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NOTE ON THE ORIGIN OF WHITE DWARFS

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Summary

A theory of the origin of white dwarfs proposed by Eddington in 1939 is examined. It is shown that the difficulties encountered by Eddington in attempting to explain the synthesis of the heavy elements from hydrogen arise from his refusal to admit the Stoner-Anderson formula for the pressure of relativistically degenerate electrons. If Eddington's theory is correct then we arrive at the following conclusions :—

(i) The material of the Galaxy was originally composed of pure hydrogen.
(ii) The majority of white dwarfs are composed largely of hydrogen.
(iii) The Fermi form of the β-decay interaction applies to the proton-proton reaction.

On the other hand if Eddington's theory is wrong then the following conclusions are reached :—

(i) All white dwarfs arise from nova and supernova nuclei.
(ii) Either the supply of white dwarfs is greater by a factor of order 100 than is suggested by the accepted statistics on the frequency of novae and supernovae, or the density of white dwarfs in the neighbourhood of the Sun is much larger than the average density for the Galaxy as a whole.
(iii) The high hydrogen content of Sirius B is due to accretion of interstellar hydrogen occurring after the formation of this white dwarf.

1. The observational data concerning Sirius B indicate that this white dwarf contains about sixty per cent by mass in the form of hydrogen. This conclusion is independent of the controversy between Eddington and Chandrasekhar on the formula for the pressure of degenerate electrons at very high densities.* Indeed the result seems inescapable, unless either the Einstein red shift of about 20 km. per sec. or the mass of $0{\cdot}95_{\odot}$ is called into serious question.

The high hydrogen content of Sirius B has been discussed by Marshak † who finds difficulties in explaining the *present* structure of the star. In a recent paper Schatzman ‡ has attempted to overcome these difficulties by considering suitably chosen shell models. But the question raised by Marshak (concerning the hydrogen content of Sirius B) is of a minor character compared with the remarkable argument given by Eddington § in 1939. This argument shows that the outstanding problem is to find the sequence of stellar evolution that can lead to the formation of white dwarf stars with high hydrogen content. The difficulties raised by this question are of a far deeper significance than those that arise in attempting to explain the present properties of the star.

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* Thus Eddington in *M.N.*, **99**, 595, 1939 obtains 68 per cent in the form of **hydrogen,** while Chandrasekhar in *Introduction to the Study of Steller Structure* gives 52 per cent.

† R. E. Marshak, *Ap.J.*, **92**, 321, 1940.

‡ E. Schatzman, *Ann. D'Astrophys.*, **8**, 143, 1945.

§ *Loc. cit.*

Eddington's conclusions are easily understood by considering the evolution that the Sun would undergo if thermonuclear reactions were not taking place in the interior. Then the continued radiation at the surface would lead to a slow collapse. The time taken for the radius to be reduced to a tenth of its present value would be about 10^8 years, but further contraction would proceed more slowly owing to the increasing importance of electron degeneracy. The pressure due to the degenerate electrons would prevent the radius being reduced below about one-fortieth of the present value (this being comparable with the radius of Sirius B). When this stage was reached radiation at the surface would lead not to further collapse but to a cooling of the interior. Such a cooling process has the property of being comparatively rapid at first, but slows up as the inner regions cool down owing to a marked decrease in the rate of radiation at the surface.*

During the stages of collapse before electron degeneracy becomes important the central temperature must rise. Eddington estimates that the central temperature would attain 10^8 deg. C. This is probably a lower limit, and we may adopt 2×10^8 deg. C. as a more likely value.† If we now reinstate the thermonuclear reactions we see that it is impossible for the Sun (with its present hydrogen content) to contract to a white dwarf state, because the rate of energy generation by the carbon-nitrogen cycle when operating at a temperature of 2×10^8 deg. C. would be enormously greater than the luminosity. Thus the assumed situation in which the Sun is contracted to a tenth of its present radius cannot be permanent. A rapid expansion would evidently occur and the radius would soon return to its present value. This conclusion raises the important question:—How was Sirius B formed?

