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Notes* I

ON "PRODUCTION RUNS FOR MULTIPLE PRODUCTS: THE TWO-PRODUCT HEURISTIC"†

STEPHEN C. GRAVES‡ AND ROBERT W. HAESSLER§

The recent paper [6] by Saipe considers the single-machine multi-product lot scheduling problem, this problem is concened with the determination of run sizes for a set of products which are produced on the same machine. In the paper [6], a heuristic solution procedure, based on a solution procedure for the two-product lot scheduling problem, is proposed for the multi-product problem. The intent of this note is to raise three issues concerning the proposed heuristic procedure: (a) the optimality of the procedure for the two-product problem, (b) the performance of the procedure at full capacity for more than two products, and (c) the effectiveness of the procedure relative to alternative solution procedures.

Optimality of Two-Product Procedure

The procedure proposed for the two-product problem need not give the optimal solution, as is purported in [6]. This follows from the fact that the condition $n_2 = kn_1$ [see (2)] is *not* a necessary condition for optimality. This is best seen from the two-product counterexample given in Table 1, where R is annual demand, P is the annual production rate, c is the setup cost, H is the inventory holding cost and EOQ is the economic order quantity. Note that a feasible schedule can be constructed using the respective economic order quantitities for each product. Suppose the time-horizon is divided into quarters; then product 1 would be produced in quarters 1, 7, 13, . . . , while product 2 is produced in quarters 2, 6, 10, ... This schedule is feasible, and hence optimal due to the optimality of EOQ for the separate single-product problems. Now, however, for n_i being the number of production runs per year for product i, we have $n_1 = 2/3$, $n_2 = 1$; clearly here the ratio of n_2 to n_1 is not integer as required in [5], equation (2).

TABLE 1

Product	R_i	P_i	c_i	H_i	EOQ_i
1	32	192	15	0.5	48
2	16	64	3	0.5	16

Performance at Full Capacity

The unwary reader may also be left with the impression that the two-product heuristic will lead to a good, if not optimal, solution in the multi-product case if the

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machine is at full capacity because it gives the same result as the complete-cycle heuristic. A simple example given in Table 2 can be used to demonstrate that this is not the case. In this example, the independently determined *EOQ*'s again lead to a feasible, and therefore, optimal schedule. Product 1 should be produced in quarters 1, 3, 5, 7, . . . , product 2 should be produced in quarters 2, 6, 10, . . . , and product 3 should be produced in quarters 4, 8, 12, ... This solution is clearly superior to producing each product once each complete cycle as the two-product heuristic indicates should be done.

TABLE 2

Product	R_i	P_i	c_i	H_i	EOQ_i
1	50	100	25	8	25
2	25	100	75	8	25
3	25	100	75	8	25

Comparison with Other Procedures

The proposed two-product heuristic is compared with the complete-cycle heuristic or pure-rotation policy in [6] on two test problems. The two-product heuristic is shown to yield substantial improvements over the complete-cycle heuristic. This is really not surprising because the complete-cycle heuristic is the least effective procedure ever seriously suggested for solving the problam. Over the last fifteen years, a number of other solution procedures for the multi-product problem have been proposed in the management science literature (see [3] for a bibliography and complete overview). Indeed, it would be useful to compare Saipe's procedures with these alternatives on a set of test problems that originated with Bomberger [1] and have been used for a comparison of alternative solution procedures (see [2], [3], [4], [5], [7]).[1] Table 3 presents the comparison. Based on these test problems, the procedure of Saipe does not compare well. It should be noted though, that these test problems have extreme parameter variation across products, and hence may not be realistic. However, it is interesting to compare the results of Saipe with those of Doll and Whybark and to note that the set of solutions considered by Saipe is a limited subset of those considered by Doll and Whybark. Therefore it is not possible for the two-product heuristic to outperform the Doll and Whybark procedure. The only possible justification for proposing the two-product heuristic is that it is computationally simpler. Assuming that some computational aid is available, it is difficult to imagine that the saving in computational time is enough to warrant the degradation in solution quality that would occur if the two-product heuristic is used in place of one of the more powerful procedures described in [3].

TABLE 3

Problem	Bomberger	Stankard & Gupta	Madigan	Doll & Whybark	Saipe
1	17.02	n.a.*	17.00	16.99	18.54
2	29.91	n.a.*	28.37	28.14	31.13
3	36.65	36.24	33.94	32.07	35.24

* not available

[1] The original test problems assumed nonzero setup times; these setup times have been ignored to allow comparison with Saipe's procedure.

