Note on shock-wave velocity in high-speed liquid-solid impact

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A better relationship between the shock-wave and impact velocities in high-speed water-solid impact is determined from the fundamental equations of continuity, momentum, and state. A second-order relationship is hypothesized and the necessary coefficients then found using the Tait equation of state. Very good agreement is found with available experimental data for water.

I. INTRODUCTION

The pressure developed in the high-speed impact between a liquid and a solid surface is of interest and concern in the problems of high-speed air craft and missiles flying in rain and of turbine blades operating in moist vapors.

The simplest approximation to the maximum impact pressure rise, $p - p_0$, developed in liquid-solid collision is the one-dimensional waterhammer pressure for a rigid target, $\rho_0 C_0 V_0$, where ρ_0 and C_0 are the density and acoustic velocity of the undisturbed liquid and V_0 is the impact velocity. The above expression has been derived from momentum considerations for the idealized case where the parameters are assumed invariant. The approximation is valid if the impact velocity is relatively small, so that the velocity of propagation of the pressure wave or shock wave in the liquid, C, can be reasonably approximated by the acoustic velocity of the liquid, C_0. However, in high-speed liquid-solid impact, when the impact Mach number $M = V_0/C_0$ becomes significant, the waterhammer pressure, $\rho_0 C_0 V_0$, needs to be corrected for the effect of compressibility.

The compressibility can be taken into account in the variation of density ρ and/or shock-wave velocity C. In essence, the waterhammer pressure is corrected for the mass transport across the shock front due to compressibility during the momentum exchange.

Heymann[1] found that it would be desirable to have an approximate relationship for water for shock-wave velocity C as a function of liquid particle velocity change V so that impact pressure could be calculated more closely. He then proposed a relationship by a simple combination of two limiting results, namely,

$$C \cong C_0, \quad \text{for small Mach numbers} \tag{1}$$

$$C \cong kV, \quad \text{for large Mach numbers,} \tag{2}$$

therefore

$$C \cong C_0 + kV, \tag{3}$$

where k is some constant. He then deduced the constant $k = 2$ by experimental data for water. Equation (3) is limited to Mach number $M \leqq 1.2$. Actually k is not a constant. For very large Mach numbers, k approaches unity, since k is equal to $\rho/(\rho - \rho_0)$ where ρ is the liquid density in the compressed state.

The objective of the present paper is to show a concise relationship for C as a function of V over an entire range of impact Mach numbers.

II. ANALYSIS

If a shock front is regarded as a mathematical discontinuity and its thickness is assumed negligible, the governing equations applying to such a shock front, derived from continuity and momentum considerations, based on the one-dimensional model, are

$$\rho_0 C = \rho(C - V), \tag{4}$$

$$p - p_0 = \rho_0 C V. \tag{5}$$

The impact relationship between the density ρ, the shock-wave velocity C, and the impact pressure p require, in addition to the equation of continuity and momemtum, relationship of thermodynamic porperties, namely, the equation of state.

Tait[2] proposed the following equation of state for water:

$$\frac{p + B}{p_0 + B} = \left(\frac{\rho}{\rho_0}\right)^A, \tag{6}$$

where B and A are two empirical functions of temperature. By exhaustive examination of the published experimental data, Li[3] concluded that the relationship of Tait's equation represents the thermodynamic relationship of water very well. At 20 °C, $B = 3047$ bar and $A = 7.15$ was given by Cole.[4]

To obtain a relationship for C and V, equating density from from Eqs. (4) and (6) yields

$$\left(\frac{\rho_0 C}{C - V}\right) = \rho_0 \left(\frac{p + B}{p_0 + B}\right)^{1/A}. \tag{7}$$

Raising both sides to the power A and replacing p from Eq. (5), Eq. (7) becomes

$$\left(\frac{\rho_0 C}{C - V}\right)^A = \rho_0^A \left(1 + \frac{\rho_0 C V}{p_0 + B}\right). \tag{8}$$

Rearranging Eq. (8) for C^2 and V/C gives

$$C^2 \left(\frac{p_0 + B}{\rho_0}\right)^{-1} = \left[1 - \left(1 - \frac{V}{C}\right)^A\right]\left[\left(\frac{V}{C}\right)\left(1 - \frac{V}{C}\right)^A\right]^{-1}. \tag{9}$$

Note that $1 \geqslant [V/C = (\rho - \rho_0)/\rho] \gtreqless 0$, or ρ becomes negative.

As V/C or V approaches zero, the right-hand side of Eq. (9) approaches the constant A. Therefore, C approaches a constant, namely, acoustic velocity C_0 which is $[A(p_0 + B)/\rho_0]^{1/2}$.

On the other hand, as V/C approaches unity, the right-hand side of Eq. (9) approaches infinity. Hence, C and V both approach infinity.