Eddington in attempting to answer this question finds that the initial carbon and nitrogen content of the material of Sirius B cannot be greater than ‡ about one part in 10^{16}. We can regard this result as being equivalent to saying that carbon and nitrogen must be wholly absent. Furthermore if carbon and nitrogen are wholly absent it is reasonable to suppose that other heavy elements are also absent. Eddington accepts the latter conclusion and regards Sirius B as being initially composed of pure hydrogen.

An additional problem of energy generation, not considered by Eddington, now arises. According to Bethe and Critchfield § the energy generation in pure hydrogen is about 2 ergs per g. per sec. when the density is 100 g. per cm.^{3} and the temperature is 2×10^7 deg. C. Now we have seen that a much higher central temperature of about 2×10^8 deg. C. must be attained if a body of solar mass is to contract down to a white dwarf state. At this temperature and the corresponding density of about 10^6 g. per cm.^{3} the Bethe-Critchfield theory gives an energy generation of about 5×10^6 ergs per g. per sec. This is vastly greater than the value required to maintain the luminosity. Thus if the Sun were contracted to a tenth of its present radius the luminosity would correspond to a mean energy loss of

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* It is estimated that about 10^9 years would be required for the central temperature to fall to 10^6 deg. C.

† The collapse would not necessarily occur through a series of homologous configurations. Consequently it is difficult to estimate the exact value to which the central temperature rises. It seems unlikely, however, that a reduction in radius by a factor of more than ten would lead only to the fivefold increase of temperature given by Eddington. The value here adopted corresponds to an increase by a factor of ten.

‡ This value is an upper limit since Eddington's central temperature is a lower limit.

§ Bethe and Critchfield, *Phys. Rev.*, **54**, 248, 1938.

only 6 ergs per g. per sec. The discrepancy is somewhat reduced by noting that the value of 5×10^6 ergs per g. per sec. applies to material at the centre of the star. It is known * that the mean energy production is only about two per cent of the central value, so that the mean energy production is 10^5 ergs per g. per sec. However, the difference between this value and the energy production required to maintain the luminosity is still very large. It may be concluded that the energy generation in pure hydrogen (on the Bethe-Critchfield theory) is too large to admit of the formation of white dwarfs from pure hydrogen by a factor of order 10^4.

This result is sufficient to destroy Eddington's argument if we accept the Bethe-Critchfield theory. But there is an alternative to this theory, which depends on accepting the form of the β-decay interaction given originally by Fermi, instead of the spin selection rule later proposed by Gamow and Teller. The Fermi interaction leads to a much lower energy production in pure hydrogen than the Gamow-Teller case. Some significance may possibly be attached to the fact that this reduction at a temperature of 2×10^8 deg. C. is by a factor of about 2×10^4, which is just of the magnitude required by Eddington's theory. This factor arises from a term (Mean thermal velocity of the protons/Velocity of light)^2, that appears in the Fermi but not in the Gamow-Teller form.†

In the past, astrophysicists, in their search for sources of energy generation, have tended to support the Gamow-Teller selection rules. But the issues involved in the present question are so important that it is desirable to emphasize that the decision between the two forms of β-decay interaction is by no means certain so far as nuclear physics is concerned. The interpretation of the often quoted He^6-Li^6 reaction is not without ambiguity, and even if it were the mechanism underlying the phenomenon of β-decay is so little understood that we cannot be sure of the validity of applying to the proton-proton reaction a result derived from a reaction involving many particle nuclei. Thus the decision between the two types of spin selection rule is still an open question, and it is permissible to adopt the Fermi form if the astrophysical data should require it. Accordingly Eddington's theory cannot necessarily be dismissed on the grounds of a too large energy production in pure hydrogen.

