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Note*

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ON "LOCATION OF BANK ACCOUNTS TO OPTIMIZE FLOAT: AN ANALYTIC STUDY OF EXACT AND APPROXIMATE ALGORITHMS"*

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(PROGRAMMING-INTEGER ALGORITHMS, HEURISTIC FACILITIES/EQUIP-
 MENT PLANNING-LOCATION)

In the course of the deliberations of the 1977 Lanchester Prize Committee, Alan J. Goldman brought to our attention an error in the proof of Lemma 1 of our paper [1]. We incorrectly stated that $z_D = \min_u z_D''(u)$. The lemma, however, is true and the original correct, but long and intricate, proof was provided to the Committee, see [2] for details.

We have recently found the following simpler proof:

LEMMA 1. For all $P \in \mathcal{P}_k$, $(z_D - z_g)/(z_D - z_R) \leq [(K-1)/K]^K < 1/e$.

PROOF. If $K = 1$ or $\rho_j(u^1) < 0$, the lemma is clearly true since $z_g = z_D(u^1)$.

Otherwise, let k be the number of locations selected by the greedy heuristic, $\alpha = (K-1)/K$, $c = \sum_{i \in I} \min_{j \in J} c_{ij}$, $D = \max_{j \in J} d_j$ and $\rho_j = \rho_j(u^j)$, $j = 1, \dots, k$.

The statement of the lemma is equivalent to

$$(1 - \alpha^K)z_D \leq z_g - \alpha^K z_R. \quad (1)$$

Substituting $z_R = c - KD$ and $z_g = c + \sum_{j=1}^k \rho_j$ in (1), we must show that

$$(1 - \alpha^K)z_D \leq (1 - \alpha^K)c + \sum_{j=1}^k \rho_j + \alpha^K KD. \quad (2)$$

For $s = 1, \dots, k$, $\sum_{i \in I} u_i^s = c + \sum_{j=1}^{s-1} (\rho_j + d_j)$, $K\rho_s$ is nonnegative and at least as large as the K largest $\rho_j(u^s)$, $j \notin J^*$, and $D > d_j$, $j \in J$. Using these facts and the definition of $z_D(u^s)$ we obtain

$$z_D \leq c + \sum_{j=1}^{s-1} \rho_j + K\rho_s + (s-1)D, \quad s = 1, \dots, k. \quad (3)$$

* All Notes are refereed.

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Multiply inequality s of (3) by $(1 - \alpha)\alpha^{k-s}$, sum for $s = 1, \dots, k$, and substitute

$$\begin{aligned} (1 - \alpha) \sum_{s=1}^k \alpha^{k-s} &= 1 - \alpha^k, \\ (1 - \alpha) \sum_{s=1}^k \alpha^{k-s} \sum_{j=1}^{s-1} \rho_j &= (1 - \alpha) \sum_{s=1}^{k-1} \left(\sum_{j=0}^{k-1-s} \alpha^j \right) \rho_s \\ &= \sum_{s=1}^{k-1} (1 - \alpha^{k-s}) \rho_s, \\ K(1 - \alpha) &= 1, \end{aligned}$$

and

$$(1 - \alpha) \sum_{s=1}^k (s - 1) \alpha^{k-s} = k - K + K\alpha^k$$

to obtain

$$(1 - \alpha^k)z_D < (1 - \alpha^k)c + \sum_{j=1}^k \rho_j + (k - K + K\alpha^k)D. \quad (4)$$

We will consider two cases to show that (4) implies (2). Case (a). [$k = K$]. Substituting $k = K$ in (4) gives (2). Case (b). [$k < K$]. In this case, Step 2 of the greedy heuristic is executed $k + 1$ times and $\rho_j(u^{k+1}) < 0$. Using the definition of $z_D(u^{k+1})$ and $\sum_{i \in I} u_i^{k+1} < c + \sum_{j=1}^k \rho_j + kD$ we obtain

$$z_D < c + \sum_{j=1}^k \rho_j + kD. \quad (5)$$

Multiply (4) by α^{K-k} , (5) by $1 - \alpha^{K-k}$ and sum to obtain

$$(1 - \alpha^K)z_D < (1 - \alpha^K)c + \sum_{j=1}^k \rho_j + [k - K\alpha^{K-k} + K\alpha^K]D. \quad (6)$$

Since $D > 0$ if $k < K$, (6) will imply (2) if

$$k < K\alpha^{K-k}. \quad (7)$$

We prove (7) by induction. For $k = K - 1$ we have $K\alpha^{K-k} = K - 1$ so that (7) is an equality for all K . Now assume that (7) is true for k and consider $k - 1$. We have

$$\begin{aligned} K\alpha^{K-k+1} &= (K\alpha)\alpha^{K-k} = K\alpha^{K-k} - \alpha^{K-k} \\ &> K\alpha^{K-k} - 1 > k - 1, \end{aligned}$$

where the last inequality is implied by the induction hypothesis. Q.E.D.

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