}{b-a}, \quad a < x < b \\ = 0, \quad \text{otherwise.}$$

In this case it can be seen that (7) will not directly yield the optimal order quantity. However we observe that the probability of obtaining the desired profit R , viz., $\int_{x_1}^{x_2} f(x) dx$, being a piecewise linear increasing function of the order quantity I , has unity as the least upper bound. Thus the optimal order quantity I^* is obtained from the condition

$$\int_{x_1}^{x_2} f(x) dx = 1, \quad \text{i.e.,} \quad (18)$$

$$(x_2 - x_1) = (b - a). \quad (19)$$

Substituting the value of x_1 and x_2 from (2) and (3) in (19), the optimal order quantity I^* can be seen to be given by

$$I^* = \frac{1}{(p-c)} \left[\frac{s(b-a)(p-q)}{p-q+s} + R \right].^1 \quad (20)$$

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