A minor objection to Eddington's argument may be mentioned at this stage. The observational data indicate a hydrogen content in Sirius B of about sixty per cent. It would evidently be more convenient if the hydrogen content were near a hundred per cent. The change in the Einstein red shift required to give this increase is from 20 km. per sec. to about 15 km. per sec. This change is small and may well be within the errors of observation. All re-examinations of the data seem to have been made so far with a view to reducing the hydrogen content. It would be useful to know what changes might reasonably be made for the opposite purpose of increasing the hydrogen content.

2. It is natural to consider the alternatives available to Eddington's theory. Two possibilities may be suggested:—

(1) The observational data for Sirius B are in error, and the correct values would lead to zero hydrogen content;

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* F. Hoyle and R. A. Lyttleton, *M.N.*, **102,** 177, 1942.

† For further details regarding the properties of these β-interactions see Bethe and Critchfield, *loc. cit.*

(2) At the time of condensation the hydrogen content of Sirius B was zero; and the present hydrogen content of Sirius B has risen from the subsequent accretion of interstellar hydrogen.

Of these possibilities (1) seems very unplausible, but (2) evidently demands serious consideration. The attempt to explain all hydrogen observed in white dwarfs as being due to accretion taking place *after* the central temperatures have fallen to values of order 10^6 deg. C. leads to an interesting line of argument. This will now be discussed.

If we regard the hydrogen content of all white dwarfs as being zero at the time of their condensation then we must explain the sequence of stellar evolution that leads to the exhaustion of hydrogen in these stars. Two suggestions may be rejected at the outset. Firstly, it is quite unplausible to suppose that an appreciable number of stars without hydrogen can condense directly from the interstellar material, for as Dunham [*] has shown, the hydrogen abundance in the interstellar material is about 10,000 times greater than the abundance of the commonest metals. Secondly, the generally accepted time-scale of about 10^{10} years is far too short [†] for the conversion of hydrogen to helium by thermonuclear reactions to exhaust the hydrogen supply in stars with masses less than the solar mass.

It appears therefore that the only stars in which the hydrogen supply can become exhausted are stars more massive than the Sun. The mechanism whereby such stars produce white dwarfs is fairly well understood. Thus in a recent paper [‡] it was seen that stars without hydrogen enter a collapsing phase and if the mass appreciably exceeds $\odot$ the collapse continues until rotational instability arises. Material is then thrown off to infinity until the mass in the remaining nucleus becomes less than Chandrasekhar's limit [§], which is close to $1{\cdot}44_{\odot}$. This process has been associated with the nova and supernova phenomena. [||]

The process outlined in the previous paragraph undoubtedly provides a steady supply of white dwarfs. But the important question is whether the supply is sufficiently rapid. If the observed density of white dwarfs within 10 parsecs from the Sun is regarded as typical for the Galaxy as a whole, then the total number of white dwarfs in the Galaxy is of the order of 10^9. The accepted rate of one supernova per galaxy per 500 years gives 2×10^7 supernovae per galaxy in 10^{10} years, which is too small to explain the estimated number of white dwarfs by a factor of order 100. This discrepancy could be reduced if it is assumed that each supernova is disrupted into an appreciable number of fragments.[¶] However, this assumption is open to very serious question for it has been shown [**] that the rate of contraction in the most rapidly collapsing star is only about one per cent of the average velocity of sound in the stellar material. Accordingly it appears most unlikely that supernovae can break into a number of separate pieces. This theoretical conclusion is in agreement with observation which indicates that material is showered off very

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[*] T. Dunham, Jr., *Proc. Amer. phil. Soc.*, **81**, 277, 1939.

[†] The writer withdraws the method used for calculating the time-scale in *Proc. Camb. phil. Soc.*, **35**, 603, 1939. This method, which was based on the time required for the exhaustion of hydrogen in white dwarfs, leads to unacceptable results when applied to white dwarfs with masses appreciably less than $\odot$.

