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ON "LOCATION OF BANK ACCOUNTS TO OPTIMIZE FLOAT: AN ANALYTIC STUDY OF EXACT AND APPROXIMATE ALGORITHMS"*

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(PROGRAMMING–INTEGER ALGORITHMS, HEURISTIC FACILITIES/EQUIPMENT PLANNING–LOCATION)

In the course of the deliberations of the 1977 Lanchester Prize Committee, Alan J. Goldman brought to our attention an error in the proof of Lemma 1 of our paper [1]. We incorrectly stated that $z_D = \min_u z''_D(u)$. The lemma, however, is true and the original correct, but long and intricate, proof was provided to the Committee, see [2] for details.

We have recently found the following simpler proof:

LEMMA 1. *For all* $P \in \mathcal{P}_k$, $(z_D - z_g)/(z_D - z_R) \leq [(K-1)/K]^K < 1/e$.

PROOF. If $K = 1$ or $\rho_{j_1}(u^1) < 0$, the lemma is clearly true since $z_g = z_D(u^1)$.

Otherwise, let k be the number of locations selected by the greedy heuristic, $\alpha = (K-1)/K$, $c = \sum_{i \in I} \min_{j \in J} c_{ij}$, $D = \max_{j \in J} d_j$ and $\rho_j = \rho_{i_j}(u^j)$, $j = 1, \ldots, k$.

The statement of the lemma is equivalent to

$$(1-\alpha^K) z_D \leq z_g - \alpha^K z_R. \tag{1}$$

Substituting $z_R = c - KD$ and $z_g = c + \sum_{j=1}^{k} \rho_j$ in (1), we must show that

$$(1-\alpha^K) z_D \leq (1-\alpha^K)c + \sum_{j=1}^{k} \rho_j + \alpha^K KD. \tag{2}$$

For $s = 1, \ldots, k$, $\sum_{i \in I} u_i^s = c + \sum_{j=1}^{s-1} (\rho_j + d_{i_j})$, $K\rho_s$ is nonnegative and at least as large as the K largest $\rho_j(u^s)$, $j \notin J^*$, and $D \geq d_j$, $j \in J$. Using these facts and the definition of $z_D(u^s)$ we obtain

$$z_D \leq c + \sum_{j=1}^{s-1} \rho_j + K\rho_s + (s-1)D, \qquad s = 1, \ldots, k. \tag{3}$$

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Multiply inequality s of (3) by $(1-\alpha)\alpha^{k-s}$, sum for $s=1,\dots,k$, and substitute

$$(1-\alpha)\sum_{s=1}^{k}\alpha^{k-s}=1-\alpha^{k},$$

$$(1-\alpha)\sum_{s=1}^{k}\alpha^{k-s}\sum_{j=1}^{s-1}\rho_j=(1-\alpha)\sum_{s=1}^{k-1}\left(\sum_{j=0}^{k-1-s}\alpha^{j}\right)\rho_s$$

$$=\sum_{s=1}^{k-1}(1-\alpha^{k-s})\rho_s,$$

$$K(1-\alpha)=1,$$

and

$$(1-\alpha)\sum_{s=1}^{k}(s-1)\alpha^{k-s}=k-K+K\alpha^{k}$$

to obtain

$$(1-\alpha^{k})z_D\leq(1-\alpha^{k})c+\sum_{j=1}^{k}\rho_j+(k-K+K\alpha^{k})D. \tag{4}$$

We will consider two cases to show that (4) implies (2). Case (a). $[k=K]$. Substituting $k=K$ in (4) gives (2). Case (b). $[k<K]$. In this case, Step 2 of the greedy heuristic is executed $k+1$ times and $\rho_j(u^{k+1})\leq 0$. Using the definition of $z_D(u^{k+1})$ and $\sum_{i\in I}u_i^{k+1}\leq c+\sum_{j=1}^{k}\rho_j+kD$ we obtain

$$z_D\leq c+\sum_{j=1}^{k}\rho_j+kD. \tag{5}$$

Multiply (4) by α^{K-k}, (5) by $1-\alpha^{K-k}$ and sum to obtain

$$(1-\alpha^{K})z_D\leq(1-\alpha^{K})c+\sum_{j=1}^{k}\rho_j+\left[k-K\alpha^{K-k}+K\alpha^{K}\right]D. \tag{6}$$

Since $D\geq 0$ if $k<K$, (6) will imply (2) if

$$k\leq K\alpha^{K-k}. \tag{7}$$

We prove (7) by induction. For $k=K-1$ we have $K\alpha^{K-k}=K-1$ so that (7) is an equality for all K. Now assume that (7) is true for k and consider $k-1$. We have

$$K\alpha^{K-k+1}=(K\alpha)\alpha^{K-k}=K\alpha^{K-k}-\alpha^{K-k}$$

$$\geq K\alpha^{K-k}-1\geq k-1,$$

where the last inequality is implied by the induction hypothesis. Q.E.D.

References

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