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Notes*
II

REPLY TO GRAVES AND HAESSLER'S NOTE ON THE PAPER "PRODUCTION RUNS FOR MULTIPLE PRODUCTS: THE TWO-PRODUCT HEURISTIC"†

JOHN P. MAYBERRY‡ AND ALAN L. SAIPE§

[1] presents a heuristic method for scheduling N products on one machine, by dividing them into two groups and scheduling the production of the groups by a simpler algorithm appropriate for scheduling two products. Although no claim was made that the resulting N-product schedule would be optimal, the paper did state that the simpler algorithm [1, §3] was optimal for the two-product case.

In fact, optimality of that two-product algorithm can only be assured if, in addition to the restrictions explicitly stated, the following are imposed:

(i) all runs of any one product are of the same length;

(ii) each production-depletion cycle begins at zero inventory for that product; and

(iii) in the notation of [1], $3r_1/p_1 + 2r_2/p_2 > 1$.

Restrictions similar to (i) and (ii) are often made explicitly, as they should have been in this case. In general, a feasible schedule is possible, subject to restrictions (i) and (ii), if $k = m_2/m_1$, where m_1, m_2 are relatively-prime integers and

$$m_2(r_1/p_1) + m_1(r_2/p_2) \leqslant 1. \tag{1}$$

Restriction (iii) is needed to preclude the possibility of a feasible schedule which would correspond to a nonintegral value of $k = n_2/n_1$. Such a schedule must involve (in each production cycle) m_1 runs of product no. 1 and m_2 runs of product no. 2, where $1 < m_1 < m_2$ and m_1, m_2, are relatively prime integers; this is not possible unless (iii) is satisfied.

If we assume k integral (or accept restriction (iii) above, which implies that the optimal feasible value of k must be integral), then $m_1 = 1$ and $m_2 = k$, and inequality (1) above is equivalent to condition (4) of [1]. In particular, restriction (iii) would surely be satisfied if $r_1/p_1 + r_2/p_2 \geqslant \frac{1}{2}$—i.e., if the two products together employed as much as one-half of the capacity of the machine.

Given $k = m_2/m_1$, where $m_2 \geqslant m_1 \geqslant 1$ and m_1, m_2 satisfy (1) above, we have $n_2 = kn_1$, and (except that k need not be integral) the argument of §3 of [1] may be used with slight change. [2] also notes that k need not be integral. We find that the optimal value k^* of k will be an approximation to the real value k' which minimizes $h(k) = w_1k + w_2/k$; in fact,

$$k' = (w_2/w_1)^{1/2}. \tag{2}$$

Since $w_2 \geqslant w_1$, we have $k' \geqslant 1$ so that we need only consider values of $k \geqslant 1$. (Recall that $m_1 = m_2 = 1$ is always feasible if any feasible solution exists.)

* All Notes are refereed.

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To obtain a better solution for the two-product problem in case restriction (iii) above does *not* hold, we modify the procedure of [1, §3], as follows:

(i) determine k' from (2) above;

(ii) determine (graphically), among the pairs (m_1, m_2) satisfying (1) above, the pair which gives the maximum value $k_1^* = (m_2/m_1)$ subject to $k_1^* \leqslant k'$, and the pair (if any) which gives the minimum value $k_2^* = (m_2/m_1)$ subject to $k_2^* > k'$;

(iii) choose $k^* = k_1^*$ or k_2^*, whichever gives a smaller value of $h(k) = w_1 k + w_2/k$;

(iv) determine n_1^* from (5) of [1];

(v) find $n_2^* = kn_1^*$.

When there is no slack on the machine, so that $p_1/r_1 + p_2/r_2 = 1$, the only solution to (1) above is $m_1 = m_2 = 1$, so that the complete-cycle solution is optimal; in that case, it is clear that restrictions (i) and (ii) above are satisfied. The corresponding inference, in case there are more than two products, was not suggested in [1]; as [2] states, such an inference would have been incorrect.

Concerning the comments in [2] and [3] on efficiency of the Two-Product Heuristic, it is important to note that

(i) The sum of the independent *EOQ* solutions provides a lower bound (often unattainable) on the cost.

(ii) The complete-cycle heuristic may be regarded as providing an upper bound on the cost.

(iii) The two-product heuristic also provides an upper bound on the cost. When not near full-capacity, the new bound often reduces the gap between the previous bounds substantially—e.g., by 87% in the 4-product example of [1].

(iv) The two-product heuristic is computationally easy; it finds a solution by direct computation and can often be done by hand, with pencil and paper. Suggestions in [3] will make the computations even simpler and more effective.

(v) We strongly disagree with the statement of [2] that "The only possible justification for proposing the two-product heuristic is that it is computationally simpler". We feel that it has two real merits which are more important: first, it creates a simple solution that is easily understood, easily modified, easily communicated and easily used, and is thus useful for preparing a broad long-term production plan (which will inevitably be modified in response to uncontrollable day-to-day events). Second, it provides a context into which can be placed more effective methods which are only slightly more complex; a future paper will describe some of those modifications, and compare them (for a spectrum of sample problems) with the various methods mentioned in [2].

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