[‡] F. Hoyle, "Note on Equilibrium Configurations for Rotating White Dwarfs", *M.N.*, **107**, 231, 1947.

[§] Chandrasekhar's limit is $5{\cdot}75\odot/\mu_e^2$, and $\mu_e \simeq 2$ when there is no hydrogen.

[||] F. Hoyle, *M.N.*, **106**, 343, 1946.

[¶] G. Gamow, *Phys. Rev.*, **59**, 617, 1939.

[**] F. Hoyle, *loc. cit.*

steadily in a supernova outburst. In particular, Minkowski's estimate of 15$\odot$ for the mass of the *diffuse* material in the Crab nebula shows that the material thrown off by the supernova of A.D. 1054 did not consist of a series of dense "bolts" with masses of order $\odot$.

At first sight it might seem that the supply of white dwarfs from novae appreciably exceeds the supply from supernovae on account of the greater frequency of novae. But this naïve point of view ignores the result that the mass of material thrown off in a nova outburst is only of order $10^{-5}\odot$. Thus a collapsing star with mass appreciably greater than $\odot$ requires at least 10^4 outbursts of a nova type in order to reach permanent stability. This is in contrast with the supernova case where there is only one outburst. On the basis that the supply of white dwarfs is the same from novae as from supernovae, a rate of one supernova per galaxy per 500 years corresponds to about 20 novae per galaxy per year. This rate of novae is in accordance with observation. Thus it seems likely that the supply of white dwarfs from novae is of the same order as the supply from supernovae.

The above discussion suggests that there is a discrepancy between the observed number of white dwarfs and the number supplied by stars with masses appreciably greater than $\odot$. If this is the case, then neither of the possibilities mentioned at the beginning of the present section is sufficient to dispose of Eddington's theory. The alternatives presented are:—

(*a*) *Eddington's theory*:
  (i) The material of the Galaxy was originally composed of pure hydrogen.
  (ii) The majority of white dwarfs are composed largely of hydrogen.
  (iii) The Fermi form of the β-decay interaction applies to the proton-proton reaction.

(*b*) If Eddington's theory is incorrect then
  (i) All white dwarfs arise from nova and supernova nuclei.
  (ii) Either the supply of white dwarfs is greater by a factor of order 100 than is suggested by the accepted statistics on the frequency of novae and supernovae, or the density of white dwarfs in the neighbourhood of the Sun is much larger than the average density for the Galaxy as a whole.
  (iii) The high hydrogen content of Sirius B is due to accretion of interstellar hydrogen occurring after the formation of this white dwarf.

The decision between these two lines of evolution is evidently one of the most important problems of cosmogony. In particular (i) of (*a*) implies that the heavy elements are formed from hydrogen. The acceptance of this view has hitherto been a matter of aesthetic preference, but if Eddington's theory is correct we are compelled to accept this result.

3. In the present section we proceed on the assumption that Eddington's theory is correct. We have then to explain the formation of the heavy elements from hydrogen by processes taking place within the Galaxy. Eddington finds * a number of serious difficulties in attempting to satisfy this requirement, but it can be shown that these difficulties arise from his refusal to admit the Stoner-Anderson formula for the pressure of relativistically degenerate electrons. On the basis of this formula Chandrasekhar has shown that no *spherically symmetric* degenerate equilibrium configuration is available when the mass of a collapsing star exceeds

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* Eddington, *loc. cit.*

$5{\cdot}75\odot/\mu_e^2$. For stars composed of hydrogen $\mu_e=1$ and the limiting mass is $5{\cdot}75\odot$. When the mass exceeds $5{\cdot}75\odot$ the star must contract until either:

(1) The energy generation in pure hydrogen becomes sufficiently large to compensate for the radiation at the surface. A sufficient degree of contraction must lead to this condition being satisfied even if the Fermi form of the β-decay interaction is adopted;

or

(2) The star becomes rotationally unstable.

