This article was downloaded by: [106.51.226.7] On: 09 August 2022, At: 00:50
Publisher: Institute for Operations Research and the Management Sciences (INFORMS)
INFORMS is located in Maryland, USA


Management Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

# Note—On Madigan's Approach to the Deterministic Multi-Product Production and Inventory Problem

Kenneth R. Baker,

To cite this article:
Kenneth R. Baker, (1970) Note—On Madigan's Approach to the Deterministic Multi-Product Production and Inventory Problem. Management Science 16(9):636-638. https://doi.org/10.1287/mnsc.16.9.636

Full terms and conditions of use: https://pubsonline.informs.org/Publications/Librarians-Portal/PubsOnLine-Terms-and-Conditions

This article may be used only for the purposes of research, teaching, and/or private study. Commercial use or systematic downloading (by robots or other automatic processes) is prohibited without explicit Publisher approval, unless otherwise noted. For more information, contact permissions@informs.org.

The Publisher does not warrant or guarantee the article's accuracy, completeness, merchantability, fitness for a particular purpose, or non-infringement. Descriptions of, or references to, products or publications, or inclusion of an advertisement in this article, neither constitutes nor implies a guarantee, endorsement, or support of claims made of that product, publication, or service.

© 1970 INFORMS

Please scroll down for article—it is on subsequent pages

With 12,500 members from nearly 90 countries, INFORMS is the largest international association of operations research (O.R.) and analytics professionals and students. INFORMS provides unique networking and learning opportunities for individual professionals, and organizations of all types and sizes, to better understand and use O.R. and analytics tools and methods to transform strategic visions and achieve better outcomes.
For more information on INFORMS, its publications, membership, or meetings visit http://www.informs.org

Published in Management Science on May 01, 1970 as DOI: 10.1287/mnsc.16.9.636.
This article has not been copyedited or formatted. The final version may differ from this version.

Downloaded from informs.org by [106.51.226.7] on 09 August 2022, at 00:50 . For personal use only, all rights reserved.

MANAGEMENT SCIENCE
Vol. 16, No. 9, May, 1970
*Printed in U.S.A.*

# NOTES

## On Madigan's Approach to the Deterministic Multi-Product Production and Inventory Problem*

The deterministic multi-product single-machine production and inventory problem is one of the simplest models in which production and inventory factors interact. There is at present no algorithm available which solves the problem optimally, and several different types of approaches have been presented in the literature.[1] Madigan's heuristic method [4] avoided some of the drawbacks characteristic of other methods, but there are some errors and limitations in his original presentation, to which this note is addressed.

The notation to be used is as follows:

$i$ = product designation, $i = 1, 2, \cdots, n$
$s$ = setup cost
$h$ = inventory carrying cost
$p$ = production rate
$r$ = demand rate
$q$ = lot size.

Madigan's procedure makes use of the familiar single-product economic lot size formula:

$$q_i^* = [2s_ir_i/h_i(1-\rho_i)]^{1/2}, \quad \text{where } \rho_i = r_i/p_i. \tag{1}$$

The average time cost associated with this lot size is:

$$c_i^* = [2s_ir_ih_i(1-\rho_i)]^{1/2}. \tag{2}$$

The average time cost associated with any other lot size is:[2]

$$c_i = (c_i^*/2)(q_i/q_i^* + q_i^*/q_i). \tag{3}$$

The method begins by calculating the optimal rotation cycle lot sizes:

$$q_i = r_i[\textstyle\sum_{i=1}^n 2s_i/\sum_{i=1}^n h_ir_i(1-\rho_i)]^{1/2}$$

and their associated costs, determined from (3). Improvements in the rotation cycle pattern are made with the following heuristic: if the lot size of any item in the cycle gives rise to costs unacceptably higher than the corresponding single-product cost of (2), possible changes of the lot size in a cost reducing direction are then considered. For "simple scheduling," as Madigan calls it, the only changes to be considered are integer multiples or integer fractions of the rotation cycle lot size. The possible changes are then evaluated by (3), and the best alternative is chosen.

If the altered lot size cannot be accommodated by the cycle (i.e., if the idle time in the cycle is too small to absorb the increased utilization), then Madigan proposes examining modifications which achieve feasibility. One such modification is the extension of all production intervals and the cycle length, but with the restriction that each lot size remain an integer multiple or integer fraction of cycle demand for that

* Received March 1969; revised September 1969.

[1] The most successful appear to be the heuristic approach of Eilon [3, Ch. 14] and of Madigan [4], and the dynamic programming approach of Bomberger [2] and of Stankard and Gupta [5].

[2] A derivation can be found in [3, pp. 243–244].

Published in Management Science on May 01, 1970 as DOI: 10.1287/mnsc.16.9.636.
This article has not been copyedited or formatted. The final version may differ from this version.

Downloaded from informs.org by [106.51.226.7] on 09 August 2022, at 00:50 . For personal use only, all rights reserved.



item. Another modification, available at later stages of revision, is the consolidation of lot sizes of items produced several times per cycle in order to make time available for larger lots or more frequent lots of other items. Simple combinations of extension and consolidation presumably can also be evaluated, and a least cost choice made among the alternatives. Madigan's procedure relies exclusively on (3) for the cost comparisons.[3]

In order that (3) be valid, the operating policy must yield a single cycle for each product; that is, the lot size and the interval between productions must be fixed, and production must begin when inventory is reduced to zero. Such a pattern arises, for example, in the optimal single-product and rotation cycle solutions. Madigan's use of integer-multiple and integer-fraction restrictions appears to be an effort to preserve single-cycle patterns, but such conditions are not sufficient to guarantee that (3) will be valid.