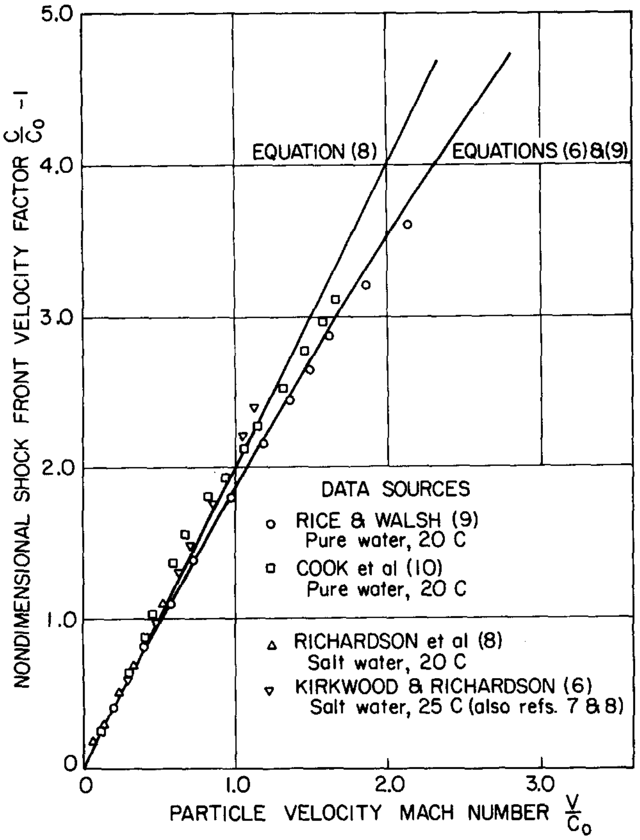

FIG. 1. Water shock-wave velocity vs Mach number.

Equation (9), therefore, contains the entire spectrum of relationships of C and V between the two limiting results of Eqs. (1) and (2).

Equation (9) shows that although shock-wave velocity C is an explicit function of V/C, it is an implicit function of V. According to Ruoff,[5] the shock-wave velocity can be expressed as a Maclaurin expansion in the form.

$$C = C_0 + aV + bV^2 + cV^3 + \cdots . \tag{10}$$

Here it is possible that $V/C_0 > 1$. The appropriate coefficients in Eq. (10) can then be determined from Eq. (9).

In a quadratic equation, it is found that $a = 1.925$ and $b = -0.083$ by a least-square-fit computer program, for impact Mach number up to 3.

$$C/C_0 = 1 + 1.925(V/C_0) - 0.083(V/C_0)^2. \tag{11}$$

Since this is only an approximation, it might be well to make it more convenient yet and still retain an acceptable accuracy, by choosing $a = 2.0$ and $b = -0.1$ within the stated ranges of application

$$C/C_0 = 1 + 2\,(V/C_0) - 0.1(V/C_0)^2. \tag{12}$$

III. RESULTS

Results calculated from Eq. (9) agree extremely well with the whole spectrum of experimental data[6-9] as shown in Fig. 1. The asymptotic value (acoustic velocity = C_0) calculated from Eq. (9) is about 4850 ft/sec. This is slightly lower than that recommended by Heymann,[1] i.e., 4900 ft/sec.

The relations

$$C/C_0 = 1 + kV/C_0, \quad k = 2.0 \tag{13}$$

and

$$C/C_0 = 1 + a(V/C_0) + b(V/C_0)^2, \quad a = 2.0, \; b = -0.1 \tag{14}$$

have been evaluated and compared with the numerical results of

$$c^2\left(\frac{p_0 + B}{\rho_0}\right)^{-1} = \left[1 - \left(1 - \frac{V}{C}\right)^A\right]\left[\left(\frac{V}{C}\right)\left(1 - \frac{V}{C}\right)^A\right]^{-1}. \tag{15}$$

Our calculations show that the percentage deviation of $C/C_0 = 1 + 2(V/C_0)$ increases as V/C_0 increases. It is about 5% at $V/C_0 = 1.0$, 11% at $V/C_0 = 2.0$, and 16% at $V/C_0 = 3.0$. On the other hand the percentage deviation of $C/C_0 = 1 + 2(V/C_0) - 0.1(V/C_0)^2$ is less than 3% over the whole range of V/C_0 up to 3.

IV. SUMMARY

A concise implicit relationship between shock-wave velocity and particle velocity change in water over the entire spectrum of impact Mach numbers, Eq. (9), is derived from fundamental equations (4)–(6) based on a plane-wave model. The shock-wave velocity can also be expressed as an explicit function of particle velocity change in the form of series expansion Eq. (10). Simplified equations with a limited range of application can be deduced for convenience. Equation (12), which includes a negative second-order term, is found to be more accurate than Eq. (13), as Heymann[10] had also pointed out.

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[1]F. J. Heymann, J. Basic Engr. D **90**, 400 (1968).
[2]P. G. Tait, Phys. Chem. **2**, 1 (1888).
[3]Y. H. Li, J. Geophys. Res. **32**, 10 (1967).
[4]R. H. Cole, *Underwater Explosions* (Dover, New York, 1965).
[5]A. L. Ruoff, J. Appl. Phys. **38**, 4976 (1967).
[6]J. G. Kirkwood and J. M. Richardson, OSRD Report No. 813, 1942 (unpublished).
[7]J. M. Richardson, A. B. Arons, and R. R. Halverson, J. Chem. Phys. **15**, 784 (1947).
[8]M. H. Rice and J. M. Walsh, J. Chem. Phys. **26**, 824 (1957).
[9]M. A. Cook, R. T. Keyes, and W. O. Ursenbach, J. Appl. Phys. **33**, 3413 (1962).
[10]F. J. Heymann, J. Basic Engr. D **91**, 134 (1969).