If the angular momentum of the star is small enough (1) must occur before (2).

It is important to notice the difference between stars with masses of order $\odot$ and stars with masses exceeding $5{\cdot}75\odot$. For small masses the contraction is limited by the occurrence of a degenerate equilibrium configuration, and if we accept the Fermi form of the β-interaction the energy generation is less than the luminosity when this stage is reached. Thus such stars form white dwarfs in the manner discussed by Eddington. On the other hand when the mass exceeds $5{\cdot}75\odot$ there is no such limit to the contraction and even on the Fermi form of the β-interaction a stage must be reached, provided the angular momentum of the star is sufficiently small, where the energy generation balances the luminosity at the surface. When this balance is reached the rate of conversion of hydrogen to helium is very rapid on account of the large luminosity that must then prevail. Indeed if no further material is added by accretion the hydrogen supply will be exhausted in a time less than 10^9 years. As soon as an appreciable quantity of helium is formed, carbon will be synthesized in triple collisions between α-particles. The star expands as the amount of carbon increases. This effect has been described by Eddington.

The above discussion shows how the hydrogen can become entirely converted into helium when the mass exceeds $5{\cdot}75\odot$. The star then follows a line of evolution that has already been described in detail.* This evolution provides for the synthesis of the heavy elements, and the eventual onset of rotational instability leads to the heavy elements being distributed in interstellar space. The crucial difference between this theory and the discussion given by Eddington is that no attempt is here made to synthesize the elements in material containing hydrogen. Thus by synthesizing the heavy elements *after* the hydrogen has been entirely converted to helium the difficulties encountered by Eddington are avoided.

The chronological history of the Galaxy according to this theory may be briefly outlined:—

(i) The Galaxy consists initially of pure hydrogen and consequently the first stars are composed of pure hydrogen.

(ii) If the mass of such a star is appreciably less than Chandrasekhar's limit a white dwarf is formed.

(iii) If the mass is appreciably greater than Chandrasekhar's limit, and the angular momentum of the star is small enough, contraction continues until the energy generation balances the luminosity. Further contraction is then arrested until the hydrogen is exhausted. When the contraction is resumed the heavy elements are synthesized and are finally distributed in interstellar space by rotational instability. The time

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* F. Hoyle, *M.N.*, **106**, 343, 1946.

required for the evolution of these stars is not appreciably greater than 10^9 years.

(iv) About 10^9 years after the formation of the first massive stars the interstellar material contains appreciable quantities of heavy elements. The first trace of carbon and nitrogen is sufficient to prevent further white dwarfs being formed by direct condensation. Thus white dwarfs are formed in large numbers only during the earliest phase in the history of the Galaxy. Subsequent stellar condensations follow the main sequence. As little as one part in 10^{10} of carbon and nitrogen in the stellar material is sufficient to produce this change.

(v) The white dwarfs remain stable once their central temperatures have fallen to values of order 10^6 deg. C., even if they subsequently acquire carbon and nitrogen by accretion of interstellar material.

4. In this section it will be assumed that Eddington's theory is incorrect, but the interstellar material will still be regarded as initially composed of pure hydrogen. The difference between the present discussion and that of Section 3 is that we here examine the consequences of accepting the Gamow-Teller form instead of the Fermi form of β-interaction. In this case it is easy to show that the first stars will contract until the luminosity is balanced by energy generation in pure hydrogen. For stars with masses less than $\odot$, this stage will be reached for radii of the same order as the values occurring in the normal main sequence. But as the mass increases the equilibrium radii become appreciably less than the main-sequence values. The evolution of stars with masses greater than $5{\cdot}75\odot$ is essentially the same as in Section 3. Thus the heavy elements are synthesized and distributed in interstellar space in a similar manner. Once the interstellar carbon and nitrogen becomes appreciable subsequent stellar condensations are of the main sequence type.

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