Consider the implementation of Madigan's procedure when the rotation cycle yields a pattern for some product in which inventory costs are higher than setup costs. The procedure calls for evaluating the production of two or more lots per cycle. Suppose that an integer fraction of the rotation cycle lot size is preferred, based on (3), and that there is sufficient idle time in the cycle to accommodate the multiple setups. The method assumes that the production intervals of the smaller lot can be evenly spaced. But in the case where the idle time remaining in the cycle is very small compared to any of the production intervals, it is quite possible that production intervals of all other products cannot be placed so that the item produced several times can be scheduled for evenly spaced productions. In such situations, formula (3) is an incorrect evaluator of costs, and a close reading of Madigan's procedure reveals that the inconsistency between (3) and fixed production lots is never resolved.[4] To overcome this problem—and to improve Madigan's algorithm—one must relax the superfluous integer-type restrictions and depart from the use of single-cycle inventory patterns. Costs will then have to be calculated directly from the geometrical patterns which result. The alternative is to rely on single-cycle patterns for all products, even if the lot sizes are each very different from the corresponding $q_i^*$. Such an approach has been proposed by Stankard and Gupta [5], though they have not treated the disadvantages of the single-cycle constraint in sufficient detail.

The reliance on (3) in seeking low-cost patterns is actually a reliance on the lot size as an indicator of associated costs, but when single-cycle patterns do not apply, the lot size may be an unreliable indicator. In the single-product situation, the use of a lot size less than $q_i^*$ yields setup costs which are greater than inventory costs, and the additional total cost associated with this imbalance is validly indicated by (3). In the multi-product case, when single-cycle patterns may not apply, inventory and setup costs might be equally balanced with the use of a lot size below $q_i^*$. The emphasis of the heuristic approach should be not on scheduling lot sizes which are near individual values of $q_i^*$, but rather on balancing inventory and setup costs equally for each product.[5]

[3] It should be noted that although Madigan did not permit backlogging in his model, his procedure applies without any basic changes when penalty costs are added to the model.

[4] That Madigan did not deal specifically with this inconsistency is also evident in one of his examples [4, p. 716], which contains two products which are both produced in alternating subcycles of different lengths.

[5] An algorithm which employs this equal-balance heuristic, and which admits variable-lot production, has been described in [1, Chapter 3].

Published in Management Science on May 01, 1970 as DOI: 10.1287/mnsc.16.9.636.
This article has not been copyedited or formatted. The final version may differ from this version.

Downloaded from informs.org by [106.51.226.7] on 09 August 2022, at 00:50 . For personal use only, all rights reserved.



Another matter which is unsatisfactory in Madigan's procedure is the question of the order in which product lot sizes are to be adjusted. The algorithm includes no mechanism by which to choose the next item to be modified. The use of an arbitrary order would be naive and might well reduce the potential power of the algorithm.

The optimum rotation cycle, with which the algorithm begins, is well-defined as long as the variables of the problem are continuous. In certain cases, it may be convenient or even necessary to restrict the solution to a cycle of integral length; but even if such restrictions are realistic, it is still possible to question Madigan's prescription to round off the non-integral optimum rotation cycle length to the nearest integer. Rounding down will always yield less idle time than rounding up, and, in addition, the cost function is not symmetric around the optimal rotation cycle length.[6]

With refinements of Madigan's procedure based on these observations, the approach can be more effectively implemented, as well as more fairly compared with other solution techniques.

**References**

1. Baker, K. R., "Control Policies for an Integrated Production and Inventory System," Technical Report No. 84, Department of Operations Research, Cornell University, 1969.
2. Bomberger, Earl, "A Dynamic Programming Approach to a Lot Size Scheduling Problem," *Management Science*, Vol. 12, No. 11 (July 1966), pp. 778–784.
3. Eilon, Samuel, *Elements of Production Planning and Control*, Macmillan, New York, 1962.
4. Madigan, J. G., "Scheduling a Multi-Product Single Machine System for an Infinite Planning Period," *Management Science*, Vol. 14, No. 11 (July 1968), pp. 713–719.
5. Stankard, M. F. and Gupta, S. K., "A Note on Bomberger's Approach to Lot Size Scheduling: Heuristic Proposed," *Management Science*, Vol. 15, No. 7 (March 1969), pp. 449–452.

Kenneth R. Baker
*The University of Michigan*

[6] An example of this property, as well as a discussion of the idle time implications of rounding off, can be found in [1, Chapter 3].

## The Distribution Problem With Upper and Lower Bounds on the Node Requirements*†

This paper considers a distribution model with upper and lower bounds on the number of units shipped from an origin or to a destination. Our problem differs from the classical distribution model in which the node shipping amounts are, by contrast, specified exactly. This generalization of the classical distribution problem not only makes the model more versatile from a theoretical standpoint but also makes the model more usable from an applications viewpoint. Furthermore, this seemingly innocent difference gives rise to some paradoxical solution properties (see [4]), and has in fact occasioned an incorrect treatment of the problem by two highly reputable texts on linear programming and network flows (see [3], [5], and [6]).

In many applications of the distribution model, the firm is only interested in shipping an approximate number of units from each origin and in receiving an approximate number of units at each destination. Thus, what has commonly been done is to use these approximate amounts as the exact amounts to be shipped or received at the

* Received June 1969; revised December 1969.
† Partly supported by O.N.R. contract N00014-67-A-0126-0009.

Published in Management Science on May 01, 1970 as DOI: 10.1287/mnsc.16.9.636.
This article has not been copyedited or formatted. The final version may differ from this version.

Copyright 1970, by INFORMS, all rights reserved. Copyright of Management Science is the property of INFORMS: Institute for Operations Research and its content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.

Downloaded from informs.org by [106.51.226.7] on 09 August 2022, at 00:50 . For personal use only, all rights reserved.